

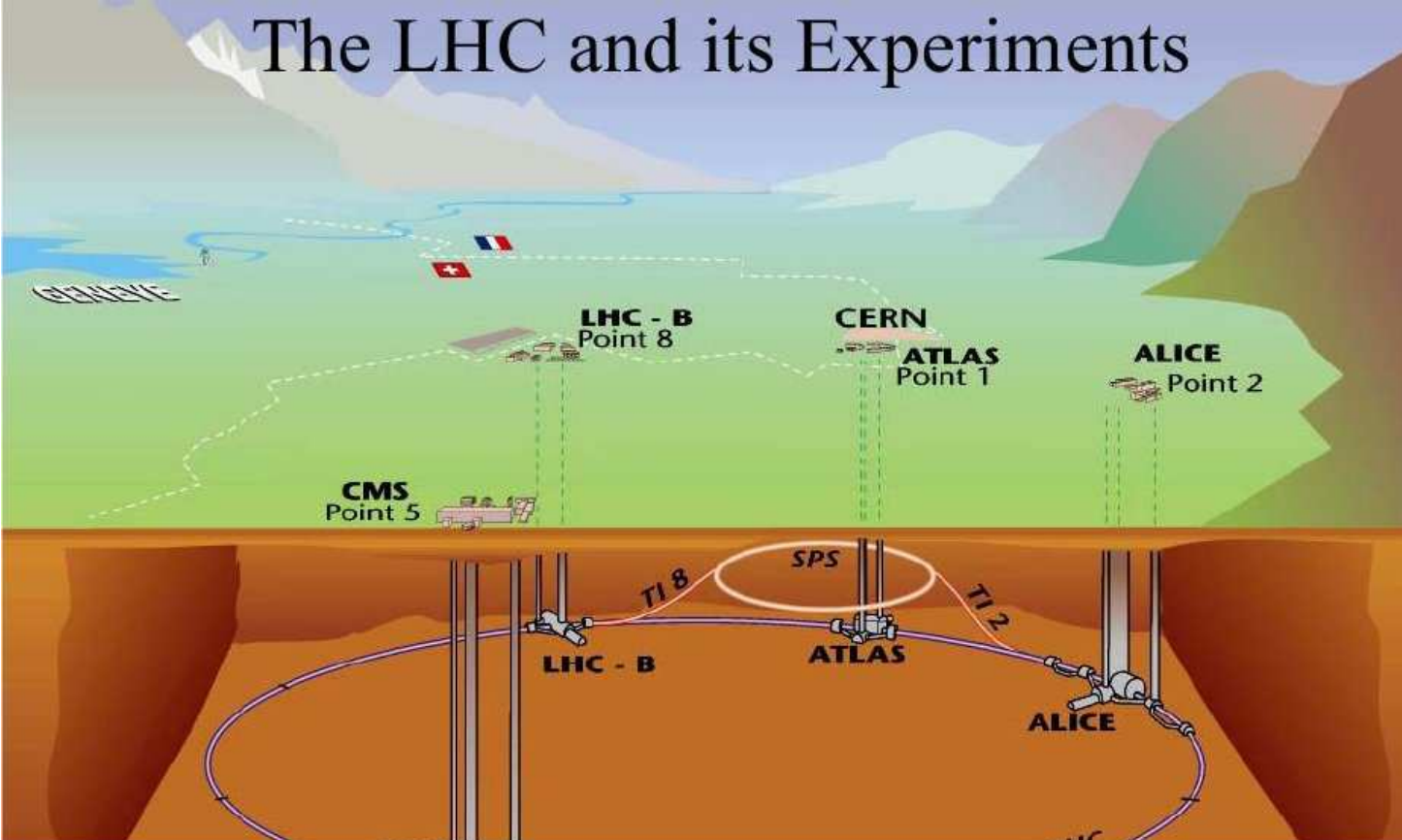


INGREDIENTS FOR ACCURATE COLLIDER PHYSICS

Gavin Salam, CERN

PSI Summer School Exothiggs,
Zuoz, August 2016

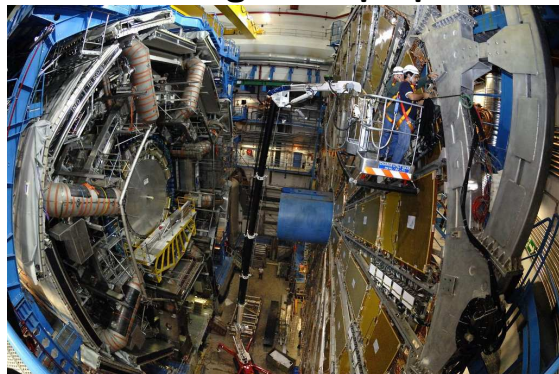
The LHC and its Experiments



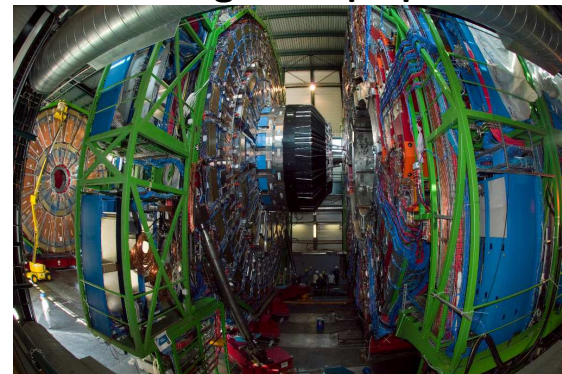
- ~16.5 mi circumference, ~300 feet underground
- 1232 superconducting twin-bore Dipoles (49 ft, 35 t each)
- Dipole Field Strength 8.4 T (13 kA current), Operating Temperature 1.9K
- Beam intensity 0.5 A ($2.2 \cdot 10^{-6}$ loss causes quench), 362 MJ stored energy



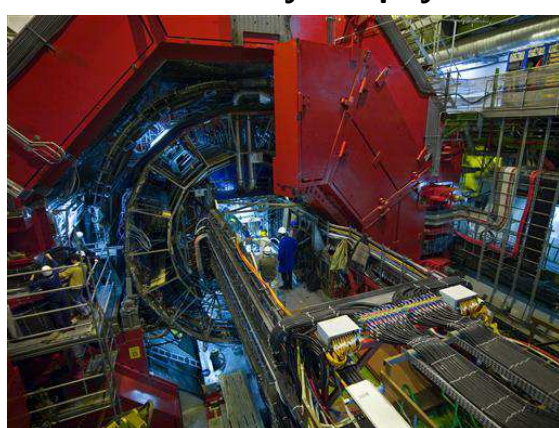
ATLAS: general purpose



CMS: general purpose



ALICE: heavy-ion physics



LHCb: B-physics



+ TOTEM, LHCf

LHC – TWO ROLES – A DISCOVERY MACHINE AND A PRECISION MACHINE

Today

- 20 fb⁻¹ at 8 TeV
- 13 fb⁻¹ at 13 TeV

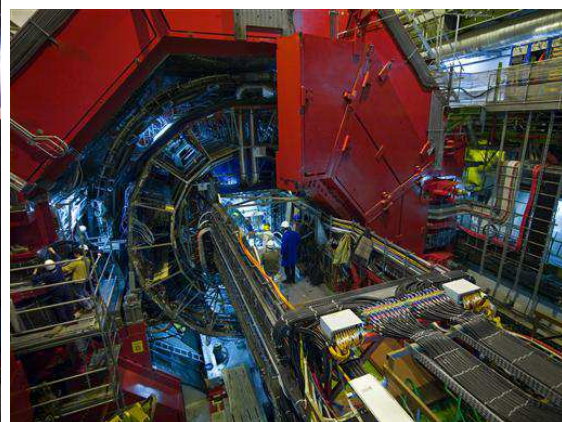
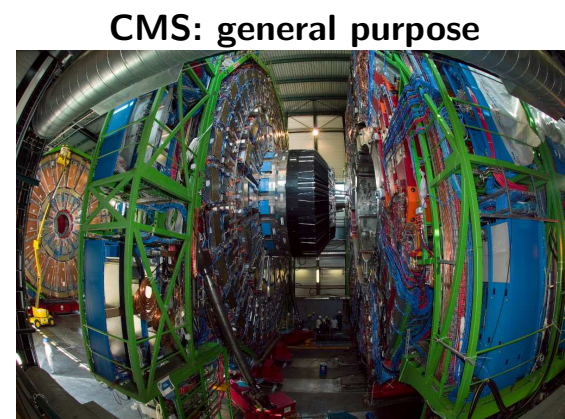
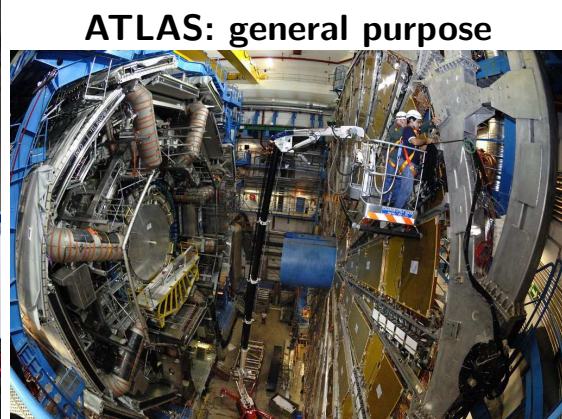
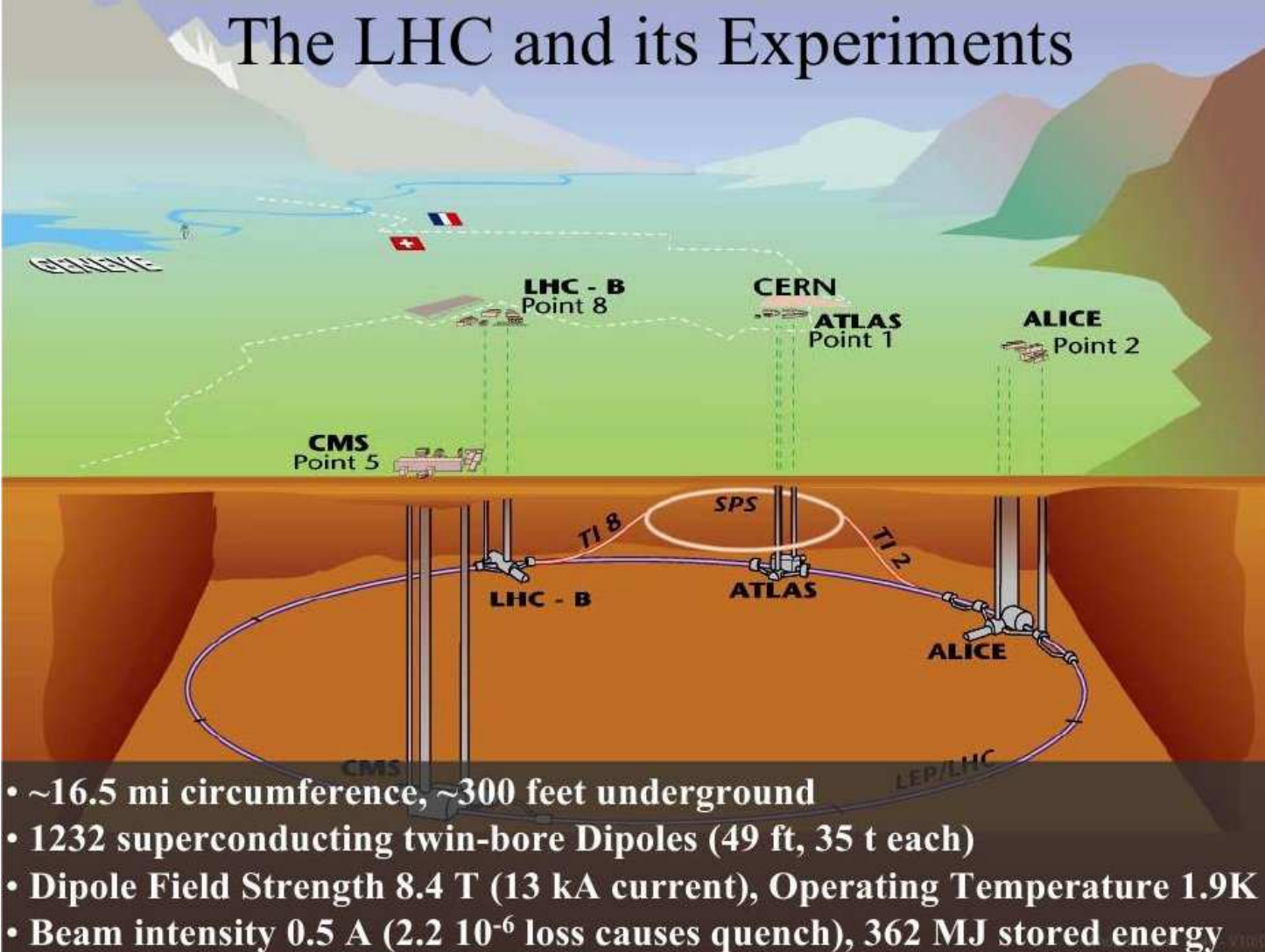
Future

- 2018: 100 fb⁻¹ @ 13 TeV
- 2023: 300 fb⁻¹ @ 1? TeV
- 2035: 3000 fb⁻¹ @ 14 TeV

1 fb⁻¹ = 10^{14} collisions

Increase in luminosity brings
discovery reach and precision

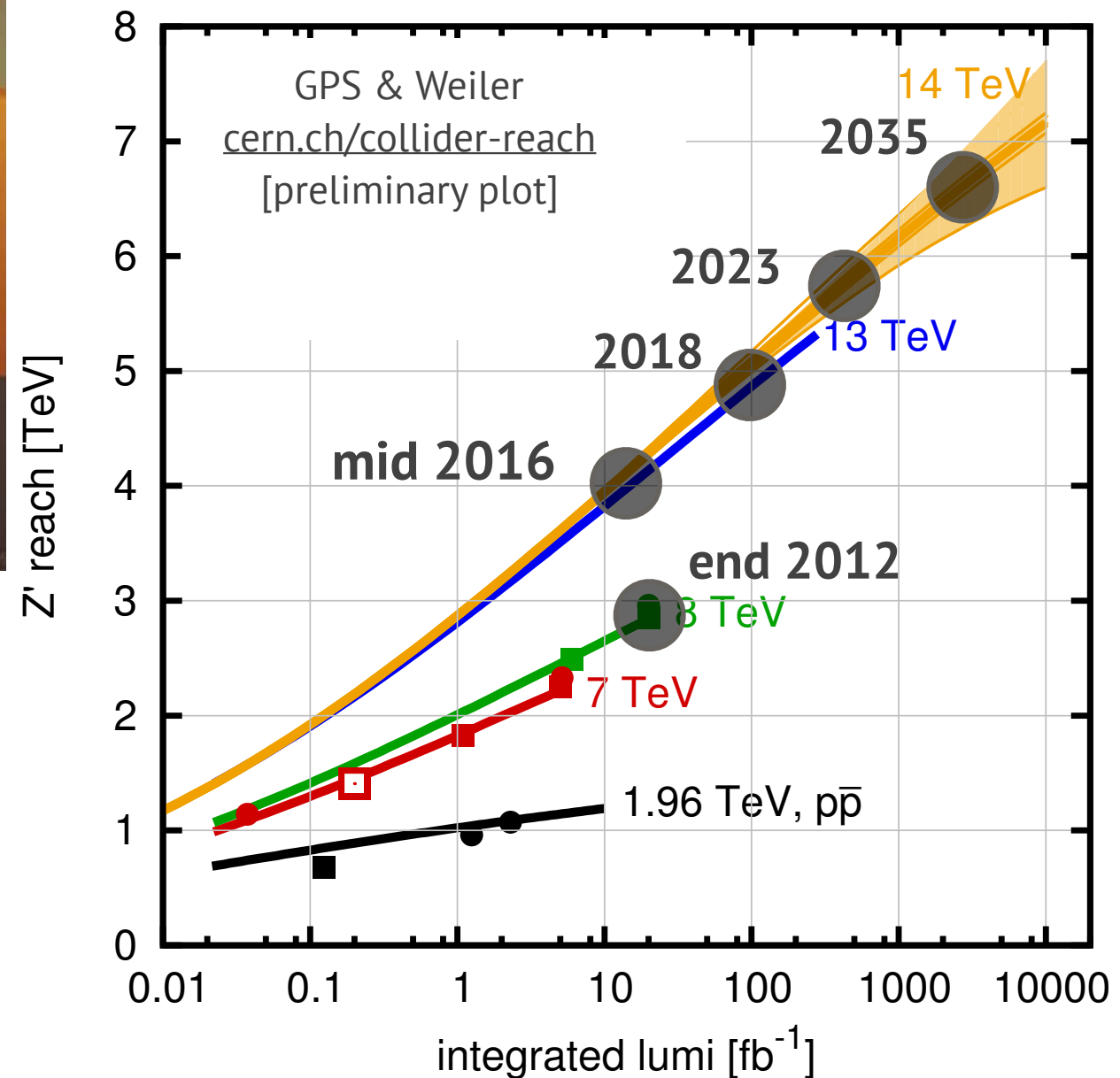
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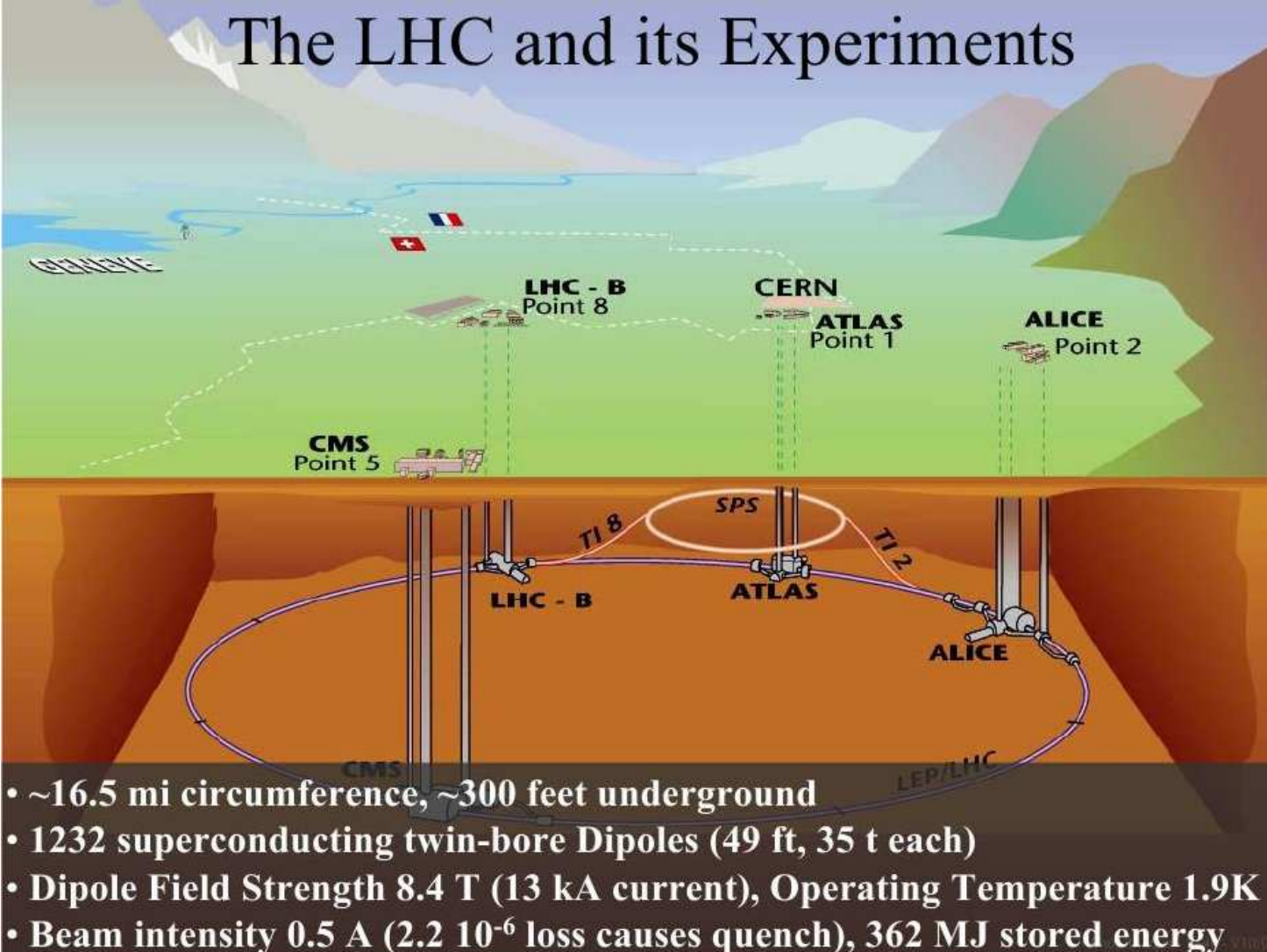
LHC – TWO ROLES – A DISCOVERY MACHINE AND A PRECISION MACHINE

Z' exclusion reach v. lumi

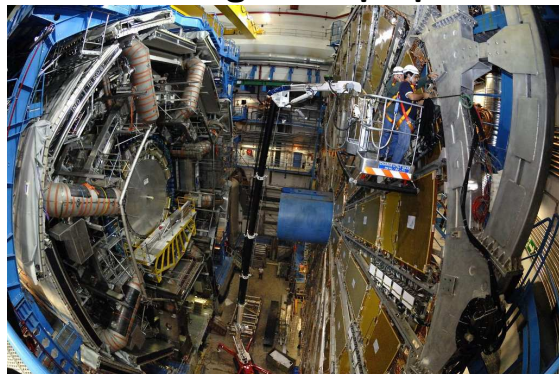


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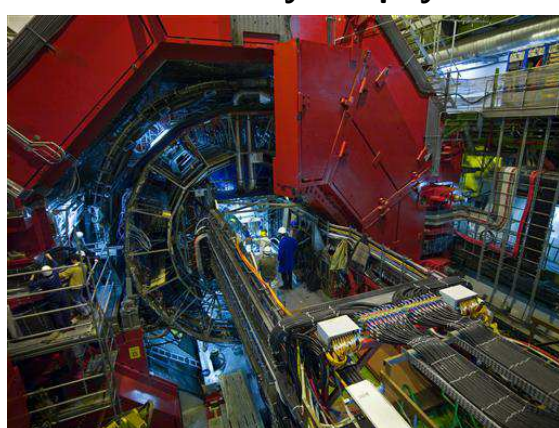
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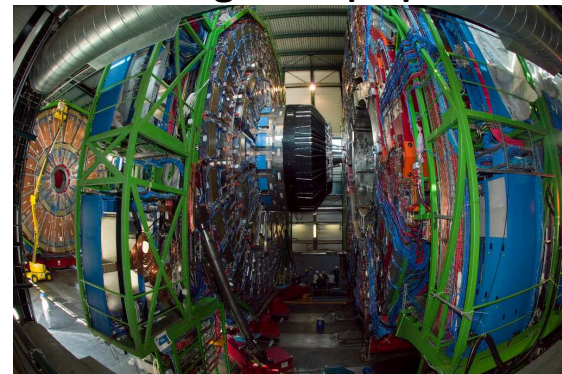
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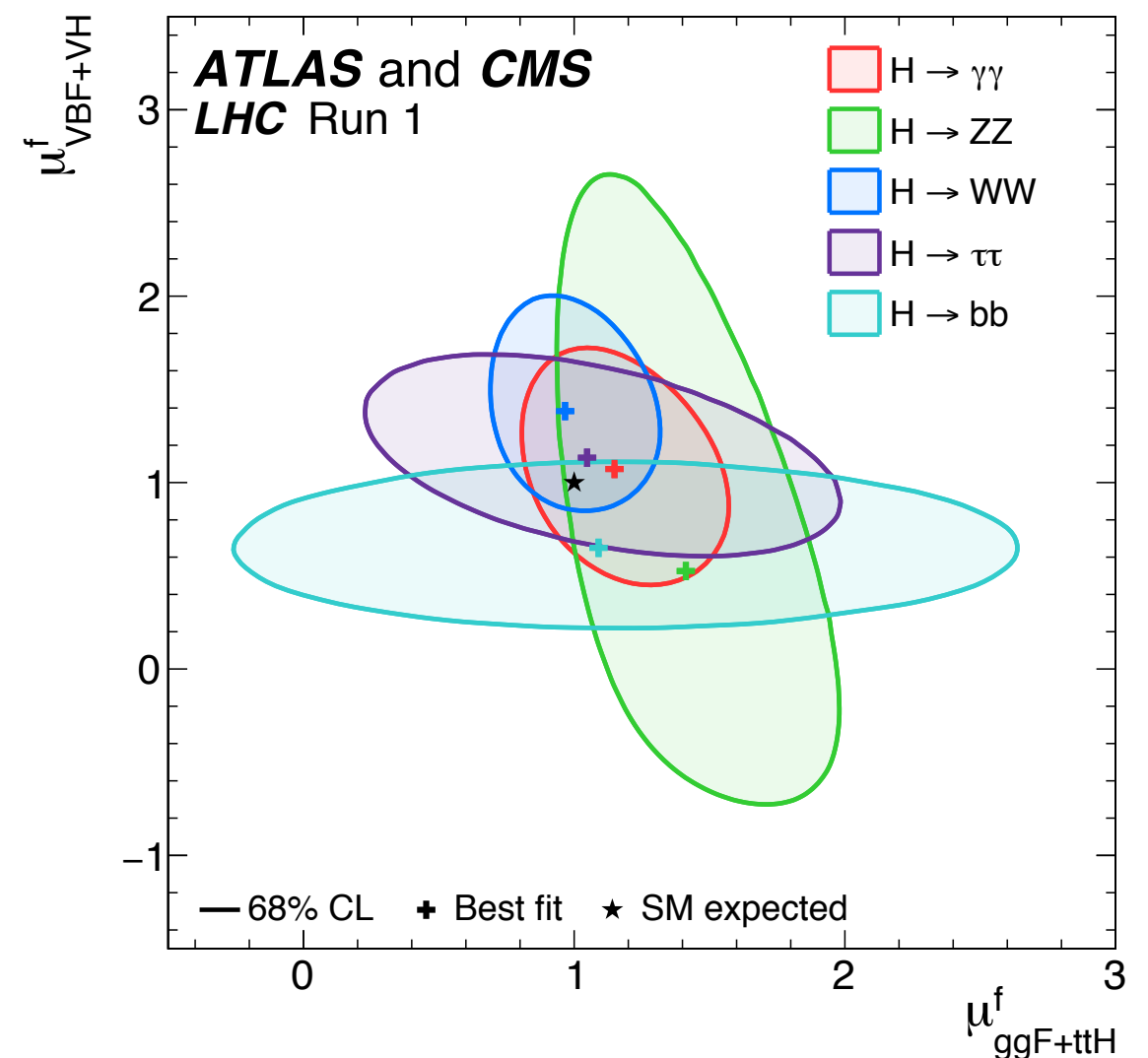
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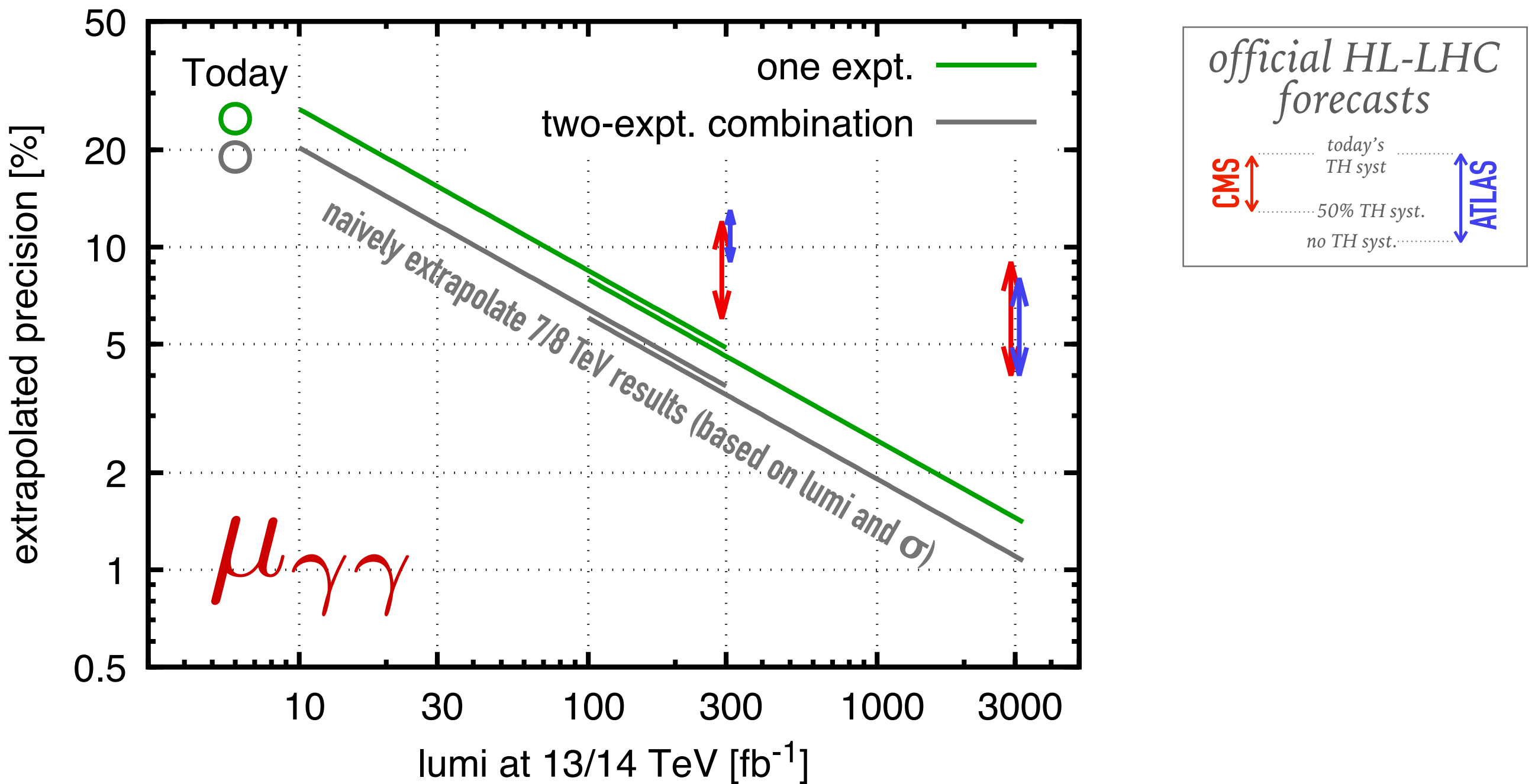
LHC – TWO ROLES – A DISCOVERY MACHINE AND A PRECISION MACHINE

Higgs couplings



Increase in luminosity brings discovery reach and precision

LONG-TERM HIGGS PRECISION?



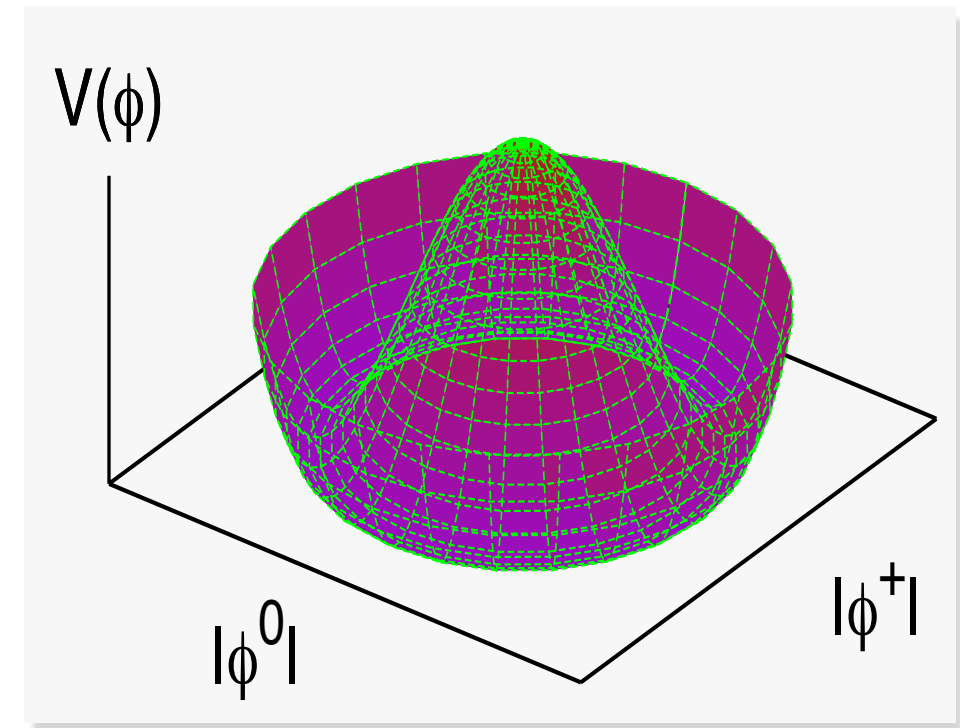
Naive extrapolation suggests LHC has long-term potential to do Higgs physics at **1% accuracy**

THE HIGGS SECTOR

The theory is old (1960s-70s).

But the particle and it's theory are unlike anything we've seen in nature.

- A fundamental scalar ϕ , i.e. spin 0 (all other particles are spin 1 or 1/2)
- A potential $V(\phi) \sim -\mu^2 (\phi\phi^\dagger) + \lambda(\phi\phi^\dagger)^2$, which until now was limited to being theorists' "toy model" (ϕ^4)
- "Yukawa" interactions responsible for fermion masses, $y_i \phi \bar{\psi} \psi$, with couplings (y_i) spanning 5 orders of magnitude

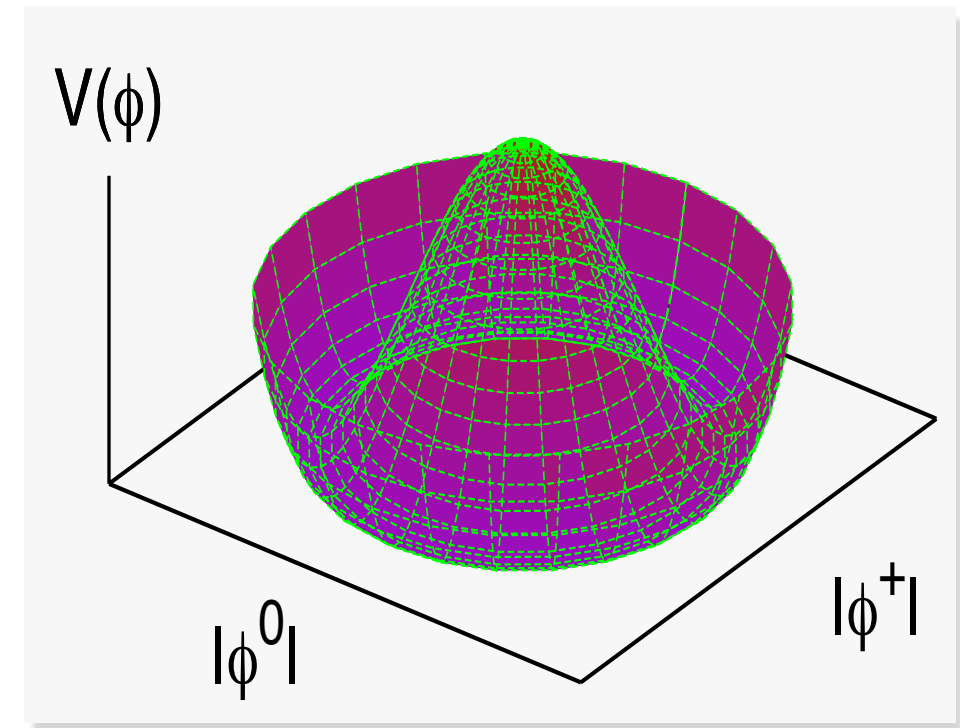


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Higgs sector needs stress-testing

Is Higgs fundamental or composite?

If fundamental, is it "minimal"?

Is it really ϕ^4 ?

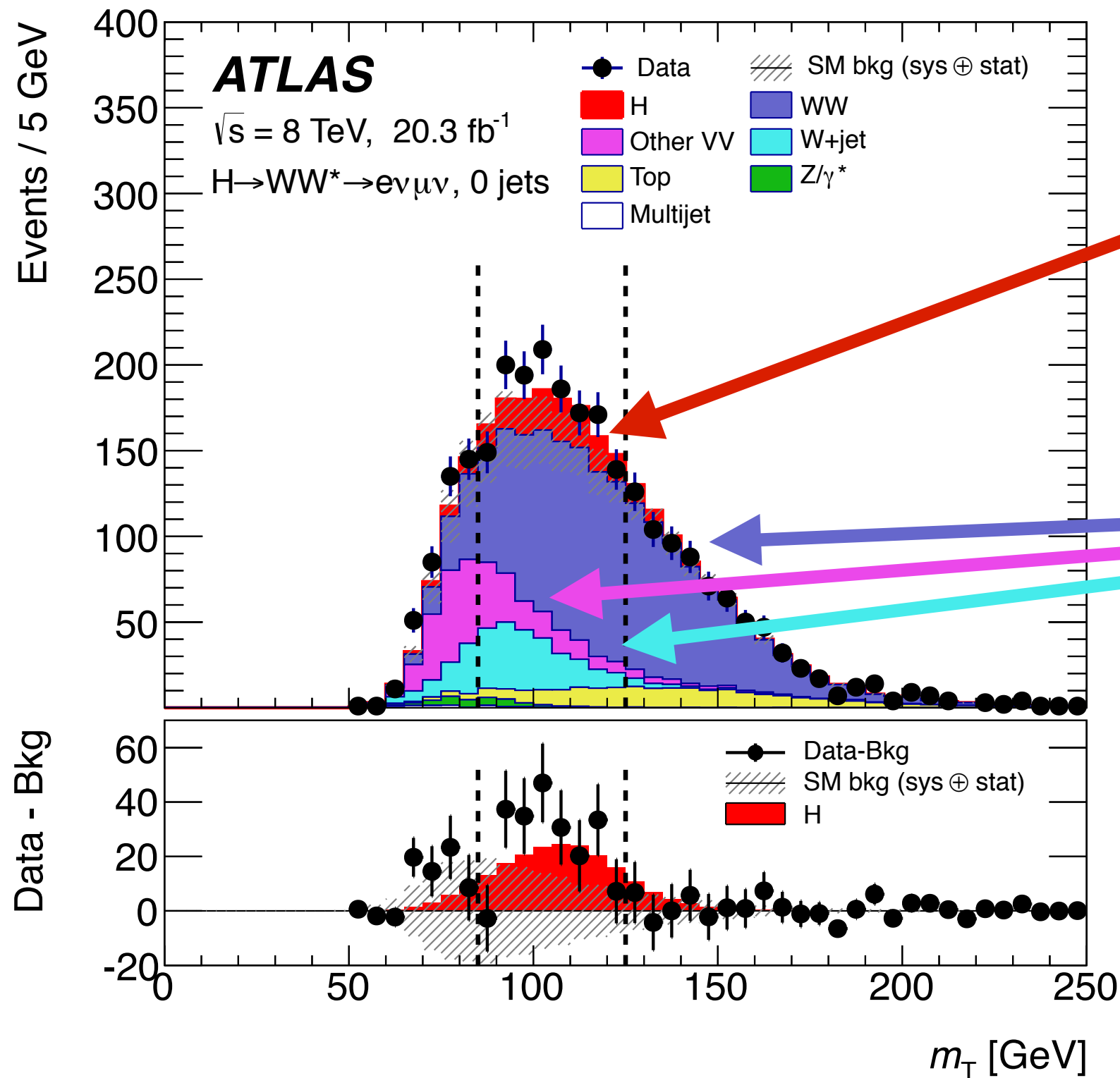
Are Yukawa couplings responsible for all fermion masses?

3 Signal and background models

The ggF and VBF production modes for $H \rightarrow WW^*$ are modelled at next-to-leading order (NLO) in the strong coupling α_s with the PowHEG MC generator [22–25], interfaced with PyTHIA8 [26] (version 8.165) for the parton shower, hadronisation, and underlying event. The CT10 [27] PDF set is used and the parameters of the PyTHIA8 generator controlling the modelling of the parton shower and the underlying event are those corresponding to the AU2 set [28]. The Higgs boson mass set in the generation is 125.0 GeV, which is close to the measured value. The PowHEG ggF model takes into account finite quark masses and a running-width Breit–Wigner distribution that includes electroweak corrections at NLO [29]. To improve the modelling of the Higgs boson p_T distribution, a reweighting scheme is applied to reproduce the prediction of the next-to-next-to-leading-order (NNLO) and next-to-next-to-leading-logarithm (NNLL) dynamic-scale calculation given by the HRES 2.1 program [30]. Events with ≥ 2 jets are further reweighted to reproduce the p_T^H spectrum predicted by the NLO PowHEG simulation of Higgs boson production in association with two jets ($H + 2$ jets) [31]. Interference with continuum WW production [32, 33] has a negligible impact on this analysis due to the transverse-mass selection criteria described in Section 4 and is not included in the signal model.

Jets are reconstructed from topological clusters of calorimeter cells [50–52] using the anti- k_t algorithm with a radius parameter of $R = 0.4$ [53]. Jet energies are corrected for the effects of calorimeter non-

ATLAS $H \rightarrow WW^*$ ANALYSIS [1604.02997]



(a) $N_{\text{jet}} = 0$

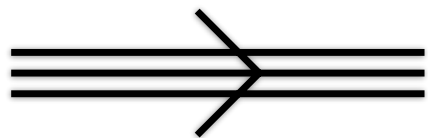
That whole paragraph was just for the red part of this distribution (the Higgs signal).

Complexity of modelling each of the backgrounds is comparable

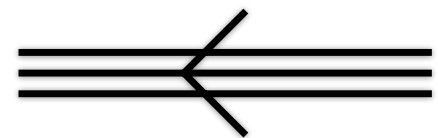
AIMS OF THESE LECTURES

- Give you basic understanding of the “**jargon**” of theoretical collider prediction methods and inputs
- Give you insight into the **power & limitations of different techniques** for making collider predictions

A proton-proton collision: INITIAL STATE

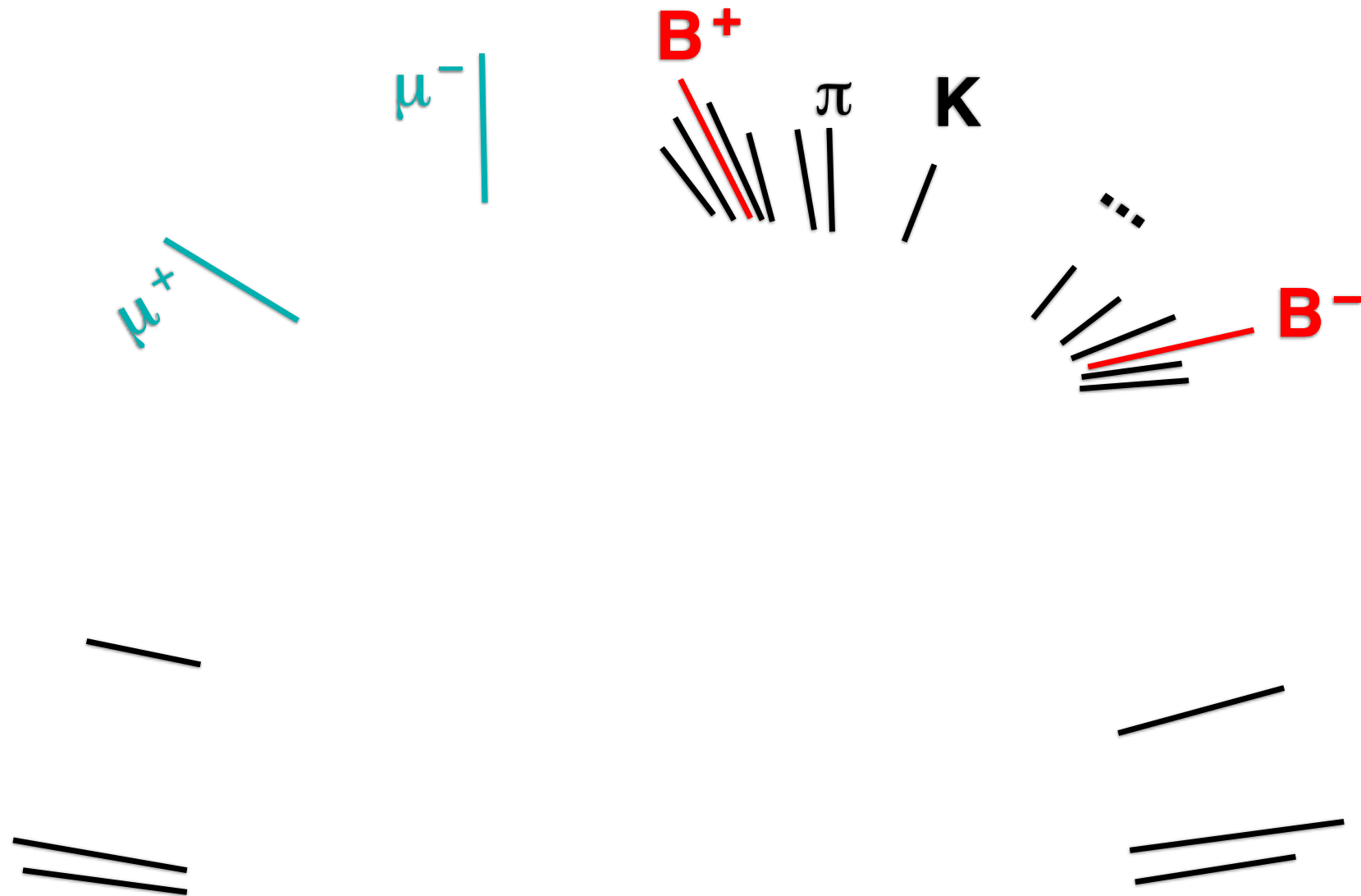


proton



proton

A proton-proton collision: FINAL STATE



(actual final-state multiplicity \sim several hundred hadrons)

IT'S MOSTLY QUANTUM CHROMODYNAMICS (QCD)

Quarks — 3 colours: $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Quark part of Lagrangian:

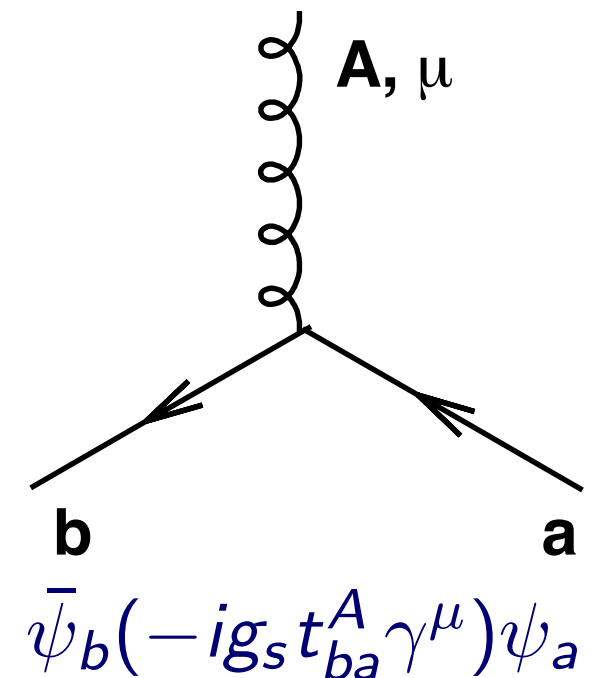
$$\mathcal{L}_q = \bar{\psi}_a (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t_{ab}^C \mathcal{A}_\mu^C - m) \psi_b$$

$SU(3)$ local gauge symmetry \leftrightarrow 8 ($= 3^2 - 1$) generators $t_{ab}^1 \dots t_{ab}^8$ corresponding to 8 gluons $\mathcal{A}_\mu^1 \dots \mathcal{A}_\mu^8$.

A representation is: $t^A = \frac{1}{2}\lambda^A$,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \end{pmatrix},$$

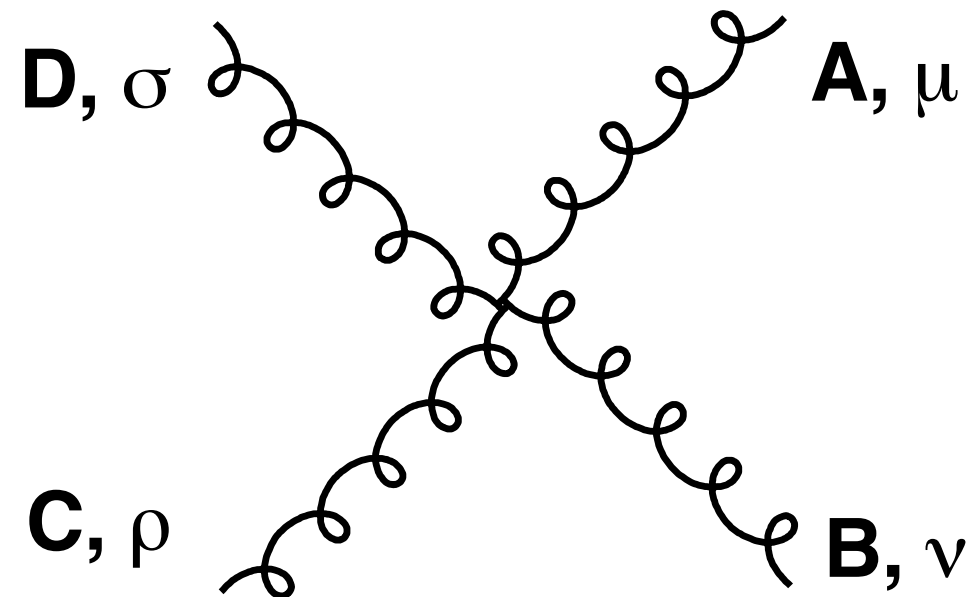
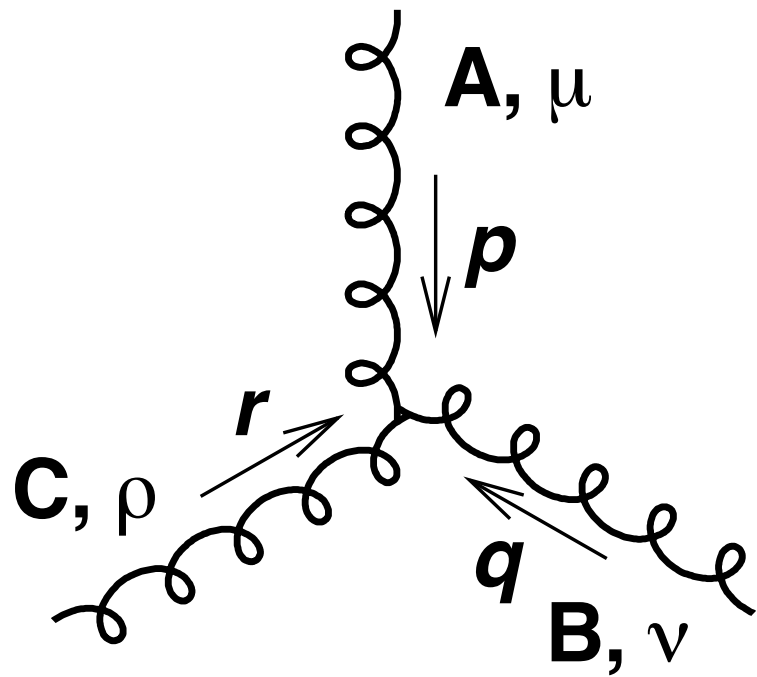


IT'S MOSTLY QUANTUM CHROMODYNAMICS (QCD)

Field tensor: $F_{\mu\nu}^A = \partial_\mu \mathcal{A}_\nu^A - \partial_\nu \mathcal{A}_\mu^A - g_s f_{ABC} \mathcal{A}_\mu^B \mathcal{A}_\nu^C$ $[t^A, t^B] = if_{ABC} t^C$

f_{ABC} are structure constants of $SU(3)$ (antisymmetric in all indices — $SU(2)$ equivalent was ϵ^{ABC}). Needed for gauge invariance of gluon part of Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4} F_A^{\mu\nu} F^A_{\mu\nu}$$



IT'S MOSTLY QUANTUM CHROMODYNAMICS (QCD)

The only complete solution uses **lattice QCD**

- put all quark & gluon fields on a 4d lattice
(NB: imaginary time)
- Figure out most likely configurations
(Monte Carlo sampling)

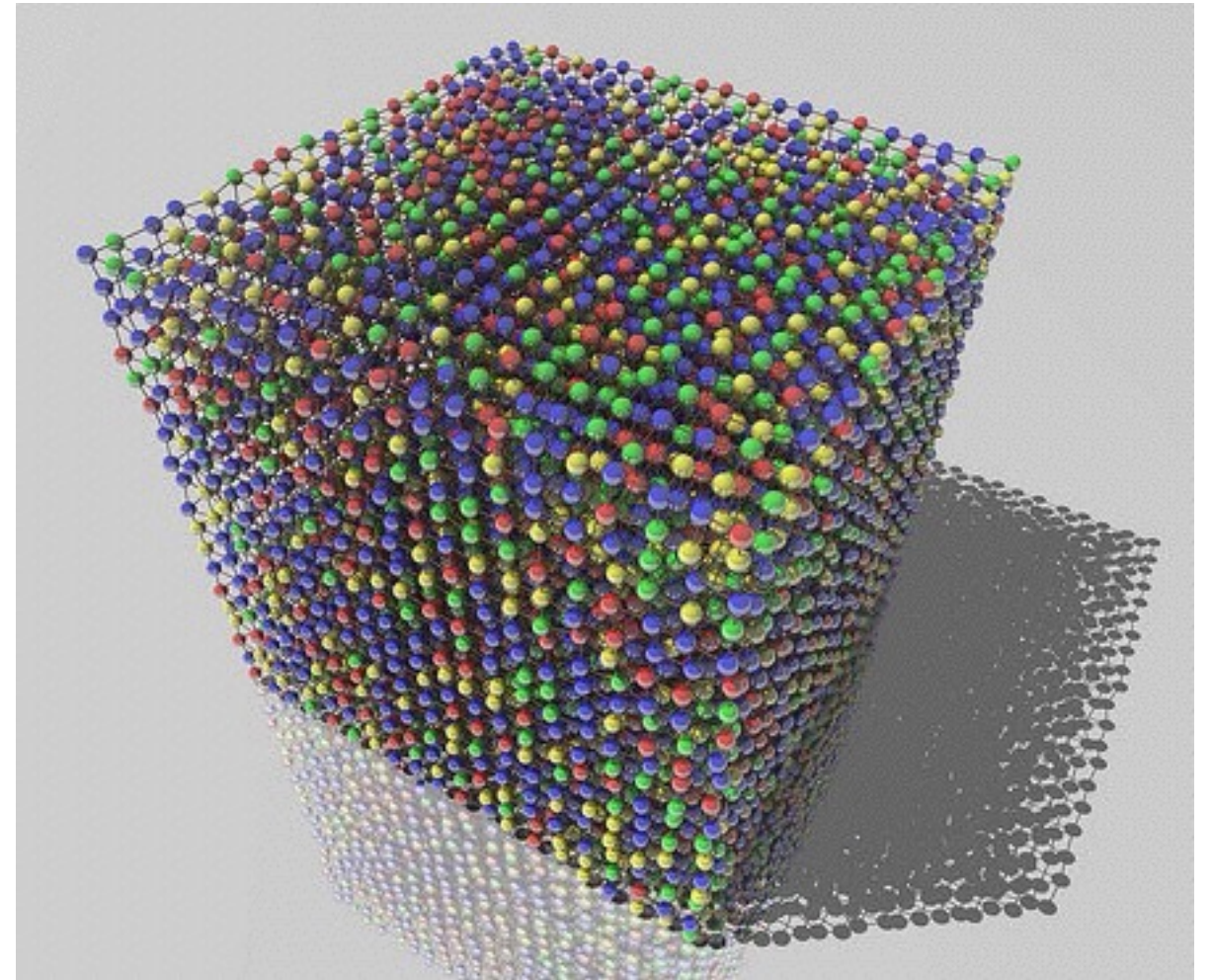


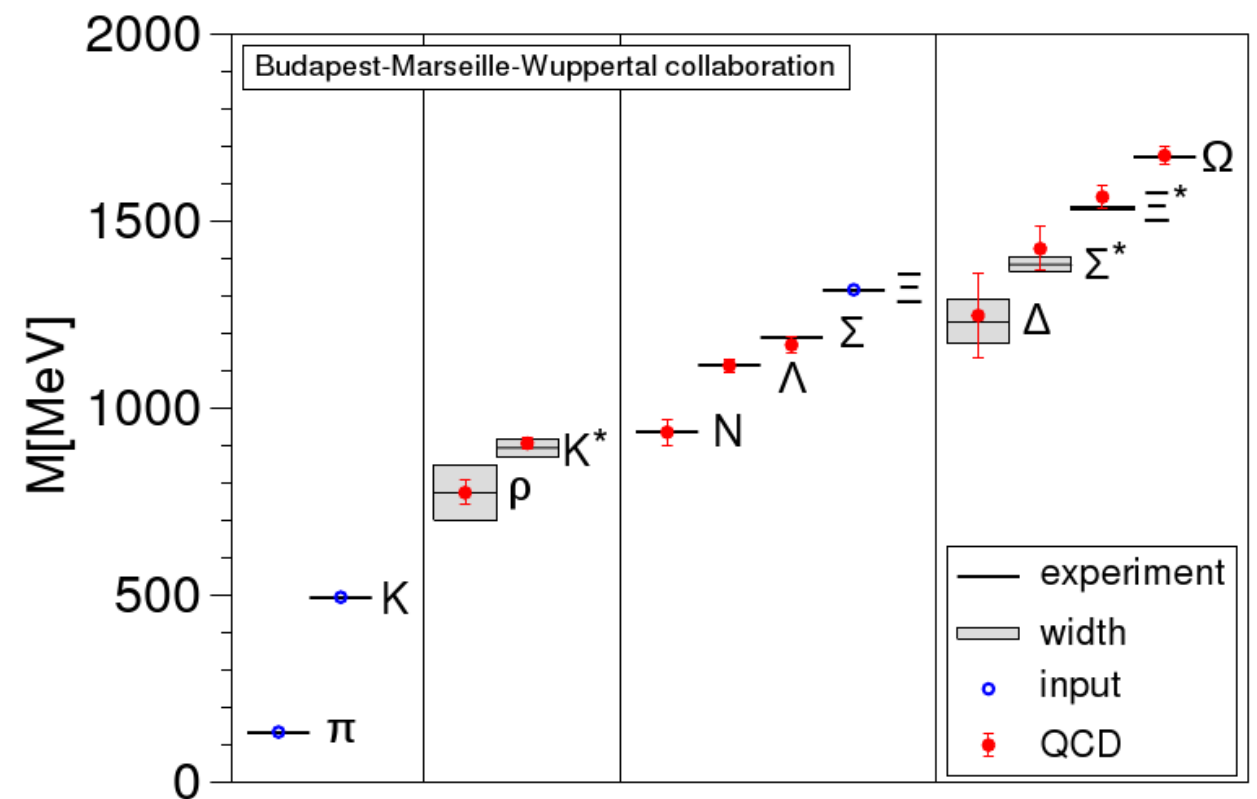
image credit [fdecomite](#) [[flickr](#)]

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hadron spectrum from lattice QCD



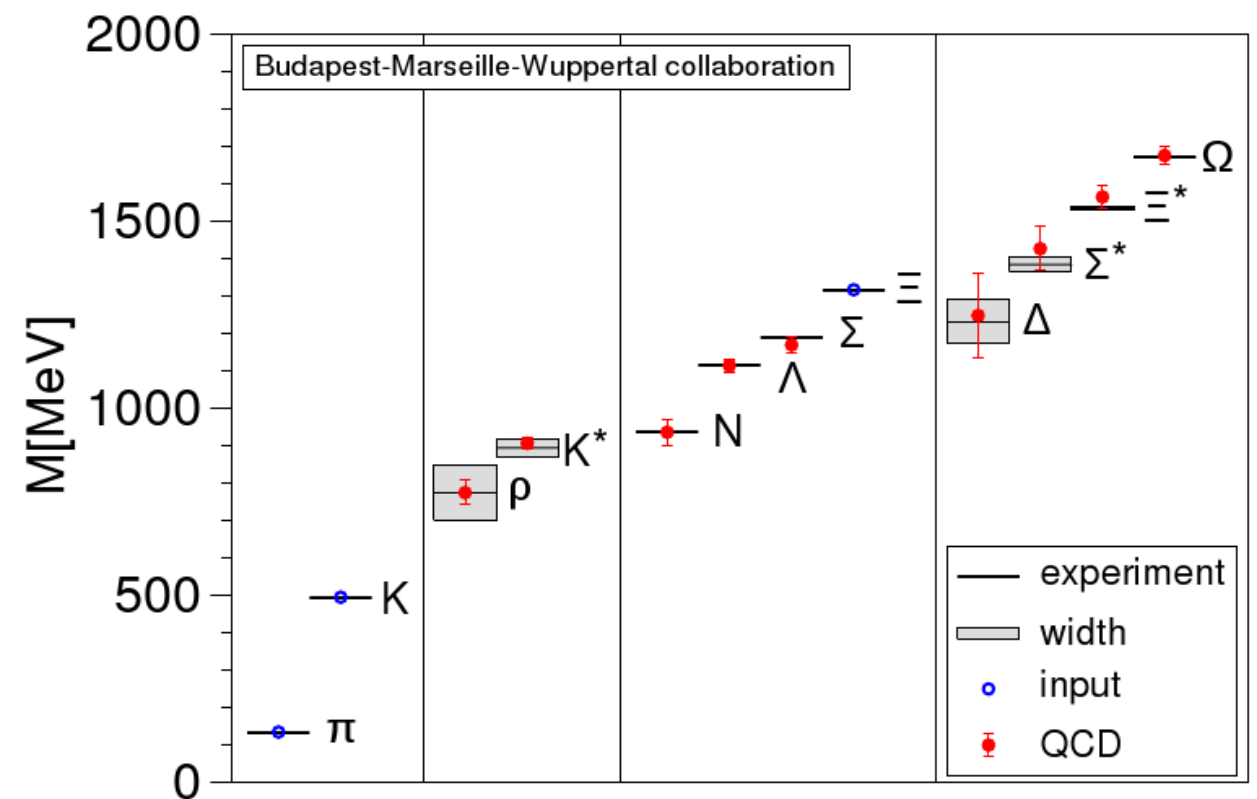
Durr et al, arXiv:0906.3599

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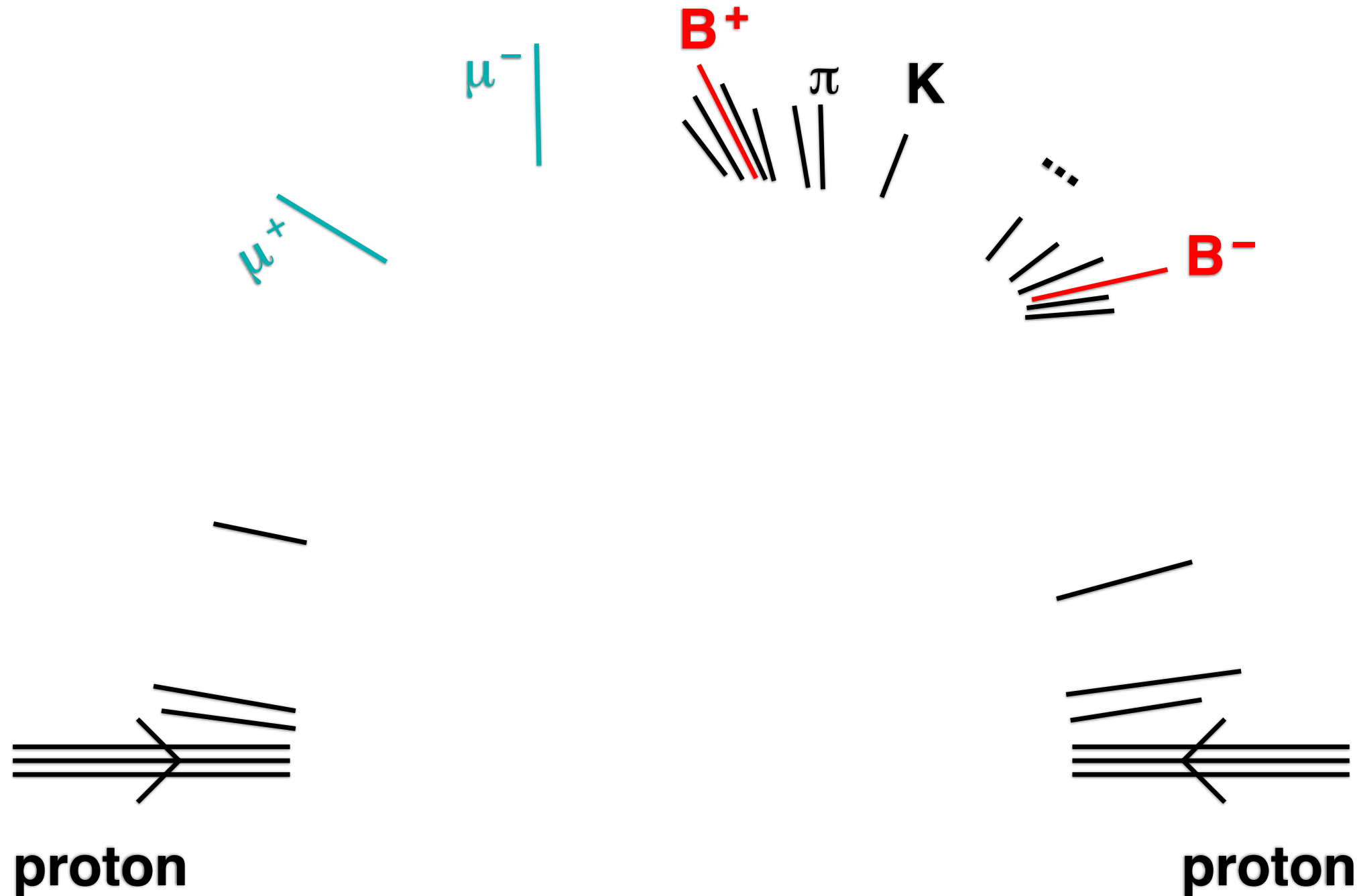
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For LHC reactions, lattice would have to

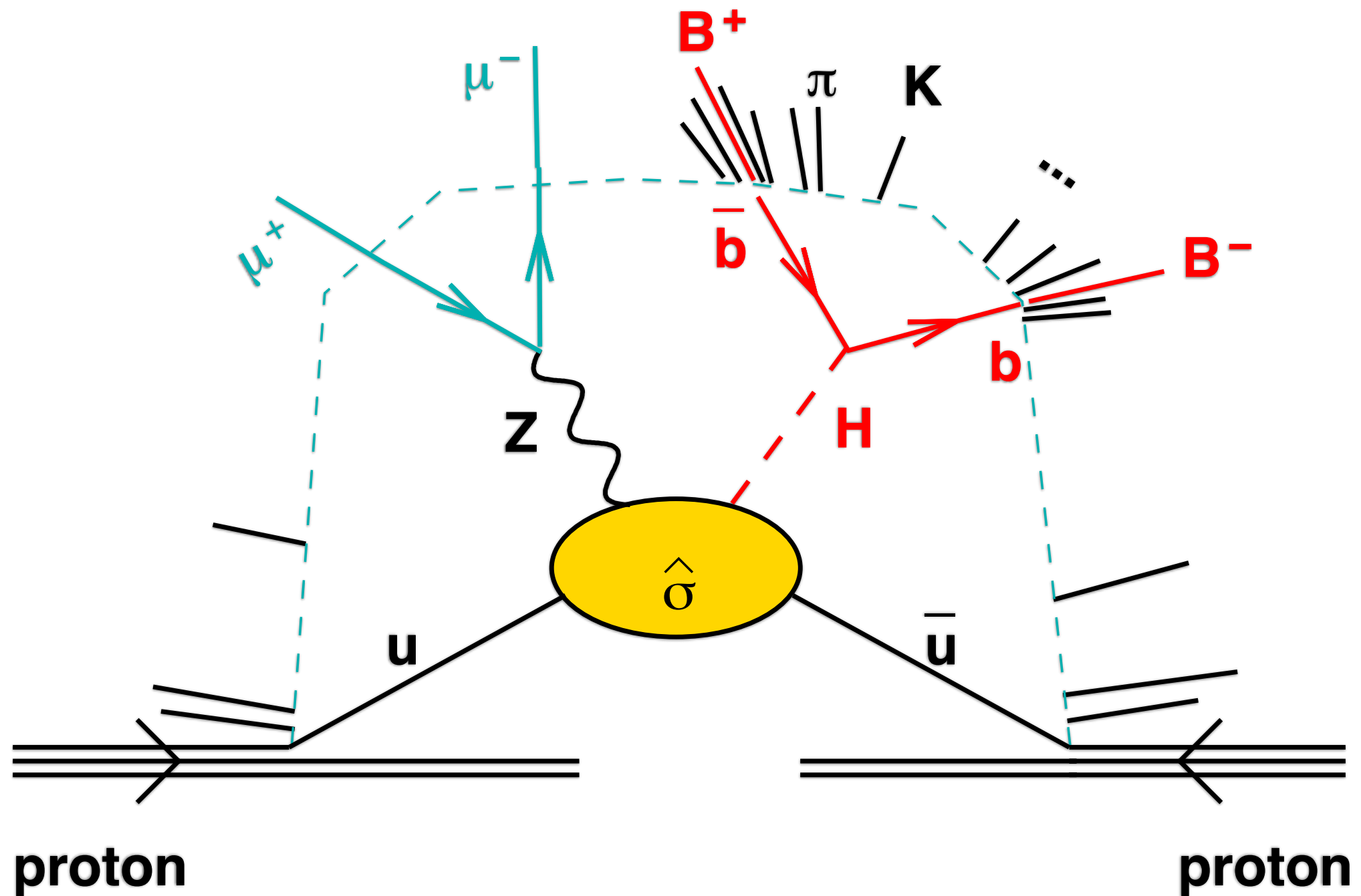
- Resolve smallest length scales ($2 \text{ TeV} \sim 10^{-4} \text{ fm}$)
- Contain whole reaction (pion formed on timescale of 1 fm , with boost of 10000 — i.e. 10^4 fm)

That implies 10^8 nodes in each dimension, i.e. 10^{32} nodes — **unrealistic**

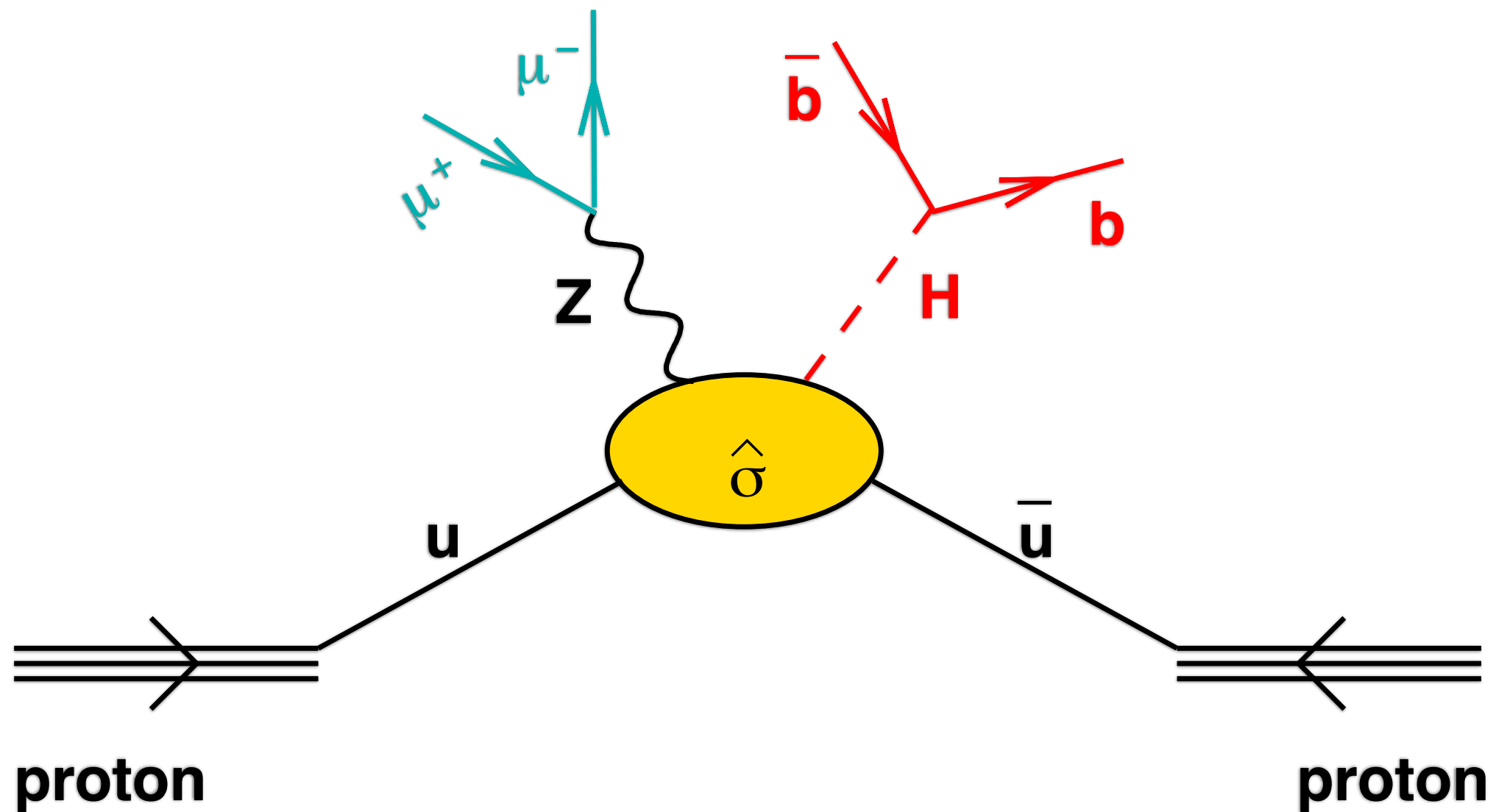
A proton-proton collision: FILLING IN THE PICTURE



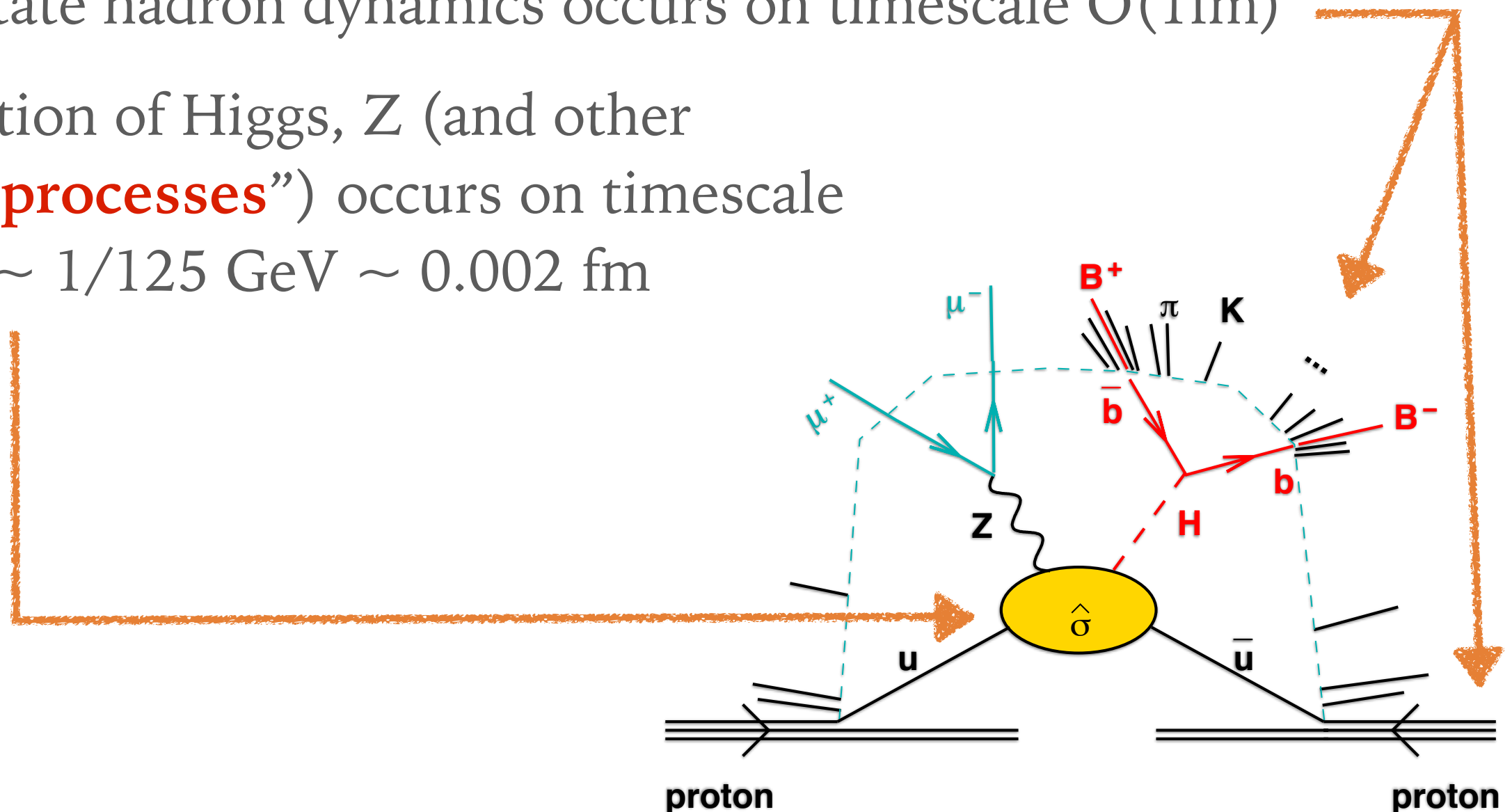
A proton-proton collision: FILLING IN THE PICTURE



A proton-proton collision: SIMPLIFYING IN THE PICTURE



- Proton's dynamics occurs on timescale $O(1 \text{ fm})$
Final-state hadron dynamics occurs on timescale $O(1 \text{ fm})$
- Production of Higgs, Z (and other “**hard processes**”) occurs on timescale $1/M_H \sim 1/125 \text{ GeV} \sim 0.002 \text{ fm}$



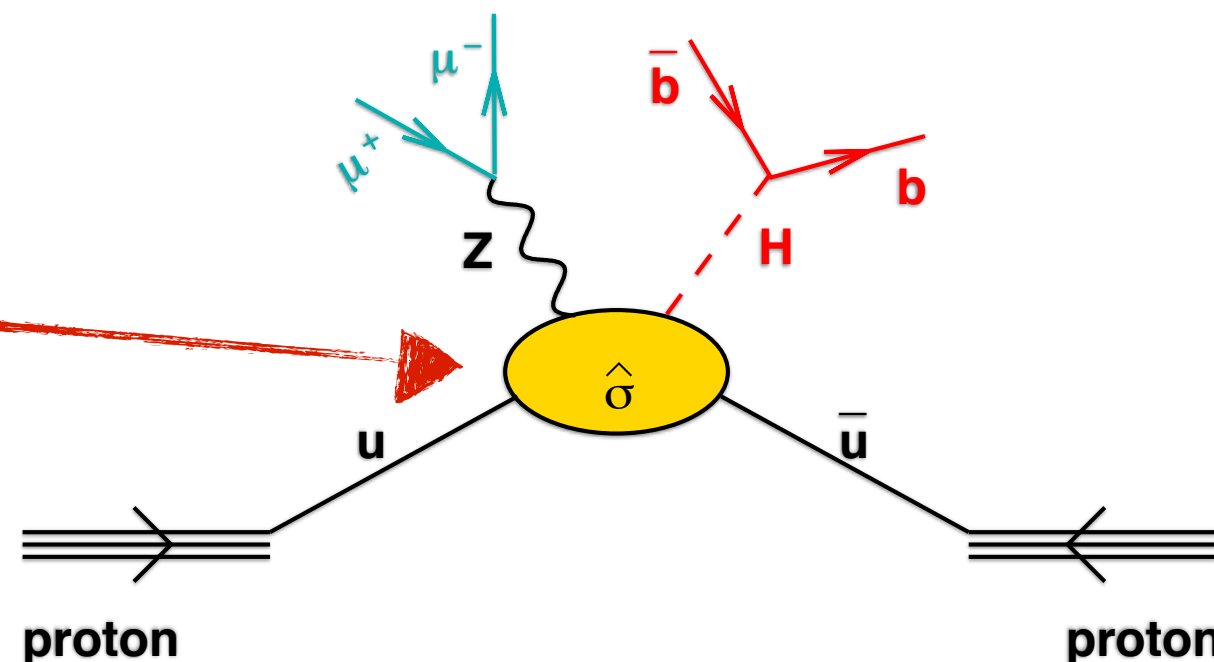
That means we can separate — “**factorise**” — the hard process, i.e. treat it as independent from all the hadronic dynamics

SHORT-DISTANCE QCD CORRECTIONS ARE PERTURBATIVE

- On timescales $1/M_H \sim 1/125 \text{ GeV} \sim 0.002 \text{ fm}$ you can take advantage of **asymptotic freedom**
- i.e. you can write results in terms of an expansion in the (*not so*) strong coupling constant $\alpha_s(125 \text{ GeV}) \sim 0.11$

$$\hat{\sigma} = \hat{\sigma}_0 (\boxed{1} + c_1 \alpha_s + c_2 \alpha_s^2 + \dots)$$

LO
(Leading Order)

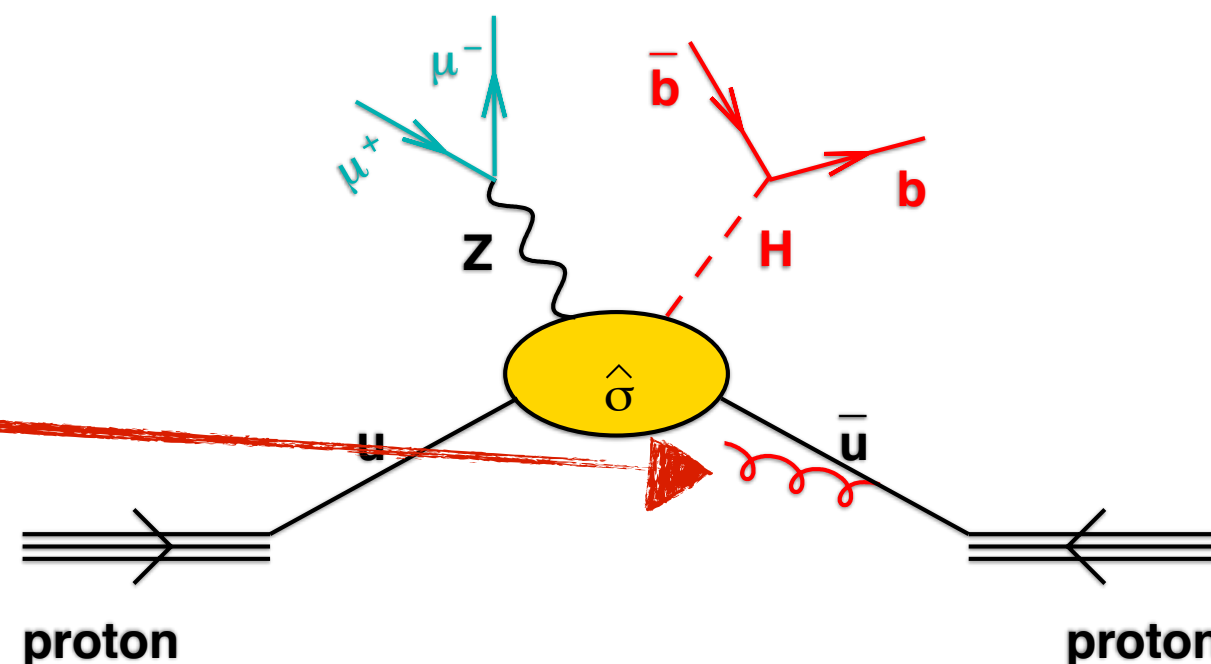


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NLO
(Next-to-Leading Order)

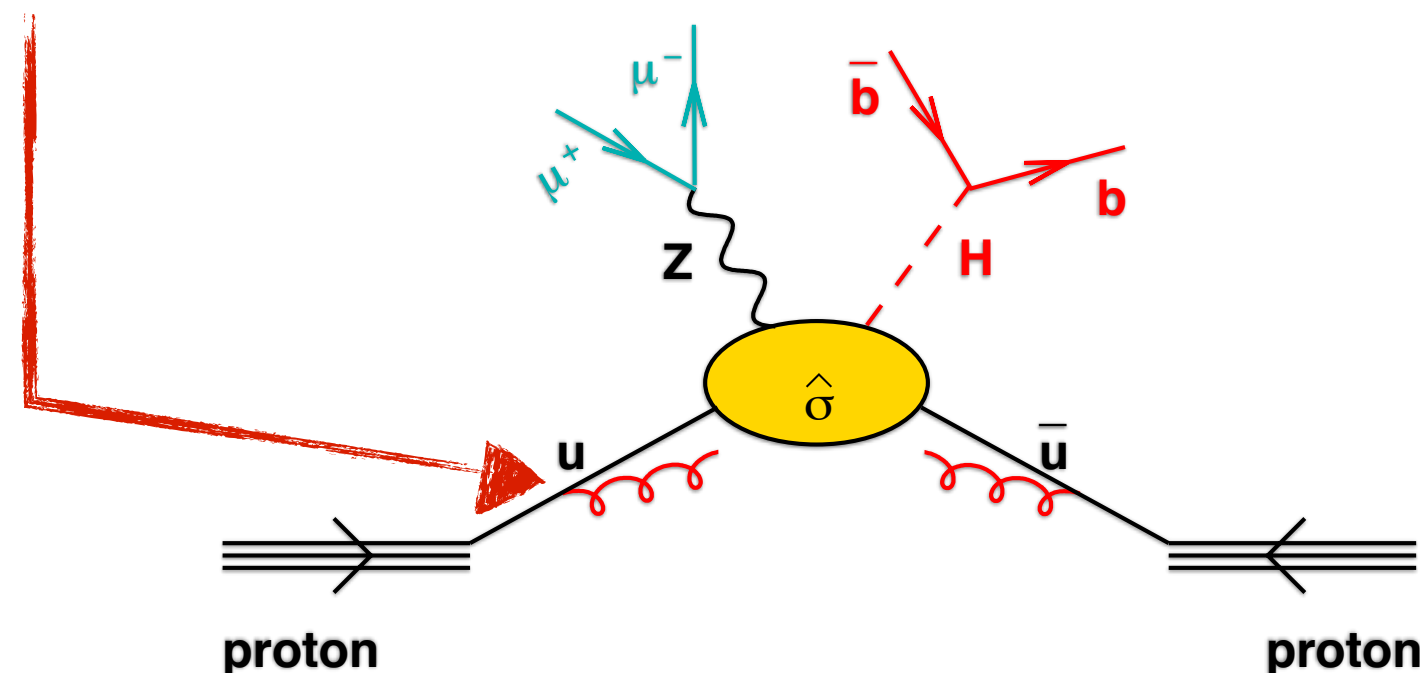


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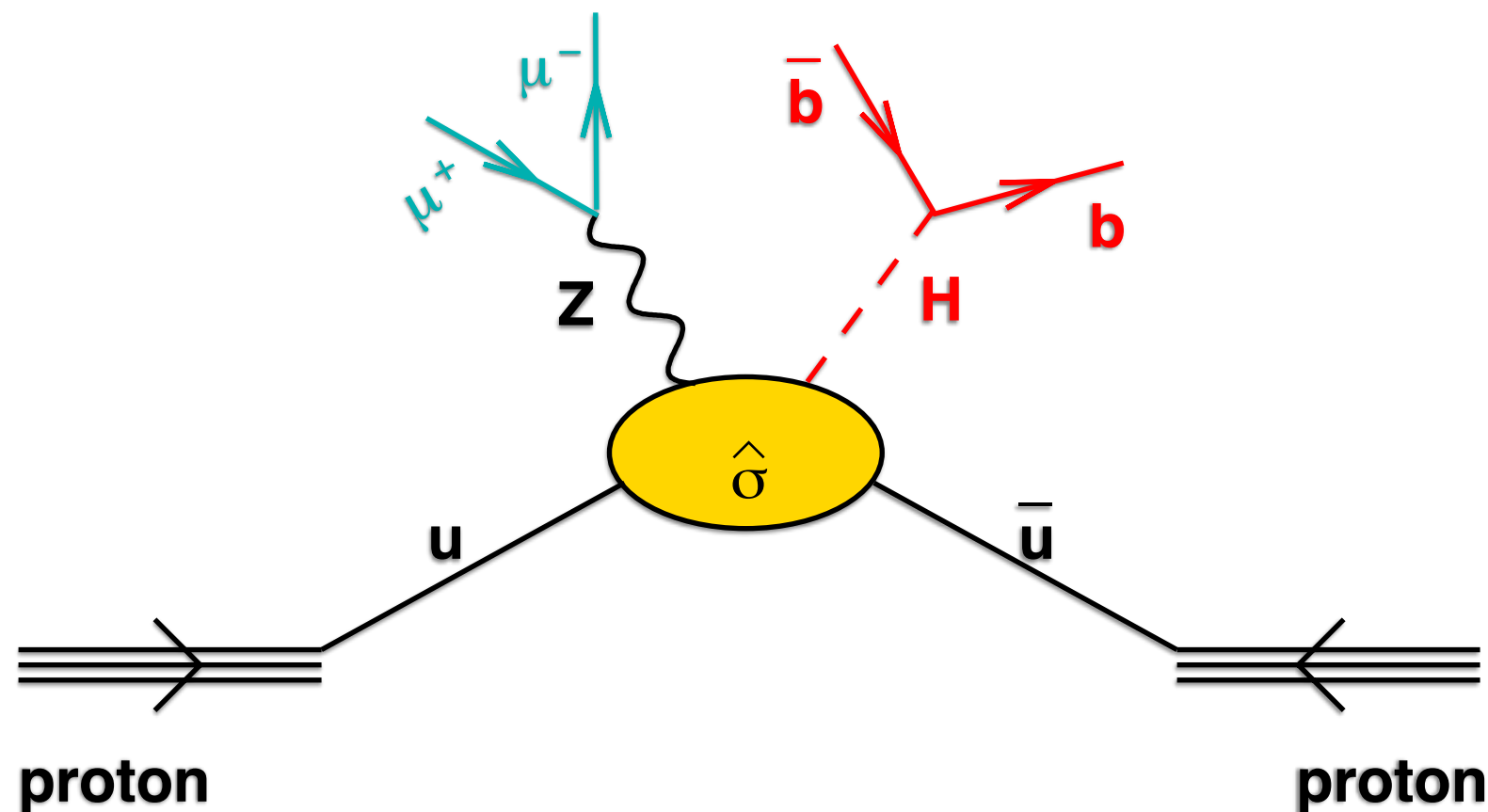
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NNLO
(Next-to-next-to-Leading Order)



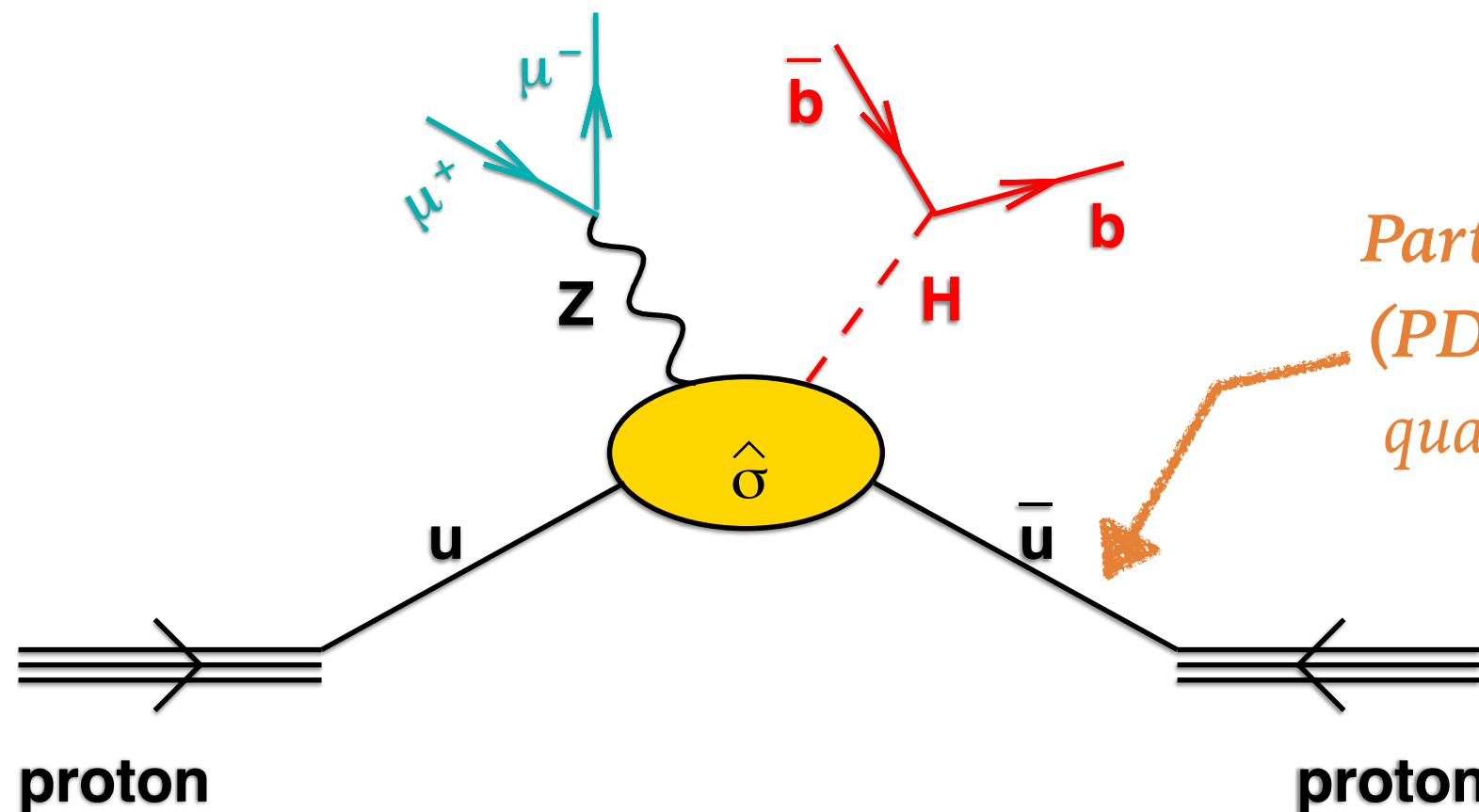
THE MASTER EQUATION

$$\sigma(h_1 h_2 \rightarrow ZH + X) = \sum_{n=0}^{\infty} \alpha_s^n(\mu_R^2) \sum_{i,j} \int dx_1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \\ \times \hat{\sigma}_{ij \rightarrow ZH+X}^{(n)}(x_1 x_2 s, \mu_R^2, \mu_F^2) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$



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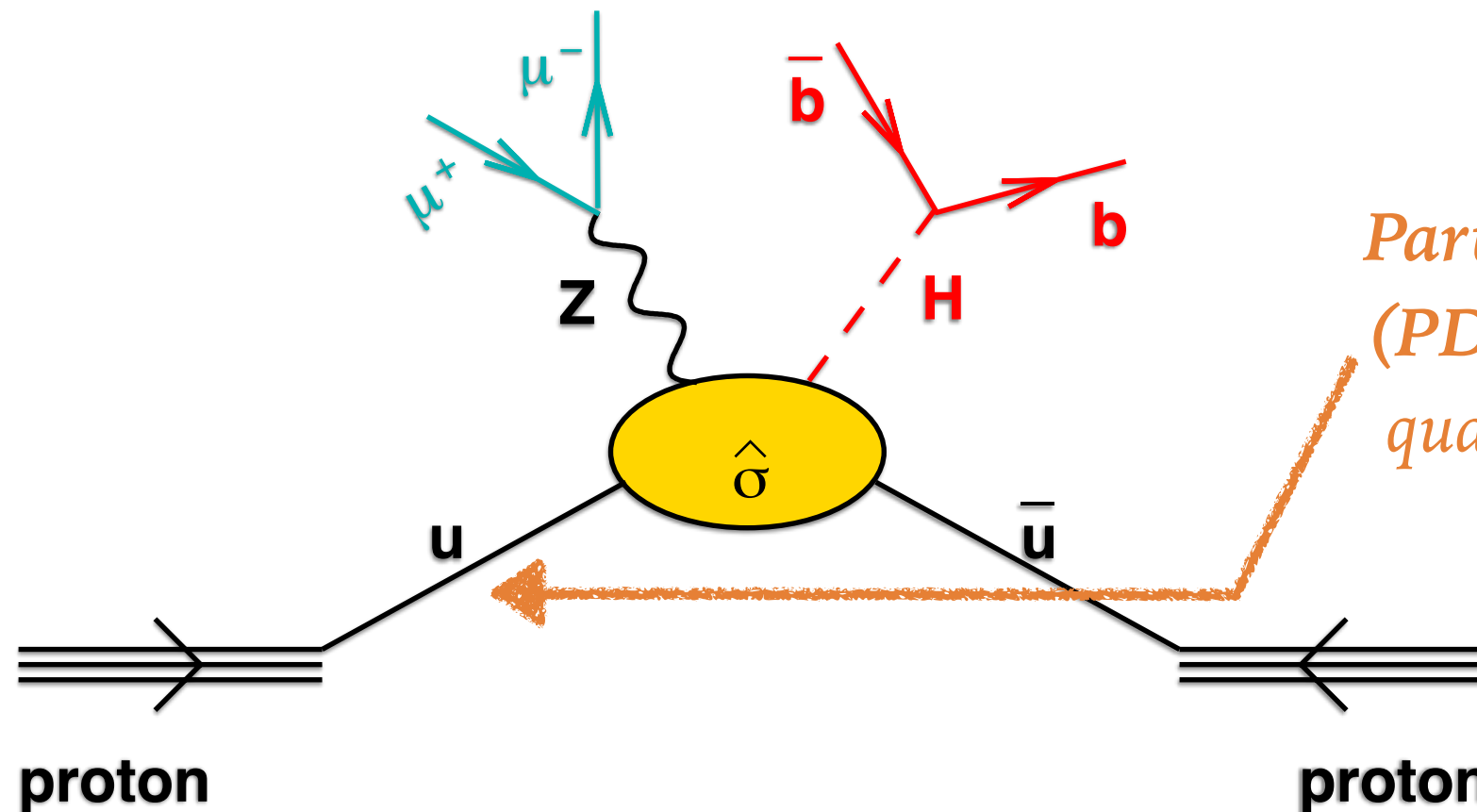
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Parton distribution function (PDF): e.g. number of up anti-quarks carrying fraction x_2 of proton's momentum

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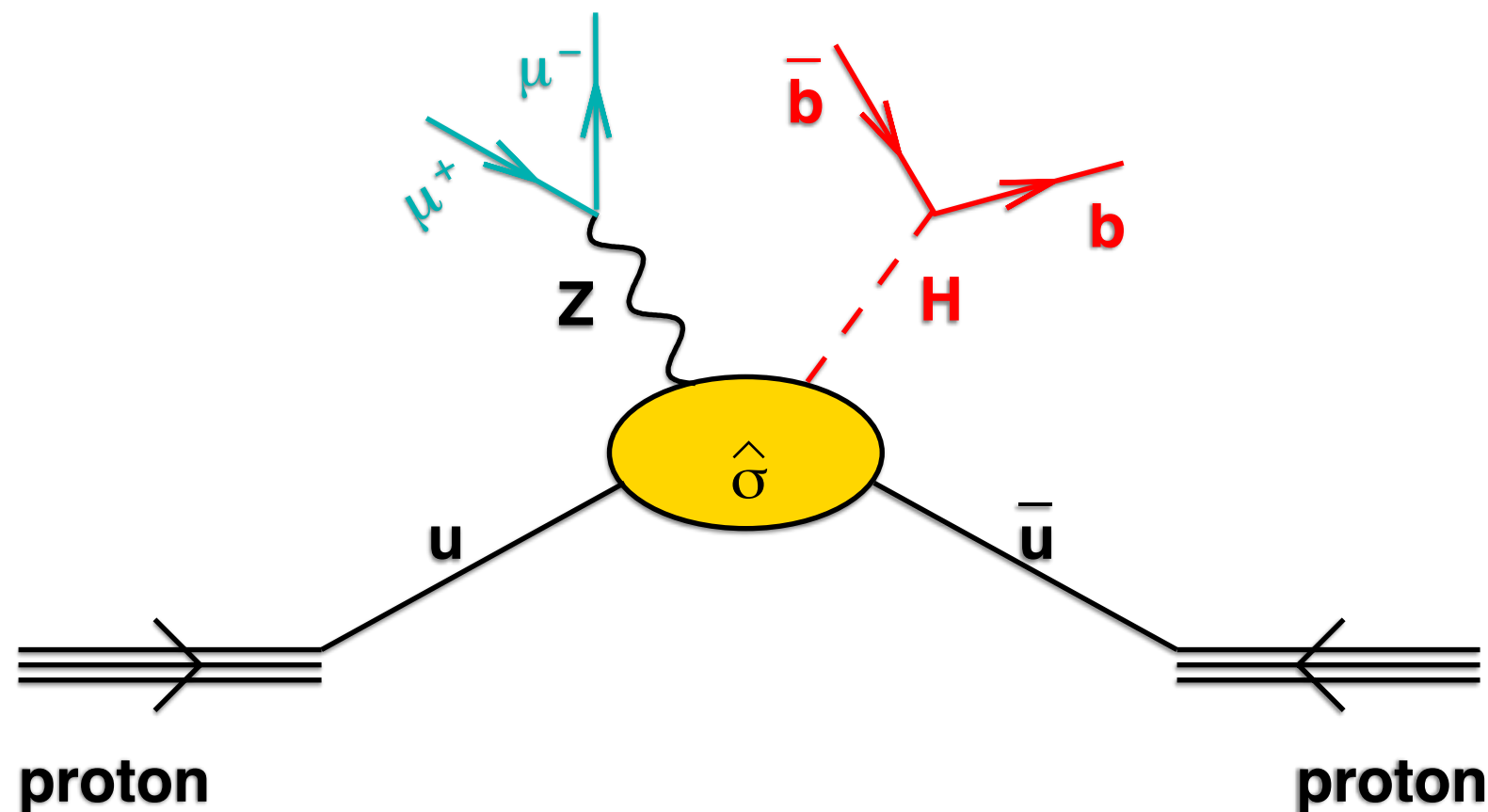


Parton distribution function (PDF): e.g. number of up quarks carrying fraction x_1 of proton's momentum

THE MASTER EQUATION

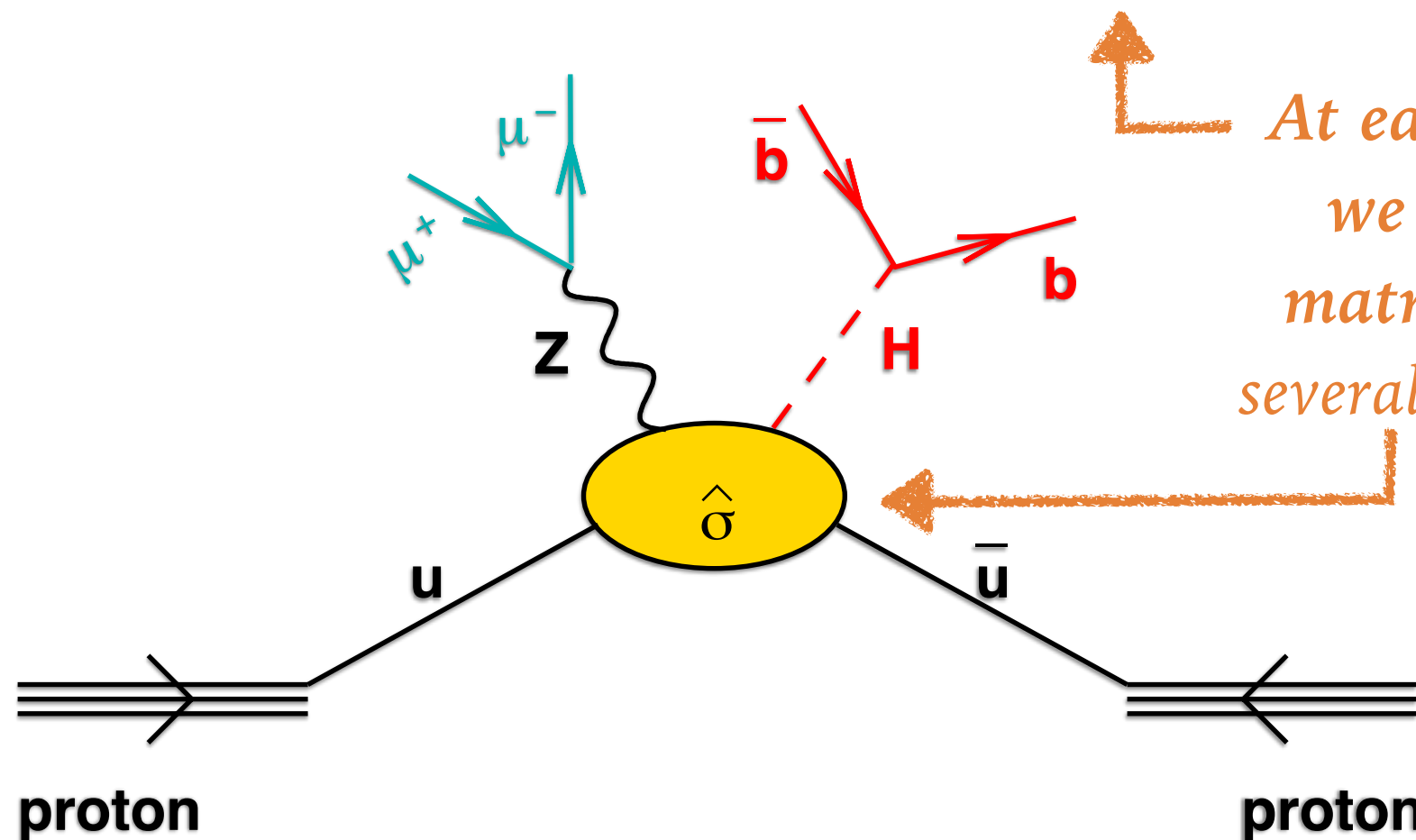
Perturbative sum over powers of the strong coupling: typically we use first 2-3 orders

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THE MASTER EQUATION

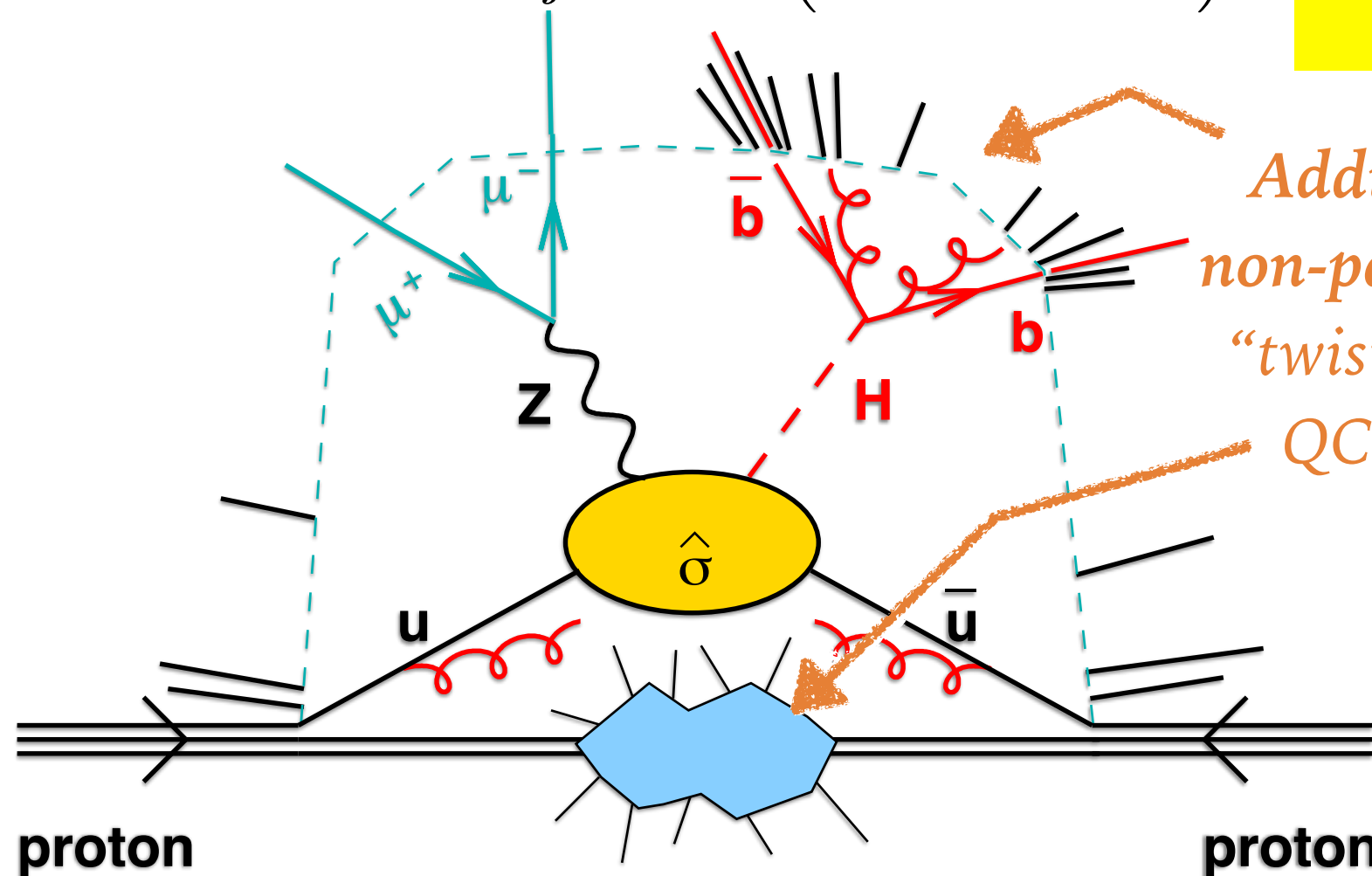
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At each perturbative order n we have a specific “hard matrix element” (sometimes several for different subprocesses)

THE MASTER EQUATION

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Additional corrections from non-perturbative effects (higher “twist”, suppressed by powers of QCD scale (Λ) / hard scale)

THE STRONG COUPLING

RUNNING COUPLING

All couplings run (QED, QCD, EW), i.e. they depend on the momentum scale (Q^2) of your process.

The QCD coupling, $\alpha_s(Q^2)$, runs **fast**:

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s), \quad \beta(\alpha_s) = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \dots),$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}, \quad b_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2} = \frac{153 - 19n_f}{24\pi^2}$$

Note sign: **Asymptotic Freedom**, due to gluon to self-interaction

2004 Nobel prize: Gross, Politzer & Wilczek

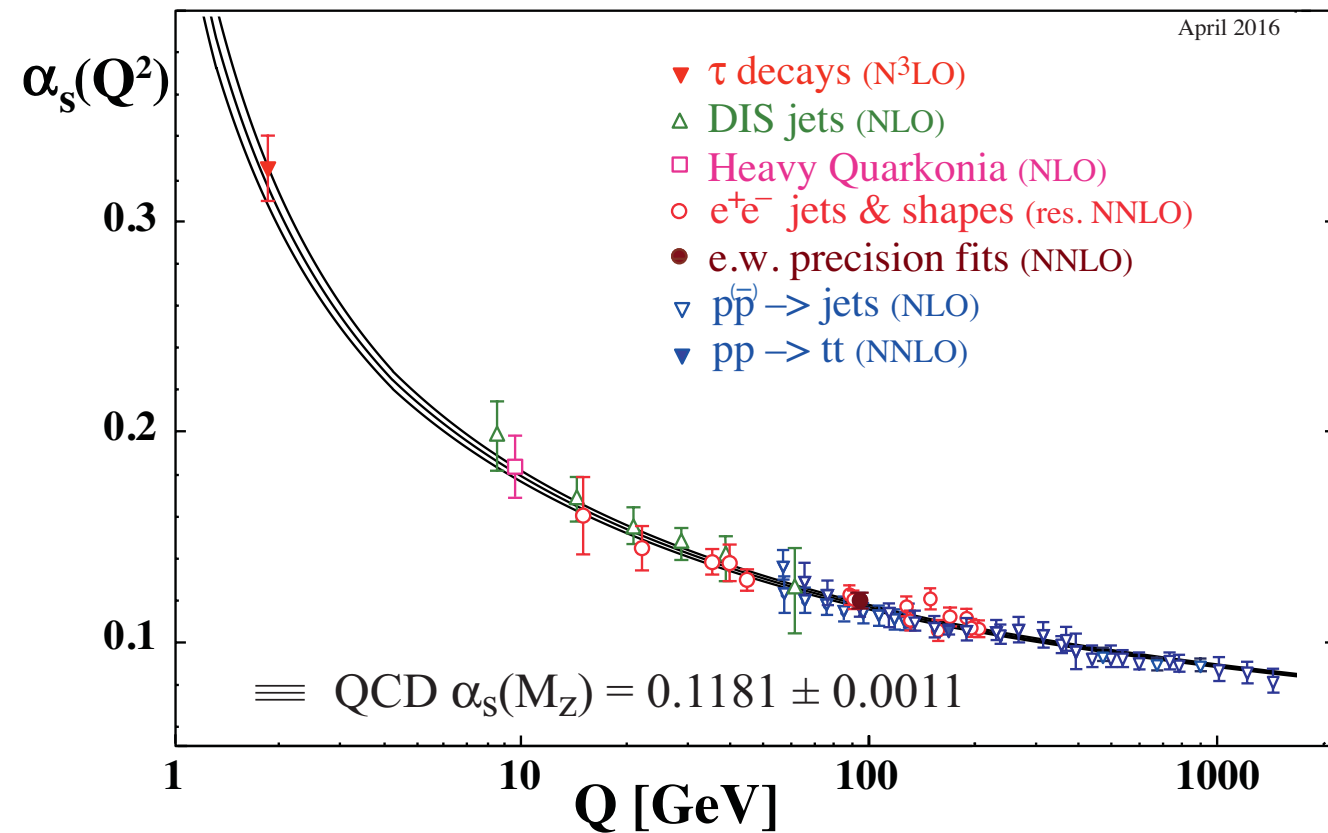
- ▶ At high scales Q , coupling becomes small
 - ➡ quarks and gluons are almost free, interactions are weak
- ▶ At low scales, coupling becomes strong
 - ➡ quarks and gluons interact strongly — confined into hadrons
 - Perturbation theory fails.

THE STRONG COUPLING V. SCALE

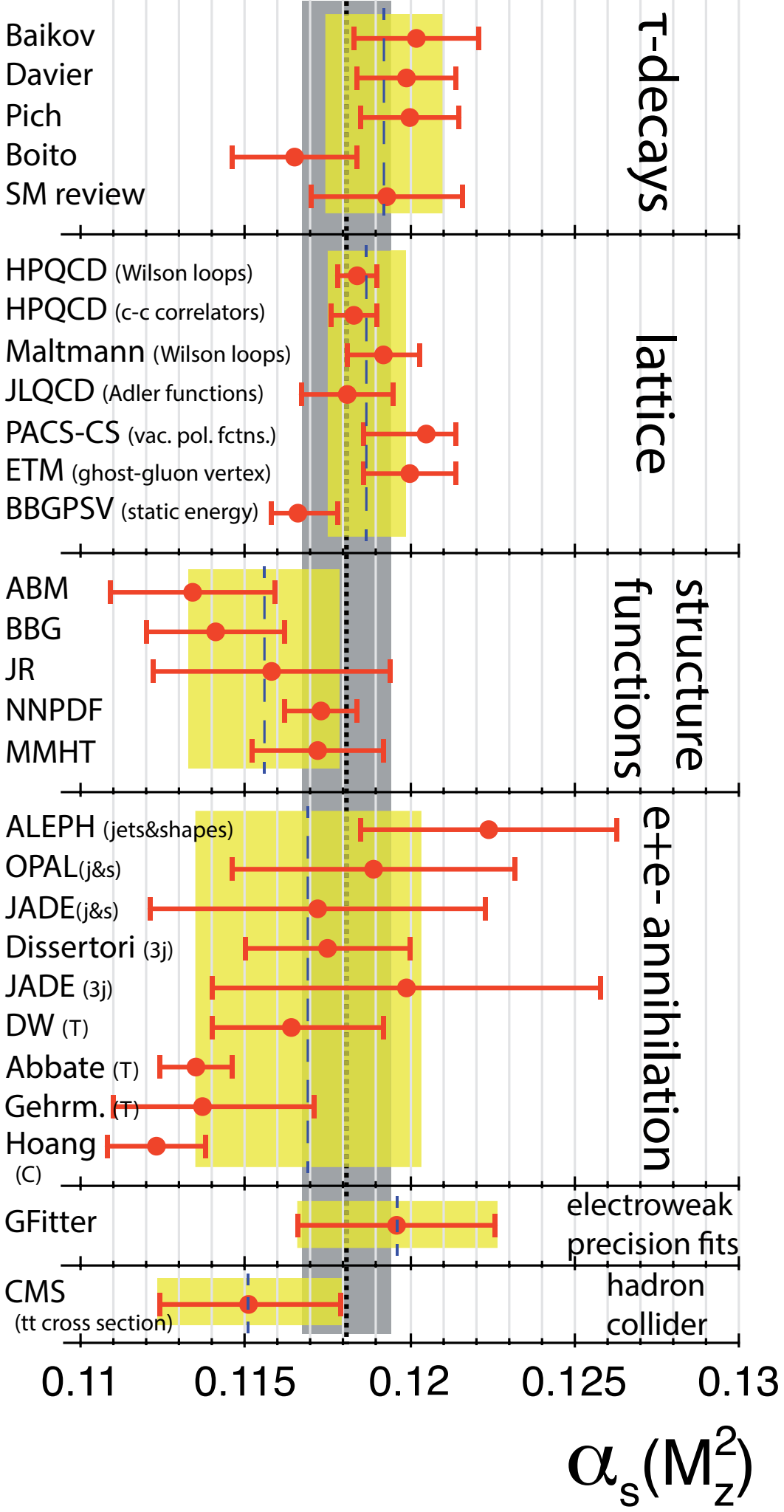
$$\text{Solve } Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \Rightarrow \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$$

$\Lambda \simeq 0.2 \text{ GeV}$ (aka Λ_{QCD}) is the fundamental scale of QCD, at which coupling blows up.

- ▶ Λ sets the scale for hadron masses (NB: Λ not unambiguously defined wrt higher orders)
- ▶ Perturbative calculations valid for scales $Q \gg \Lambda$.



PDG World Average: $\alpha_s(M_Z) = 0.1181 \pm 0.0011$ (0.9%)



STRONG-COUPLING DETERMINATIONS

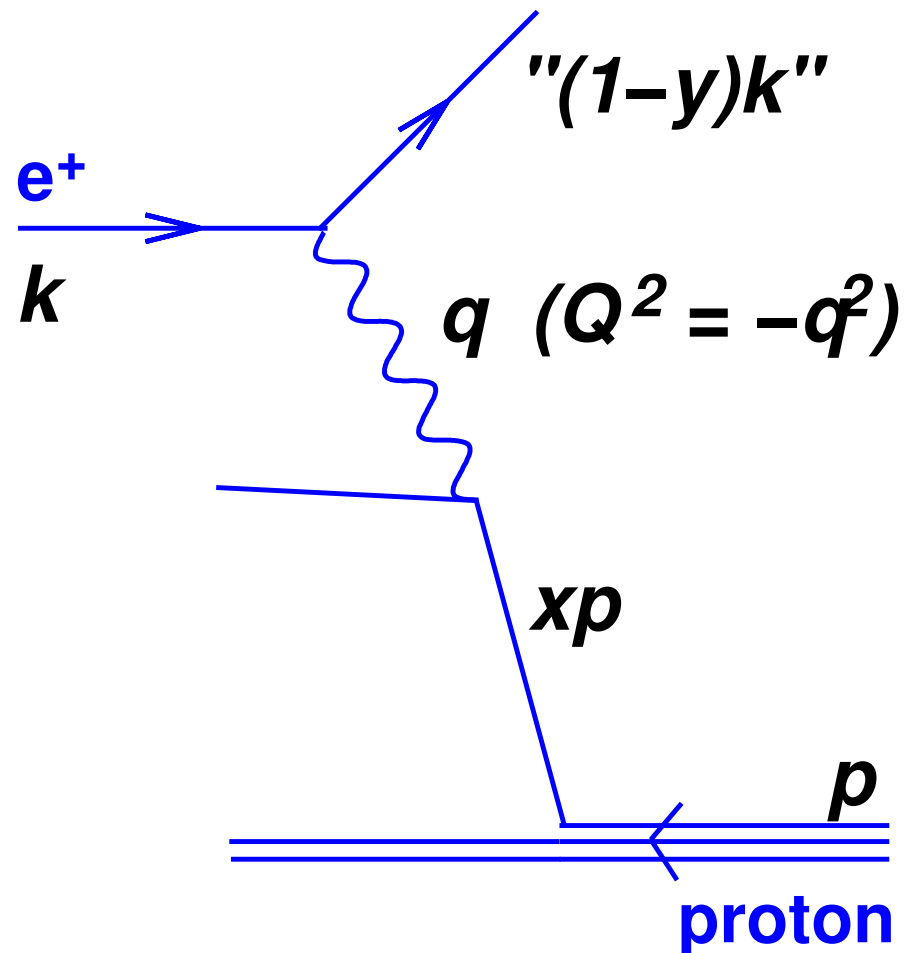
Bethke, Dissertori & GPS in PDG '16

- Most consistent set of independent determinations is from lattice
- Two best determinations are from same group (HPQCD, 1004.4285, 1408.4169)
 $\alpha_s(M_Z) = 0.1183 \pm 0.0007$ (0.6%)
 [heavy-quark correlators]
 $\alpha_s(M_Z) = 0.1183 \pm 0.0007$ (0.6%)
 [Wilson loops]
- Many determinations quote small uncertainties ($\lesssim 1\%$). All are disputed!
- Some determinations quote anomalously small central values (~ 0.113 v. world avg. of 0.1181 ± 0.0011). Also disputed

PARTON DISTRIBUTION FUNCTIONS (PDFs)

DEEP INELASTIC SCATTERING

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



Kinematic relations:

$$x = \frac{Q^2}{2p \cdot q}; \quad y = \frac{p \cdot q}{p \cdot k}; \quad Q^2 = xys$$

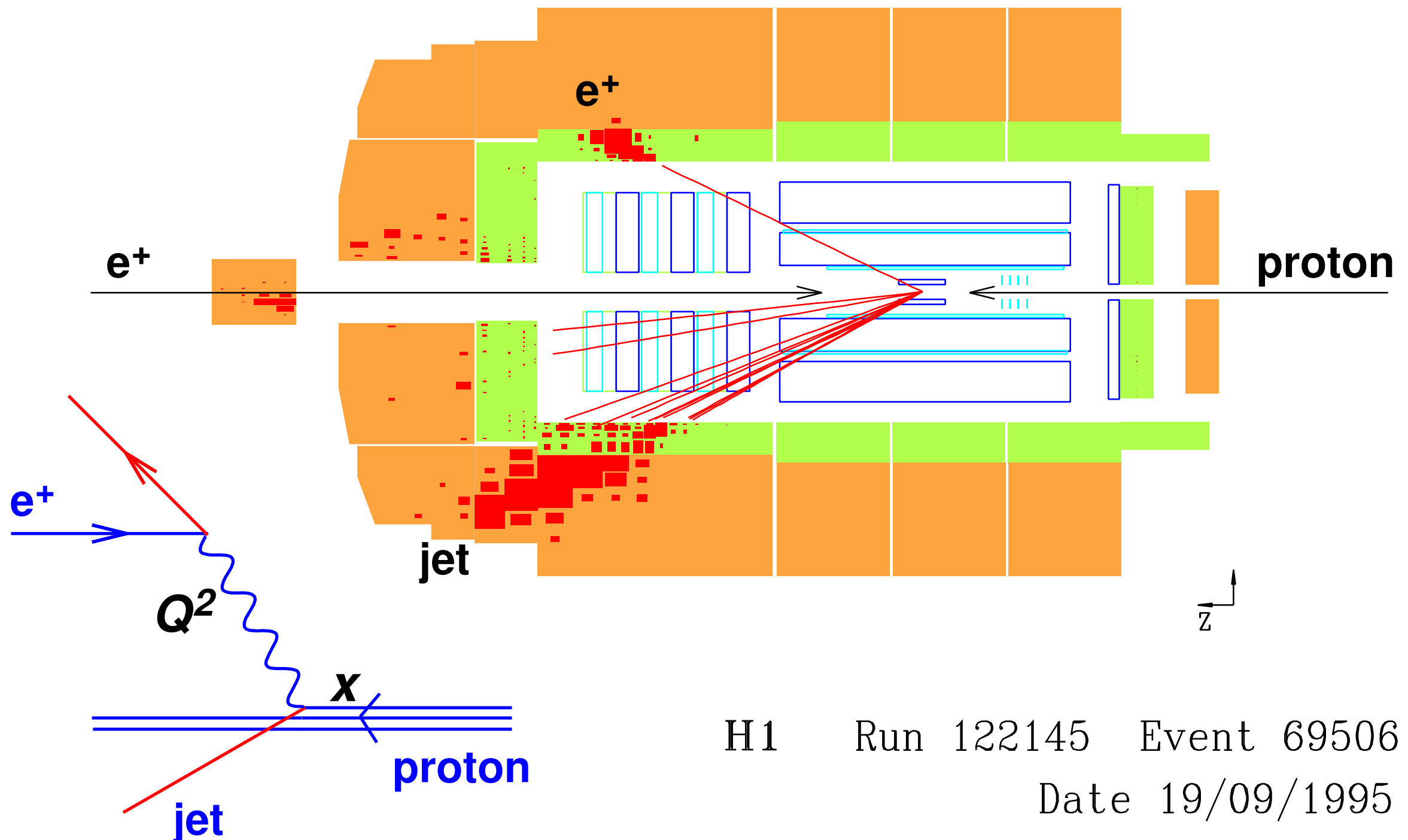
\sqrt{s} = c.o.m. energy

- ▶ Q^2 = photon virtuality \leftrightarrow *transverse resolution* at which it probes proton structure
- ▶ x = *longitudinal momentum fraction* of struck parton in proton
- ▶ y = momentum fraction lost by electron (in proton rest frame)

DEEP INELASTIC SCATTERING



$$Q^2 = 25030 \text{ GeV}^2; \quad y = 0.56; \quad \mathbf{x=0.50}$$



DEEP INELASTIC SCATTERING

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dx dQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4} \left(\frac{1 + (1-y)^2}{2} F_2^{em} + \mathcal{O}(\alpha_s) \right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x \left(\frac{4}{9} u(x) + \frac{1}{9} d(x) \right)$$

[$u(x)$, $d(x)$]: parton distribution functions (PDF)]

NB:

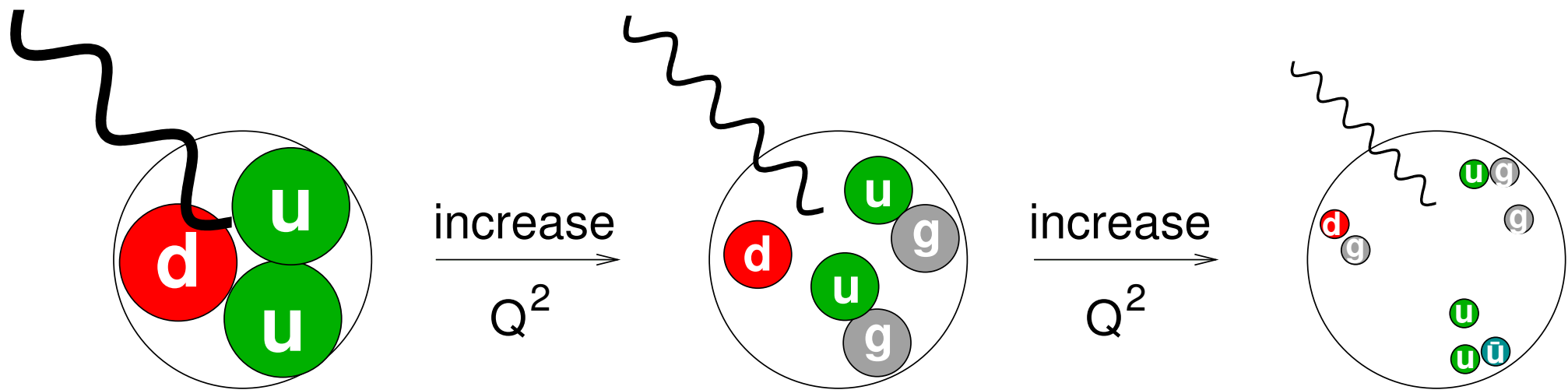
- ▶ use perturbative language for interactions of up and down quarks
- ▶ but distributions themselves have a *non-perturbative* origin.

PARTON DISTRIBUTION AND DGLAP

- Write up-quark distribution in proton as

$$u(x, \mu_F^2)$$

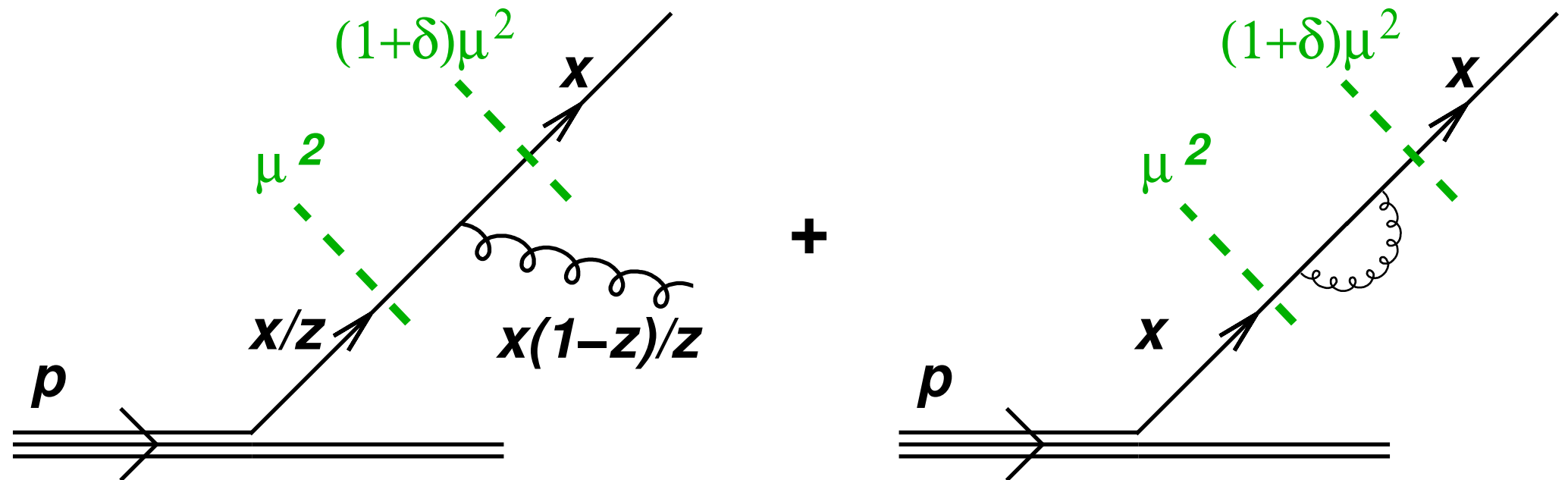
- μ_F is the **factorisation scale** — a bit like the renormalisation scale (μ_R) for the running coupling.
- As you vary the factorisation scale, the parton distributions evolve with a renormalisation-group type equation



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations

DGLAP EQUATION

take derivative wrt factorization scale μ^2



$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 dz \, p_{qq}(z) \frac{q(x/z, \mu^2)}{z} - \frac{\alpha_s}{2\pi} \int_0^1 dz \, p_{qq}(z) q(x, \mu^2)$$

p_{qq} is real $q \leftarrow q$ splitting kernel: $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

DGLAP EQUATION

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \underbrace{\int_x^1 dz \, P_{qq}(z) \frac{q(x/z, \mu^2)}{z}}_{P_{qq} \otimes q}, \quad P_{qq} = C_F \left(\frac{1+z^2}{1-z} \right)_+$$

This involves the *plus prescription*:

$$\int_0^1 dz \, [g(z)]_+ f(z) = \int_0^1 dz \, g(z) f(z) - \int_0^1 dz \, g(z) f(1)$$

$z = 1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z = 1$

DGLAP EQUATION

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space*:

$$\frac{d}{d \ln Q^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q \leftarrow q} & P_{q \leftarrow g} \\ P_{g \leftarrow q} & P_{g \leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

$$P_{qg}(z) = T_R [z^2 + (1-z)^2], \quad P_{gq}(z) = C_F \left[\frac{1 + (1-z)^2}{z} \right],$$

$$P_{gg}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z) \frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- ▶ P_{qg}, P_{gg} : *symmetric* $z \leftrightarrow 1-z$ (except virtuals)
- ▶ P_{qq}, P_{gg} : *diverge* for $z \rightarrow 1$ soft gluon emission
- ▶ P_{gg}, P_{gq} : *diverge* for $z \rightarrow 0$ Implies PDFs grow for $x \rightarrow 0$

2015 EPS HEP prize to Bjorken, Altarelli, Dokshitzer, Lipatov & Parisi

NLO:

$$P_{\text{ps}}^{(1)}(x) = 4 C_F n_f \left(\frac{20}{9} \frac{1}{x} - 2 + 6x - 4H_0 + x^2 \left[\frac{8}{3} H_0 - \frac{56}{9} \right] + (1+x) \left[5H_0 - 2H_{0,0} \right] \right)$$

$$P_{\text{qg}}^{(1)}(x) = 4 C_A n_f \left(\frac{20}{9} \frac{1}{x} - 2 + 25x - 2p_{\text{qg}}(-x)H_{-1,0} - 2p_{\text{qg}}(x)H_{1,1} + x^2 \left[\frac{44}{3} H_0 - \frac{218}{9} \right] \right. \\ \left. + 4(1-x) \left[H_{0,0} - 2H_0 + xH_1 \right] - 4\zeta_2 x - 6H_{0,0} + 9H_0 \right) + 4 C_F n_f \left(2p_{\text{qg}}(x) \left[H_{1,0} + H_{1,1} + H_2 \right. \right. \\ \left. \left. - \zeta_2 \right] + 4x^2 \left[H_0 + H_{0,0} + \frac{5}{2} \right] + 2(1-x) \left[H_0 + H_{0,0} - 2xH_1 + \frac{29}{4} \right] - \frac{15}{2} - H_{0,0} - \frac{1}{2} H_0 \right)$$

$$P_{\text{gq}}^{(1)}(x) = 4 C_A C_F \left(\frac{1}{x} + 2p_{\text{gq}}(x) \left[H_{1,0} + H_{1,1} + H_2 - \frac{11}{6} H_1 \right] - x^2 \left[\frac{8}{3} H_0 - \frac{44}{9} \right] + 4\zeta_2 - 2 \right. \\ \left. - 7H_0 + 2H_{0,0} - 2H_1 x + (1+x) \left[2H_{0,0} - 5H_0 + \frac{37}{9} \right] - 2p_{\text{gq}}(-x)H_{-1,0} \right) - 4 C_F n_f \left(\frac{2}{3} x \right. \\ \left. - p_{\text{gq}}(x) \left[\frac{2}{3} H_1 - \frac{10}{9} \right] \right) + 4 C_F^2 \left(p_{\text{gq}}(x) \left[3H_1 - 2H_{1,1} \right] + (1+x) \left[H_{0,0} - \frac{7}{2} + \frac{7}{2} H_0 \right] - 3H_{0,0} \right. \\ \left. + 1 - \frac{3}{2} H_0 + 2H_1 x \right)$$

$$P_{\text{gg}}^{(1)}(x) = 4 C_A n_f \left(1 - x - \frac{10}{9} p_{\text{gg}}(x) - \frac{13}{9} \left(\frac{1}{x} - x^2 \right) - \frac{2}{3} (1+x)H_0 - \frac{2}{3} \delta(1-x) \right) + 4 C_A^2 \left(27 \right. \\ \left. + (1+x) \left[\frac{11}{3} H_0 + 8H_{0,0} - \frac{27}{2} \right] + 2p_{\text{gg}}(-x) \left[H_{0,0} - 2H_{-1,0} - \zeta_2 \right] - \frac{67}{9} \left(\frac{1}{x} - x^2 \right) - 12H_0 \right. \\ \left. - \frac{44}{3} x^2 H_0 + 2p_{\text{gg}}(x) \left[\frac{67}{18} - \zeta_2 + H_{0,0} + 2H_{1,0} + 2H_2 \right] + \delta(1-x) \left[\frac{8}{3} + 3\zeta_3 \right] \right) + 4 C_F n_f \left(2H_0 \right. \\ \left. + \frac{2}{3} \frac{1}{x} + \frac{10}{3} x^2 - 12 + (1+x) \left[4 - 5H_0 - 2H_{0,0} \right] - \frac{1}{2} \delta(1-x) \right) .$$

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski
& Petronzio '80

NNLO DGLAP

Divergences for $x = 1$ are understood in the sense of ϵ -distributions.

The third-order pure-singlet contribution to the quark-quark splitting function (2.4), corresponding to the anomalous dimension (3.10), is given by

$$\begin{aligned} P_{qq}^{(2)} x &= 16 C_A C_F n_f \left[\frac{4}{3} x^2 \frac{13}{x} H_0 + \frac{14}{9} H_0 \frac{1}{2} H_1 \zeta_2 + H_1 \int_0^1 \frac{2H}{1-x} \right. \\ &+ H_{12} \frac{2}{3} x^2 \frac{16}{x} \zeta_2^2 H_2 + 9 \zeta_3 \frac{9}{4} H_{10} + \frac{6761}{216} H_1 \frac{10}{3} H_2 H_3 \zeta_2 + \frac{1}{6} H_{11} \\ &+ 3 H_{100} + 2 H_{110} + 2 H_{111} + 1 x \frac{185}{9} H_1 \frac{158}{3} \frac{397}{36} H_{00} + \frac{13}{2} H_{20} + 3 H_{0000} \\ &+ \frac{13}{6} H_{10} + 3 x H_{10} + H_{30} + H_{252} + 2 H_{210} + 3 H_{200} + \frac{1}{2} H_{0052} + \frac{1}{2} H_{152} + \frac{9}{4} H_{100} \\ &+ \frac{3}{4} H_{11} + H_{110} + H_{111} + 1 x \frac{7}{12} H_{052} + \frac{31}{6} \zeta_3 \frac{91}{18} H_2 + \frac{71}{12} H_3 + \frac{113}{18} \zeta_2 \frac{826}{27} H_0 \\ &+ \frac{5}{2} H_{20} + \frac{16}{3} H_{10} + 6 x H_{10} + \frac{31}{6} H_{0000} + \frac{117}{20} H_1 \frac{117}{20} \zeta_2^2 + 9 H_{052} + \frac{5}{2} H_{152} + 2 H_{210} \\ &+ \frac{1}{2} H_{100} + 2 H_{12} + H_{252} + \frac{7}{2} H_{200} + H_{110} + 2 H_{211} + H_{31} + \frac{1}{2} H_{14} + 5 H_{20} + H_{21} \\ &+ H_{0000} + \frac{1}{2} \zeta_2^2 + 4 H_{30} + 4 H_{052} + \frac{32}{9} H_{00} + \frac{29}{12} H_{10} + \frac{235}{12} \zeta_2 \frac{511}{12} \frac{97}{12} H_2 + \frac{33}{4} H_3 + H_3 \\ &+ \frac{11}{2} H_{052} + \frac{11}{2} \zeta_3 + \frac{3}{2} H_{20} + 10 H_{000} + \frac{2}{3} x^2 \frac{83}{4} H_{00} + \frac{243}{4} H_0 + 10 \zeta_2 \frac{511}{8} \frac{97}{8} H_1 + \frac{4}{3} H_2 \\ &+ 4 \zeta_3 H_{052} + H_3 + H_{20} + 6 H_{20} + 16 C_F n_f^2 \frac{2}{27} H_{20} + 2 H_2 \zeta_2 + \frac{2}{3} x^2 H_2 \zeta_2 + 3 \\ &+ \frac{19}{6} H_0 + \frac{2}{9} x^2 H_{11} + \frac{5}{3} H_1 + \frac{2}{3} + 1 x \frac{1}{6} H_{11} + \frac{7}{6} H_1 + x H_1 + \frac{35}{27} H_2 + \frac{185}{34} \\ &+ \frac{1}{3} + 1 x \frac{4}{3} H_2 + \frac{4}{3} \zeta_3 H_2 + H_{21} + 2 H_3 + 2 H_{052} + \frac{29}{9} H_{00} + H_{000} + 16 C_F^2 n_f \frac{85}{12} H_1 \\ &+ \frac{29}{6} H_{00} + H_{000} + \frac{583}{12} H_{10} + \frac{101}{54} H_2 + \frac{73}{4} H_3 + 5 H_{20} + H_{21} + H_{052} + x^2 \frac{55}{12} \\ &+ \frac{85}{4} H_1 + \frac{22}{3} H_{00} + \frac{109}{6} \frac{13}{54} H_{10} + \frac{28}{9} \zeta_2 + \frac{28}{9} H_2 + \frac{16}{3} H_{052} + \frac{16}{3} H_3 + 4 H_{20} + \frac{4}{3} H_{21} + \frac{26}{3} \zeta_3 \\ &+ \frac{22}{3} H_{000} + \frac{4}{3} x^2 \frac{23}{12} H_{10} + \frac{523}{144} H_1 + 3 \zeta_3 \frac{55}{16} \frac{1}{4} H_{100} + H_{11} + H_{110} + H_{111} \\ &+ 1 x \frac{2}{2} H_{100} + \frac{7}{12} H_{11} + \frac{2743}{72} H_{10} + \frac{53}{12} H_{00} + \frac{251}{12} H_1 + \frac{5}{2} \zeta_2 + \frac{5}{2} H_2 + \frac{8}{3} H_{10} + x H_{10} \\ &+ 3 H_{052} + 3 H_3 + H_{110} + H_{111} + 1 x \frac{1669}{216} \frac{5}{2} H_{000} + 4 H_{21} + 7 H_{20} + 10 x \zeta_3 + \frac{37}{10} \zeta_2^2 \\ &+ 7 H_{052} + 6 H_{052} + 4 H_{0000} + H_{200} + 2 H_{210} + 2 H_{211} + 4 H_{30} + H_{31} + 6 H_4 \quad (4.12) \end{aligned}$$

Due to Eqs. (3.11) and (3.12) the three-loop gluon-quark and quark-gluon splitting functions read

$$\begin{aligned} P_{qg}^{(2)} x &= 16 C_A C_F n_f p_{qg} x \frac{39}{2} H_1 \zeta_3 + 4 H_{111} + 3 H_{200} + \frac{15}{4} H_{12} + \frac{9}{2} H_{110} + 3 H_{210} \\ &+ H_{052} + 2 H_{211} + 4 H_{252} + \frac{173}{12} H_{052} + \frac{551}{72} H_{00} + \frac{64}{3} \zeta_3 + \zeta_2^2 + \frac{49}{4} H_2 + \frac{3}{2} H_{0000} + \frac{1}{3} H_{100} \end{aligned}$$

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$$\begin{aligned} &+ \frac{655}{576} H_0 + \frac{151}{6} \zeta_3 + \frac{185}{18} H_{11} + \frac{1}{6} H_{111} + \frac{95}{9} H_2 + \frac{29}{6} H_{21} + \frac{171}{4} H_{10} + 12 H_{100} + 7 H_{152} \\ &+ 16 H_{110} + \frac{5}{3} H_{20} + \frac{3}{2} H_{211} + 4 H_{0000} + 35 H_{20} + \frac{179}{27} H_{10} + \frac{2041}{144} H_{00} + \frac{19}{6} H_{052} \\ &+ 2 H_{30} + \frac{13}{2} H_{052} + 13 H_{30} + \frac{13}{2} H_{10} + \frac{15}{2} H_3 + \frac{2005}{64} \frac{157}{4} \zeta_2 + 8 \zeta_3 + \frac{1291}{432} H_1 + \frac{55}{12} H_{11} \\ &+ \frac{3}{2} H_2 + \frac{1}{4} H_{10} + \frac{27}{4} H_{10} + \frac{11}{2} H_{100} + 8 H_{200} + 4 \zeta_2^2 + \frac{3}{2} H_{12} + H_{22} + \frac{2}{3} H_{152} + 8 H_{110} \\ &+ 4 H_{20} + \frac{3}{2} H_{211} + H_{152} + 7 H_{252} + 6 H_{252} + 12 H_{210} + 6 H_{200} + 3 H_{411} + H_{0052} \\ &+ \frac{9}{2} H_{100} + \frac{35}{8} H_{100} + 2 H_4 + 3 H_{110} + H_{12} + 16 C_A^2 C_F x^2 \frac{2105}{81} \frac{77}{18} H_{00} \\ &+ 6 H_3 + \frac{16}{3} \zeta_3 + 10 H_{10} + \frac{14}{3} H_{20} + \frac{2}{3} H_{152} + \frac{14}{3} H_{000} + \frac{104}{9} H_2 + \frac{4}{3} H_{100} + \frac{37}{9} H_{11} \\ &+ \frac{4}{3} H_{110} + \frac{104}{9} \zeta_2 + \frac{8}{3} H_{21} + \frac{145}{18} H_{10} + \frac{4}{3} H_{12} + \frac{2}{3} H_{111} + \frac{109}{27} H_1 + \frac{8}{3} H_{100} + 6 H_{052} \\ &+ 4 H_{20} + \frac{584}{27} H_0 + p_{qg} x \frac{7}{2} H_{152} + \frac{138305}{2592} \frac{1}{3} H_{20} + \frac{13}{4} H_{152} + 2 H_{211} + \frac{11}{2} H_{100} \\ &+ 4 H_{31} + \frac{43}{6} H_{111} + \frac{109}{12} \zeta_2 + \frac{17}{2} H_{21} + \frac{71}{24} H_{10} + \frac{11}{6} H_{20} + 21 \zeta_3 + \frac{3}{2} H_{1000} + H_{210} \\ &+ \frac{395}{54} H_{00} + 2 H_{1052} + H_{1152} + \frac{55}{12} H_{110} + 2 H_{1100} + 4 H_{1110} + 2 H_{1111} + 4 H_{112} + \frac{55}{12} H_{12} \\ &+ 6 H_{120} + 4 H_{121} + 4 H_{13} + 3 H_{210} + 3 H_{22} + p_{qg} x \frac{23}{2} H_1 \zeta_3 + 5 H_{252} + 2 H_{210} \\ &+ \frac{109}{12} H_{10} + H_{052} + \frac{17}{5} \zeta_2^2 + \frac{1}{6} H_{152} + 2 H_{252} + \frac{65}{24} H_{11} + \frac{19}{2} H_{110} + 4 H_{30} + 3 H_{200} \\ &+ 7 H_{200} + \frac{3}{2} H_{12} + \frac{3379}{216} H_1 + 4 H_{22} + \frac{49}{6} H_{100} + \frac{1}{2} H_{1000} + 13 H_{11} + \zeta_2 + 8 H_{13} \\ &+ 6 H_{110} + \frac{1}{2} H_{10} + 12 H_{1100} + 10 H_{112} + 10 H_{1052} + 5 H_{120} + 2 H_{120} + 2 H_{121} \\ &+ \frac{11}{6} H_{052} + 1 x \frac{41699}{2592} + 3 H_{210} + \frac{3}{2} H_{252} + \frac{128}{9} \zeta_2 + 4 H_{30} + \frac{26}{3} \zeta_3 + \frac{5}{2} H_{200} \\ &+ 7 H_{152} + \frac{97}{12} H_{100} + \frac{10}{3} H_{100} + \frac{12}{3} H_{100} + \frac{84}{3} H_{0000} + 1 x H_{411} + H_{211} + \frac{29}{6} H_{12} \\ &+ \frac{1}{2} H_{20} + 12 H_{20} + \frac{31}{12} H_{211} + \frac{1}{2} H_{200} + H_{252} + \frac{61}{36} H_{10} + 4 H_{052} + \frac{13}{3} H_{10} + \frac{4}{3} H_{110} \\ &+ \frac{25}{4} H_4 + \frac{93}{4} H_{052} + \frac{55}{9} H_{11} + \frac{71}{18} H_{10} + \frac{49}{18} H_{00} + \frac{1}{2} H_{0052} + \frac{47}{40} \zeta_2^2 + \frac{6131}{40} \frac{31}{2} H_{10} + \frac{2}{3} \zeta_2 \\ &+ 15 H_{210} + \frac{9}{2} H_{100} + 3 H_{211} + \frac{9}{4} H_{21} + \frac{53}{3} H_{20} + \frac{1}{2} H_{200} + 5 H_{20} + \frac{7}{6} H_{111} + 8 H_{052} \\ &+ \frac{67}{40} \zeta_2^2 + \frac{29}{6} H_{12} + H_{10} + 8 H_{22} + 25 H_{052} + \frac{412}{9} H_1 + \frac{928}{9} H_0 + \frac{1}{4} H_4 + 65 H_3 + 38 H_{00} \\ &+ 9 H_{10} + \frac{3}{2} H_{0000} + x \frac{27}{2} H_{10} + \frac{1}{2} H_{0000} + \frac{3}{4} H_{0052} + \frac{1}{2} H_{30} + 14 H_{000} + \frac{1}{12} H_{111} \\ &+ \frac{43}{36} \zeta_2 + \frac{1}{2} H_{252} + \frac{7}{72} H_0 + \frac{749}{54} H_1 + \frac{135}{4} \zeta_3 + \frac{97}{24} H_{10} + \frac{43}{12} H_{052} + \frac{84}{12} H_{152} + \frac{13}{3} H_{100} \end{aligned}$$

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$$\begin{aligned} &+ \frac{385}{72} H_{10} + \frac{31}{2} H_{11} + \frac{113}{12} H_1 + \frac{49}{4} H_{20} + \frac{5}{2} H_{152} + \frac{79}{4} H_{000} + \frac{173}{12} H_3 + \frac{1259}{32} \frac{2833}{216} H_0 \\ &+ 6 H_{21} + 3 H_{210} + 9 H_{1052} + 6 H_{1152} + H_{1100} + 3 H_{1110} + 4 H_{1111} + 6 H_{1121} \\ &+ 6 H_{13} + \frac{49}{4} \zeta_2 + p_{qg} x \frac{17}{2} H_{152} + \frac{5}{2} H_{110} + \frac{5}{2} H_{12} + \frac{9}{2} H_{10} + \frac{5}{2} H_{20} + \frac{3}{2} H_{121} \\ &+ 2 H_{31} + 2 H_4 + 6 H_{22} + 6 H_{210} + 6 H_{200} + 2 H_{0052} + 9 H_{252} + 3 H_{120} + 2 H_{121} \\ &+ 6 H_{1110} + 6 H_{1100} + 6 H_{1112} + 12 H_{11052} + 9 H_{1152} + 2 H_{120} + \frac{11}{2} H_{1000} \\ &+ 6 H_{13} + \frac{1}{2} x^2 \frac{55}{12} \zeta_3 + \frac{23}{9} H_{10} + \frac{4}{3} H_{110} + \frac{1}{2} x^2 \frac{2}{3} H_{100} + \frac{371}{108} H_{10} + \frac{23}{9} H_{11} \\ &+ \frac{2}{3} H_{111} + 1 x + 6 H_{210} + 3 H_{211} + \frac{5}{6} H_{111} + 7 H_{200} + 2 H_{12} + 39 H_{052} + 4 H_{252} + \frac{16}{3} \zeta_3 \\ &+ H_{110} + \frac{154}{3} H_{052} + \frac{899}{24} H_{00} + \frac{121}{10} \zeta_2^2 + \frac{607}{36} H_2 + \frac{5}{2} H_{152} + \frac{65}{6} H_{1000} + \frac{29}{12} H_{10} + \frac{13}{18} H_{11} \\ &+ 1189 H_1 + \frac{67}{108} H_{12} + 29 H_{20} + \frac{949}{36} \zeta_2 + \frac{67}{2} H_{000} + \frac{142}{3} H_3 + \frac{215}{32} \frac{3989}{48} H_0 + 2 H_{30} \\ &+ 1 x + H_{100} + 10 H_{252} + 6 H_{200} + 2 H_{0052} + 9 H_{110} + 7 H_{12} + 9 H_{20} + 2 H_{31} \\ &+ 4 H_{210} + 4 H_4 + 4 H_{30} + 4 H_{0000} + \frac{37}{2} H_{10} + \frac{5}{2} 1 x + H_{152} + 4 H_{200} + 2 H_{0052} \\ &+ H_{252} + 3 H_{110} + 2 H_{0000} + H_{30} + 9 H_{210} + \frac{9}{2} H_{211} + \frac{11}{8} H_{111} + \frac{19}{2} H_{200} + \frac{9}{2} H_{12} \\ &+ \frac{91}{2} H_{052} + 8 H_{252} + \frac{5}{2} H_{110} + \frac{5}{2} H_{12} + \frac{9}{2} H_{10} + \frac{39}{2} H_{20} + \frac{473}{12} H_{052} + \frac{1853}{48} H_{00} \\ &+ 217 \zeta_3 + \frac{59}{4} \zeta_2^2 + \frac{169}{18} H_2 + \frac{13}{4} H_{152} + \frac{2}{3} H_{100} + \frac{167}{24} H_{10} + \frac{191}{18} H_{11} + \frac{1283}{108} H_1 + \frac{185}{12} H_{12} \\ &+ \frac{75}{4} H_{20} + \frac{170}{9} \zeta_2 + 4 H_{000} + \frac{12}{5} H_{052} + \frac{7693}{192} \frac{3659}{48} H_0 + 2 x H_{22} + 4 H_{30} + 4 H_{22} \\ &+ 16 C_A n_f^2 \frac{1}{2} p_{qg} x H_{12} + H_{152} + H_{100} + H_{110} + H_{111} + \frac{229}{3} H_{20} + \frac{11}{2} x^2 \frac{1}{6} H_{20} \\ &+ \frac{53}{18} H_{00} + \frac{17}{18} H_{00} + \zeta_3 + \frac{11}{2} \zeta_2 + \frac{139}{108} \frac{1}{3} p_{qg} x + H_{100} + \frac{53}{3} \frac{1}{2} x^2 \frac{2}{9} 1 x + 6 H_{000} \\ &+ 2 x H_1 + H_{00} + 2 x H_{11} + \frac{7}{9} 1 x + H_{10} + \frac{4}{2} H_0 + \frac{19}{54} H_1 + H_{000} + \frac{5}{9} H_{10} + \frac{2}{2} 1 x + 6 H_{000} \\ &+ \frac{85}{216} 16 C_A^2 n_f p_{qg} x + 3 H_{13} + \frac{31}{6} H_{10} + \frac{17}{2} H_{21} + \frac{7}{5} \zeta_2^2 + \frac{55}{12} H_{10} + \frac{31}{12} H_1 + \frac{31}{12} H_{152} \\ &+ \frac{5}{12} H_{20} + \frac{63}{8} H_{10} + \frac{23}{12} H_{12} + \frac{155}{6} \zeta_2 + \frac{25}{24} H_2 + \frac{2537}{27} H_0 + \frac{867}{8} \frac{22}{2} H_{100} + 3 H_4 + H_{111} \\ &+ 383 H_{11} + \frac{25}{72} H_{20} + \frac{2}{8} \zeta_2 + \frac{2}{4} H_{152} + 3 H_{0052} + \frac{12}{12} H_{052} + \frac{103}{216} H_1 + \frac{2}{2} H_{1000} + \frac{2561}{72} H_{00} \\ &+ H_{111} + 2 H_{200} + 3 H_{210} + 5 H_{1052} + 3 H_{000} + H_{1152} + H_{1100} + 4 H_{1110} + 2 H_{1111} \\ &+ 2 H_{112} + 2 H_{210} + p_{qg} x + H_{1152} + 2 H_{12} + 6 H_{110} + H_{111} + 2 H_{152} + H_{200} \\ &+ \frac{727}{36} H_{10} + H_{152} + 2 H_{22} + \frac{5}{2} H_{152} + H_{120} + 2 H_{1100} + 2 H_{112} + \frac{2}{2} H_{1000} \end{aligned}$$

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Finally the Mellin inversion of Eq. (3.13) yields the NNLO gluon-gluon splitting function

$$\begin{aligned} P_{gg}^{(2)} x &= 16 C_A C_F n_f x^2 \frac{9}{2} H_2 + 3 H_{10} + \frac{97}{12} H_3 + \frac{8}{3} H_{20} + \frac{2}{3} H_{052} + \frac{103}{27} H_0 + \frac{16}{3} \zeta_2 + 2 H_3 \\ &+ 6 H_{10} + 2 H_{20} + \frac{127}{18} H_{00} + \frac{511}{12} p_{gg} x + 2 \zeta_3 + \frac{55}{24} 4 x^2 + \frac{17}{24} \frac{43}{18} H_0 \\ &+ \frac{521}{144} H_1 + \frac{6923}{432} \frac{1}{2} H_{21} + 2 H_{152} + H_{052} + 2 H_{100} + \frac{1}{12} H_{11} + H_{110} + H_{111} + \frac{175}{12} H_{12} \\ &+ 6 H_{10} + 8 H_{052} + 6 H_{20} + \frac{55}{6} H_{052} + \frac{49}{2} H_2 + \frac{185}{4} \zeta_2 + \frac{511}{12} \frac{1}{2} H_{20} + 3 H_{10} + 4 H_{0000} \end{aligned}$$

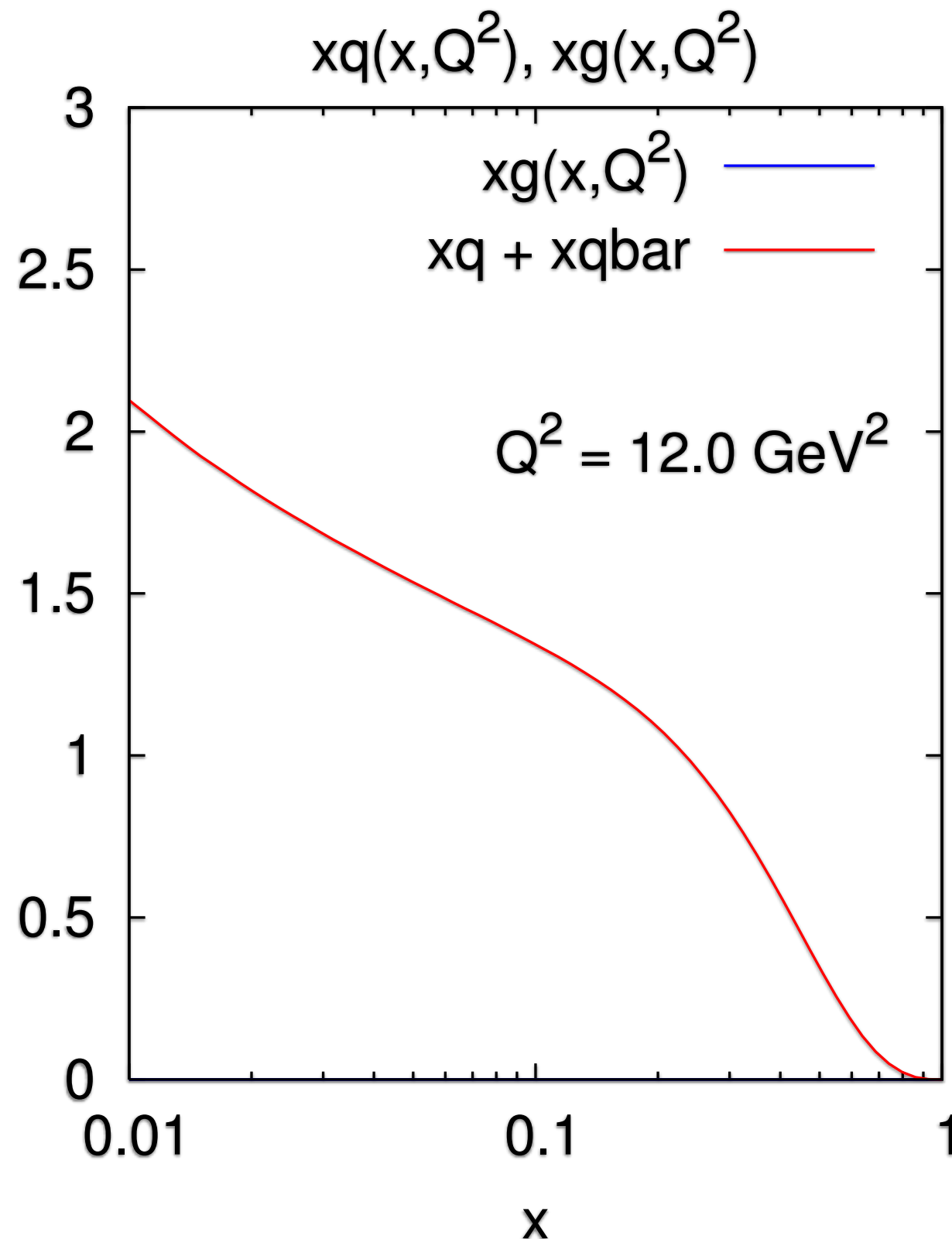
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$$\begin{aligned} &+ 6 H_{1110} + 2 H_{1120} + \frac{1}{2} x^2 \frac{2}{3} H_{21} + \frac{32}{9} \zeta_2 + 2 H_{100} + \frac{4}{3} H_{110} + \frac{10}{9} H_{11} \\ &+ \frac{8}{3} H_{100} + \frac{3}{2} H_{10} + 6 \zeta_3 + \frac{161}{36} H_1 + \frac{2351}{108} \frac{2}{3} x^2 \frac{26}{3} H_{10} + \frac{28}{9} H_{00} + 2 H_{110} \\ &+ 2 H_{12} + H_{152} + H_{152} + \frac{10}{3} H_2 + H_{111} + 1 x + 15 H_{0000} + 5 H_{252} + \frac{65}{6} \zeta_3 + \frac{11}{6} H_{111} \\ &+ \frac{3}{2} H_4 + \frac{5}{2} H_{0052} + H_{110} + \frac{31}{6} H_{20} + \frac{17}{12} H_{10} + \frac{551}{20} \zeta_2^2 + \frac{29}{4} H_{100} + \frac{113}{4} H_2 + \frac{18691}{72} H_0 \\ &+ \frac{2243}{108} \frac{265}{6} H_{100} + \frac{33}{2} H_{200} + 19 H_{21} + \frac{31}{12} H_{11} + \frac{23}{2} H_{20} + \frac{497}{36} \zeta_2 + \frac{29}{6} H_{152} + \frac{143}{12} H_3 \\ &+ \frac{11}{6} H_{111} + \frac{19}{12} H_{052} + \frac{1223}{72} H_1 + \frac{43}{6} H_{000} + \frac{3011}{36} H_{00} + 1 x + 8 H_{210} + 4 H_{12} \\ &+ 7 H_{110} + \frac{35}{6} H_{111} + 5 H_{252} + 11 H_{200} + \frac{1}{3} H_{10} + \frac{15}{2} H_{152} + 8 H_{31} + 10 H_{210} \\ &+ 5 H_{252} + 4 H_{211} + H_{30} + 36 H_{052} + 5 H_{252} + 2 H_{12} + 6 H_{110} + 6 H_{210} + 3 H_{211} \\ &+ 11 H_{0000} + 5 H_{31} + \frac{25}{4} H_{111} + \frac{13}{2} H_{252} + \frac{27}{2} H_{200} + \frac{11}{2} H_{30} + \frac{13}{2} H_{252} + \frac{17}{4} H_{100} \\ &+ 13 H_{210} + \frac{17}{12} H_{111} + \frac{3}{4} H_4 + \frac{1}{2} H_{0052} + H_{12} + \frac{11}{2} H_{110} + \frac{79}{12} H_{20} + \frac{67}{8} H_{10} + \frac{263}{8} \zeta_2^2 \\ &+ \frac{119}{3} \zeta_3 + \frac{967}{24} H_{12} + \frac{305}{12} H_{10} + 24 H_{052} + H_{152} + \frac{13375}{72} H_0 + \frac{1889}{18} \frac{38 H_{100}}{100} + \frac{21}{2} H_{21} \\ &+ \frac{79}{4} H_{200} + \frac{217}{24} H_{11} + \frac{7}{2} H_{20} + \frac{79}{72} \zeta_2 + \frac{4}{3} H_{152} + \frac{17}{12} H_{111} + \frac{17}{12} H_{052} + \frac{31}{18} H_1 + 3 H_{000} \\ &+ 145 H_3 + \frac{1553}{24} H_{00} + 16 C_F n_f^2 \frac{7}{6} H_{000} + \frac{11}{36} \frac{739}{96} \frac{163}{24} H_0 + \frac{7}{24} H_{00} + 2 H_{0000} \\ &+ \frac{5}{9} H_{11} + \frac{5}{9} H_2 + 18 H_{10} + \frac{5}{6} \zeta_2 + \frac{1}{6} p_{gg} x + H_{21} + \frac{91}{35} \frac{35}{3} H_0 + \frac{22}{3} H_{00} + H_{111} + 6 H_{000} \\ &+ \zeta_3 + 2 H_{100} + \frac{77}{81} x^2 + 1 x + \frac{1}{12} H_1 + \frac{6463}{432} \frac{16}{4000} H_{000} + \frac{16}{3} H_{000} + \frac{7}{5} H_{11} \\ &+ \frac{7}{9} H_2 + \frac{8}{9} H_{10} + \frac{7}{9} \zeta_2 + 1 x + \frac{3475}{216} H_0 + \frac{103}{12} H_{00} + 16 C_F^2 n_f p_{gg} x + 7 H_{13} + 7 H_4 \\ &+ 2 H_{30} + 7 H_{152} + 5 H_{22} + 6 H_{30} + 6 H_{31} + H_{210} + 4 H_{200} + 3 H_{21} + 2 H_{211} + \frac{5}{2} H_{20} \\ &+ \frac{61}{8} H_2 + \frac{61}{8} \zeta_2 + \frac{87}{8} H_1 + \frac{11}{2} H_{12} + \frac{61}{8} H_1 + \frac{17}{2} H_{10} + 7 H_{0052} + \frac{5}{2} H_{100} + \frac{5}{2} H_{110} + \frac{19}{2} \zeta_3 \\ &+ \frac{81}{8} \frac{11}{12} H_3 + \frac{11}{2} H_{052} + \frac{7}{2} H_{152} + \frac{15}{2} H_{000} + \frac{87}{54} H_0 + \frac{11}{5} \zeta_2^2 + 3 H_{111} + 5 H_{252} + 7 H_{052} \\ &+ 11 H_{00} + 2 H_{10} + 7 H_{1052} + 3 H_{1000} + 5 H_{152} + 4 H_{1100} + H_{1110} + 2 H_{1111} + 5 H_{112} \\ &+ 6 H_{120} + 6 H_{121} + p_{qg} x + H_{0000} + H_{20} + H_{110} + H_{200} + \frac{2}{2} H_{120} + \frac{5}{2} H_{10} \\ &+ \frac{5}{2} H_{100} + \frac{1}{2} H_{30} + \frac{1}{2} H_{152} + H_{1100} + \frac{1}{4} H_{1000} + 2 1 x + H_{210} + H_{200} + H_{22} \\ &+ H_{31} + 2 H_{30} + 2 H_{152} + H_{12} + H_{100} + H_{110} + H_{252} + \zeta_2^2 + \frac{43}{8} H_2 + \frac{49}{8} \zeta_3 + \frac{13}{8} H_{11} \end{aligned}$$

and

$$\begin{aligned} &+ \frac{67}{12} H_{00} + \frac{43}{2} \zeta_3 + H_{21} + \frac{97}{12} H_1 + 4 \zeta_2^2 + \frac{9}{2} H_3 + 8 H_{30} + \frac{33}{2} H_{000} + \frac{4}{3} x^2 \frac{1}{2} H_2 + H_{20} \\ &+ \frac{11}{3} H_{10} + H_{20} + \frac{19}{6} \zeta_2 + 2 \zeta_3 + H_{152} + 4 H_{110} + \frac{1}{2} H_{100} + H_{12} + 1 x + 9 H_{152} \\ &+ 12 H_{0000} + \frac{293}{108} \frac{61}{6} H_{052} + \frac{2}{3} H_{10} + \frac{857}{36} H_1 + 9 H_{052} + 16 H_{210} + 4 H_{200} + 8 H_{252} \\ &+ \frac{13}{2} H_{100} + \frac{2}{3} H_{11} + H_{110} + H_{111} + 1 x + \frac{1}{2} H_{20} + \frac{95}{3} H_{10} + \frac{149}{36} H_2 + \frac{3451}{108} H_0 \\ &+ 7 H_{20} + \frac{302}{9} H_{00} + \frac{19}{6} H_{10} + \frac{991}{36} \zeta_2 + \frac{163}{6} \zeta_3 + \frac{35}{3} H_{000} + \frac{17}{6} H_{21} + \frac{43}{10} \zeta_2^2 + 13 H_{152} \\ &+ 18 H_{110} + H_{31} + 6 H_4 + 4 H_{12} + 6 H_{0052} + 8 H_{252} + 7 H_{200} + 2 H_{210} + 2 H_{211} + 4 H_{30} \\ &+ 9 H_{100} + \frac{241}{288} 1 x + 16 C_F n_f^2 \frac{19}{54} H_0 + \frac{1}{24} H_0 + \frac{1}{27} p_{gg} x + \frac{13}{54} x^2 \frac{5}{3} H_1 \\ &+ 1 x + \frac{11}{72} H_1 + \frac{71}{216} \frac{2}{5} 1 x + \zeta_3 + \frac{13}{12} H_{00} + \frac{1}{2} H_{00} + H_2 + \frac{29}{288} 5 1 x \\ &+ 16 C_F^2 n_f x^2 \zeta_3 + \frac{11}{9} \zeta_2 + \frac{11}{9} H_{00} + \frac{2}{3} H_{152} + \frac{1639}{108} H_0 + 2 H_{20} + \frac{1}{3} p_{gg} x + \frac{10}{3} \zeta_2 \\ &+ \frac{209}{36} 8 \zeta_3 + 2 H_{20} + \frac{2}{3} H_{00} + \frac{10}{3} H_{00} + \frac{20}{3} H_{10} + \frac{20}{3} \zeta_3 + \frac{5443}{108} H_3 + \frac{10}{9} p_{gg} x + \zeta_2 \\ &+ 2 H_{10} + \frac{3}{16} H_{052} + H_{00} + \frac{1}{3} x^2 \frac{1}{2} H_2 + H_{052} + \frac{13}{2} H_2 + \frac{5443}{108} \frac{3 H_{152}}{36} + \frac{205}{36} H_1 \\ &+ \frac{13}{3} H_{10} + H_{100} + \frac{1}{x} x^2 \frac{151}{54} H_0 + \frac{8}{3} \zeta_3 + \frac{1}{3} H_{152} + \zeta_3 + 2 H_{110} + \frac{2}{3} H_{100} \\ &+ \frac{37}{9} H_{10} + \frac{3}{2} H_{12} + 1 x + \frac{5}{6} H_{20} + H_{30} + 2 H_{000} + \frac{269$$

DGLAP evolution (initial quarks only)

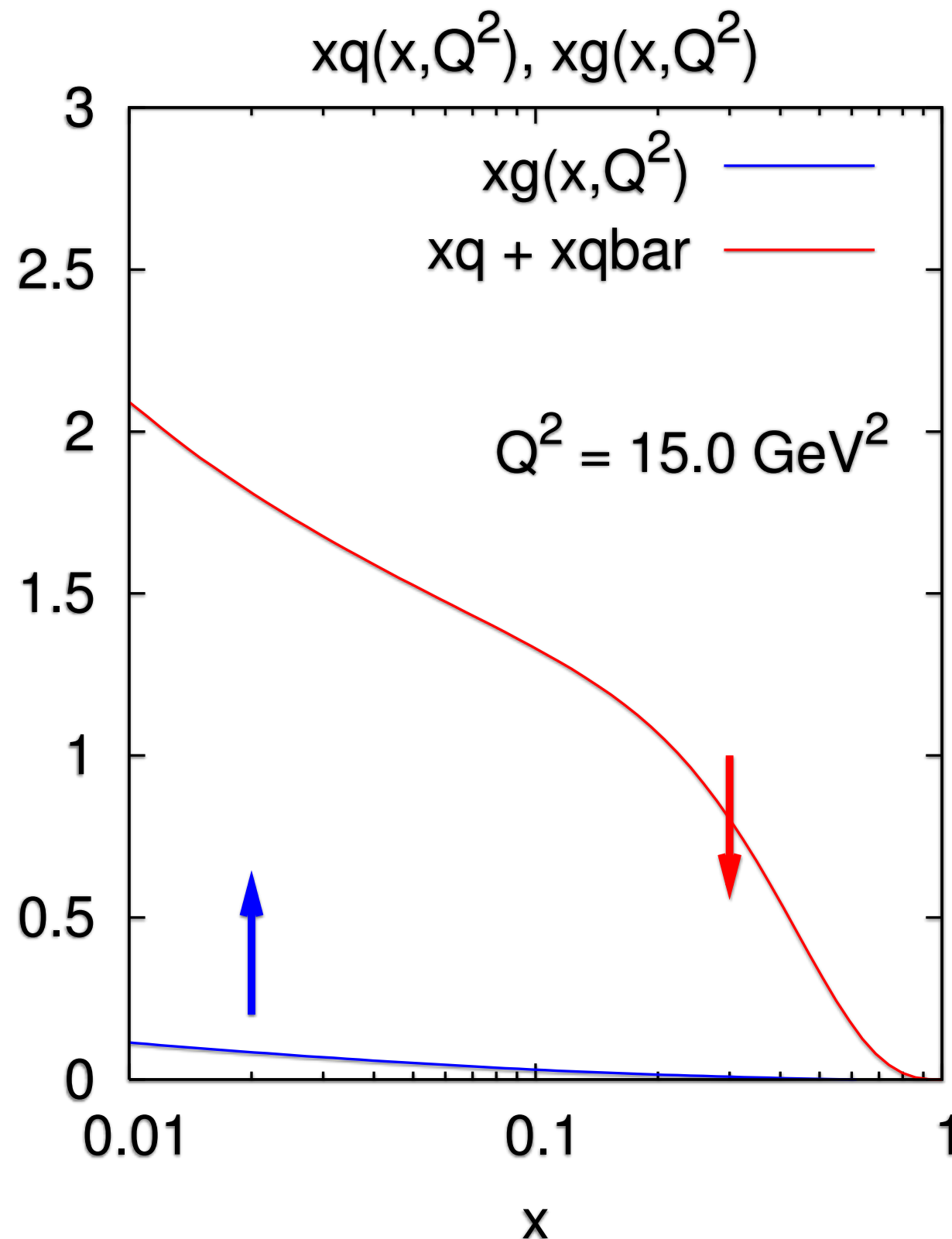


Take example evolution starting with just quarks:

$$\begin{aligned}\partial_{\ln Q^2} q &= P_{q \leftarrow q} \otimes q \\ \partial_{\ln Q^2} g &= P_{g \leftarrow q} \otimes q\end{aligned}$$

- ▶ quark is depleted at large x
- ▶ gluon grows at small x

DGLAP evolution (initial quarks only)

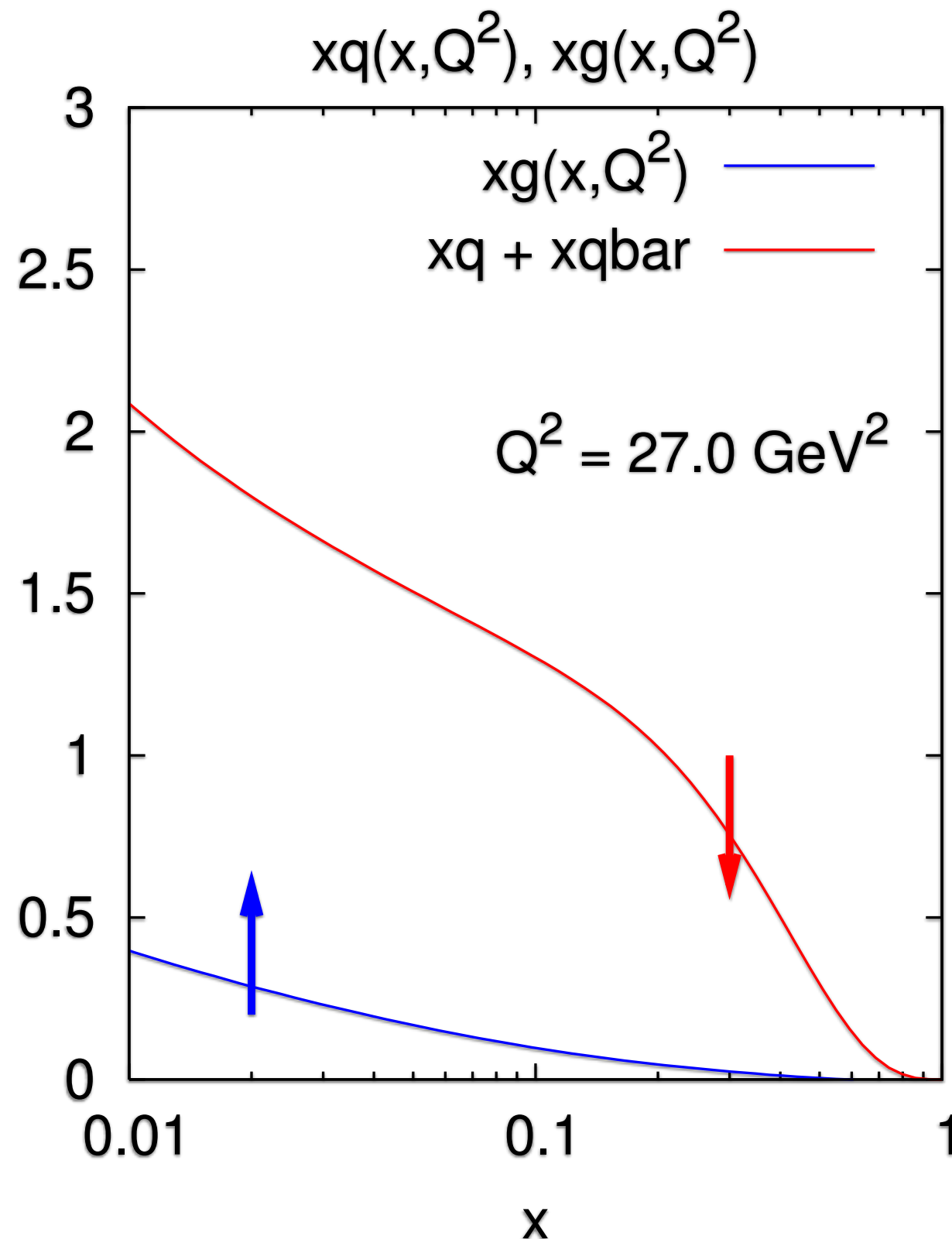


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DGLAP evolution (initial quarks only)

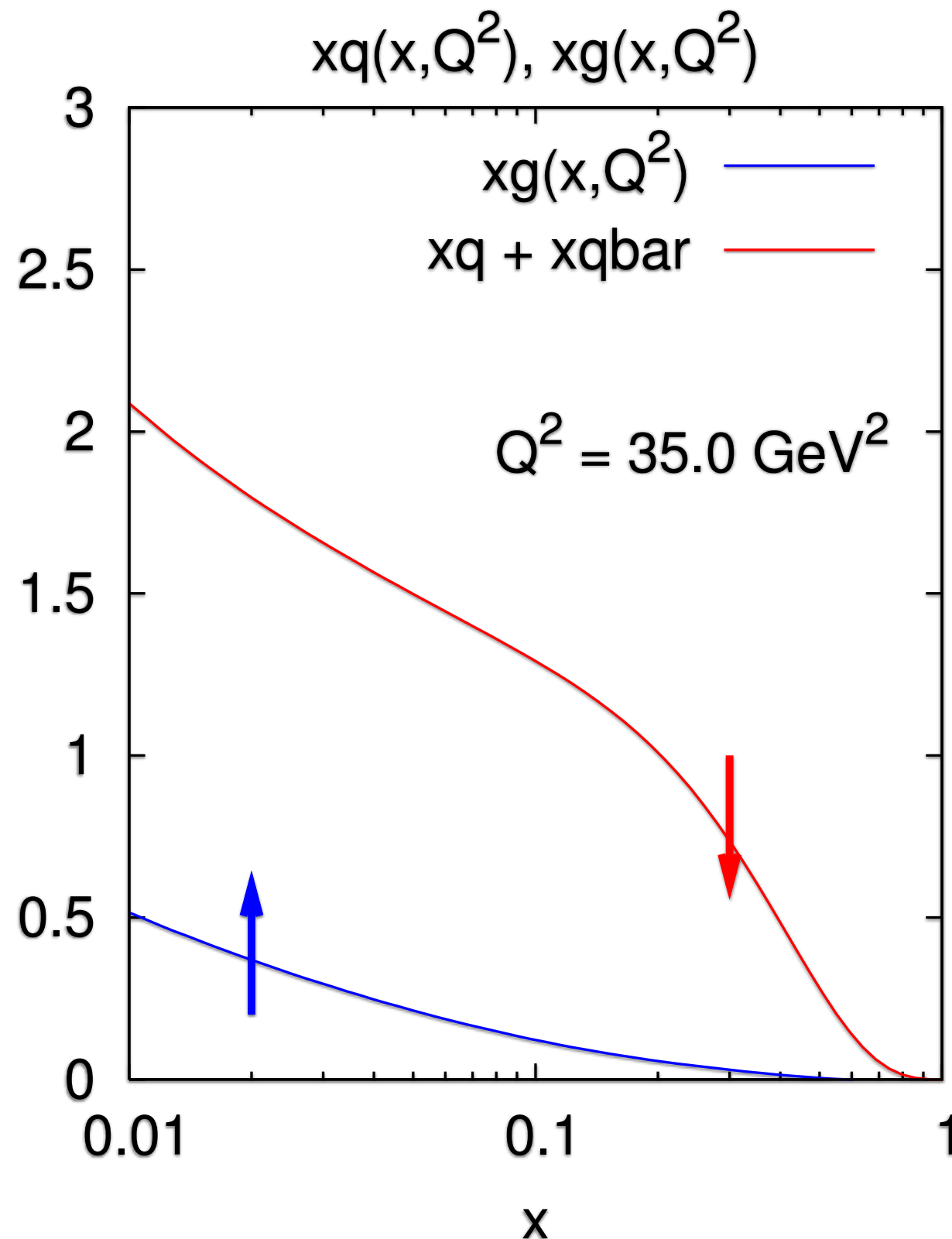


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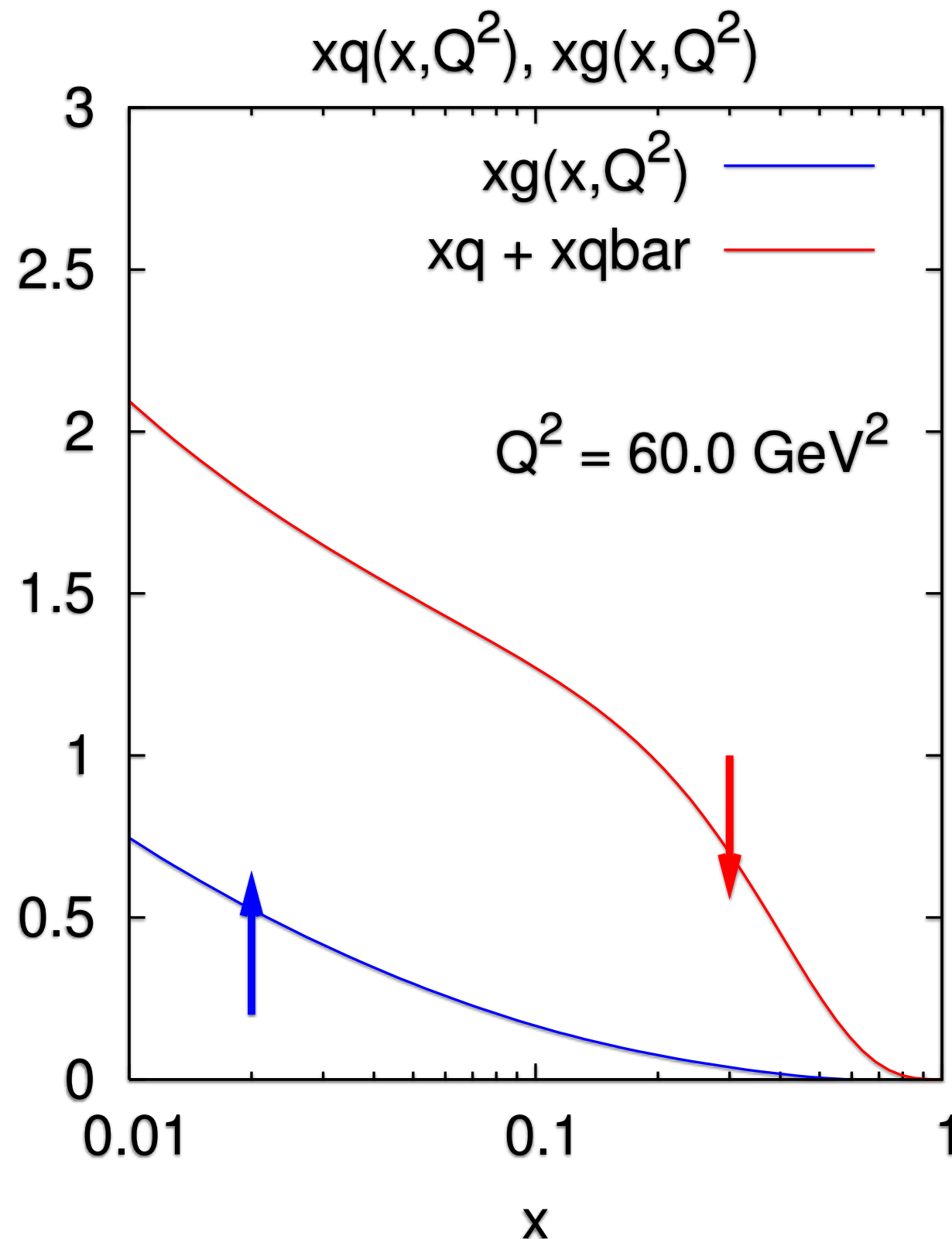


Take example evolution starting with just quarks:

$$\begin{aligned}\partial_{\ln Q^2} q &= P_{q \leftarrow q} \otimes q \\ \partial_{\ln Q^2} g &= P_{g \leftarrow q} \otimes q\end{aligned}$$

- ▶ quark is depleted at large x
- ▶ gluon grows at small x

DGLAP evolution (initial quarks only)

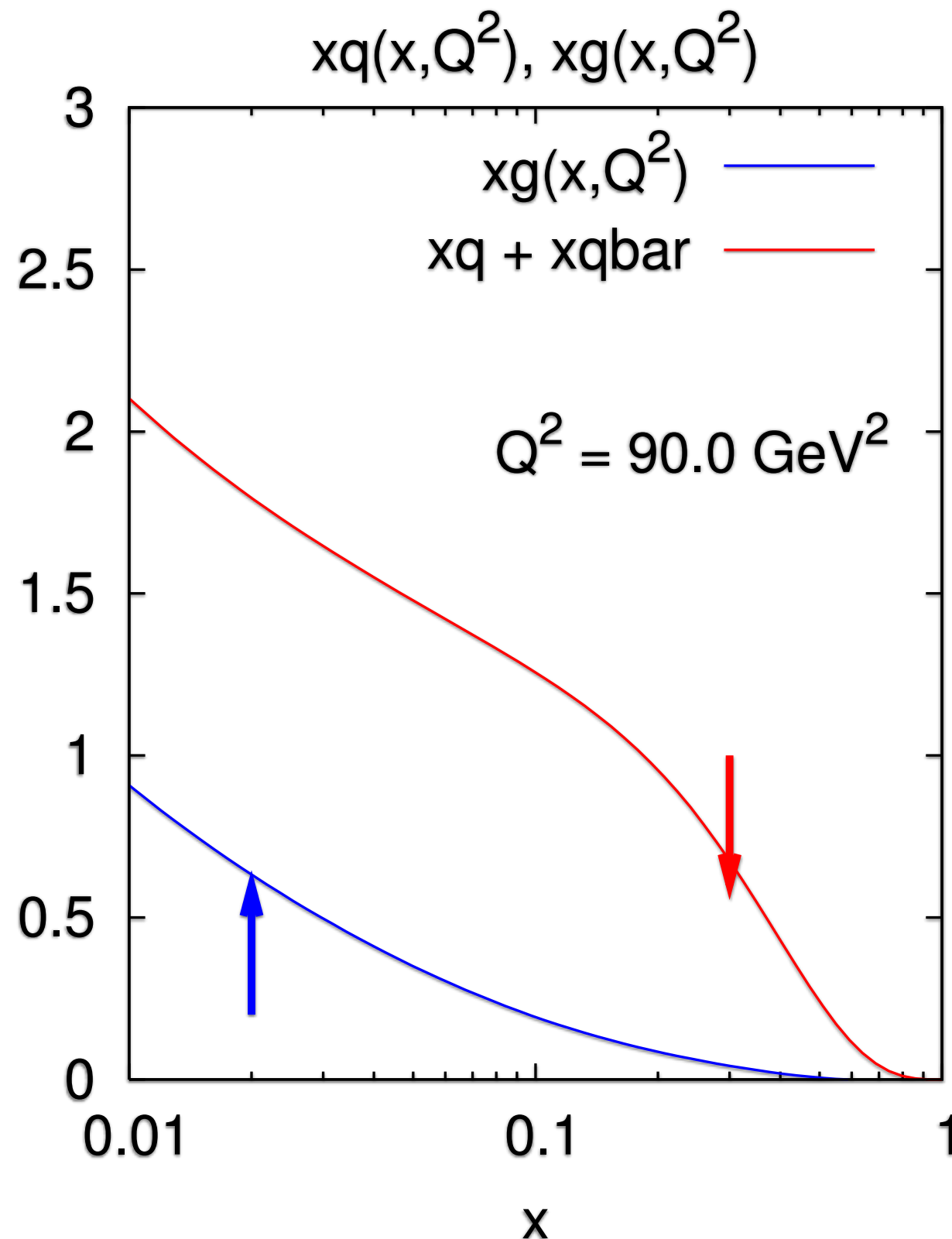


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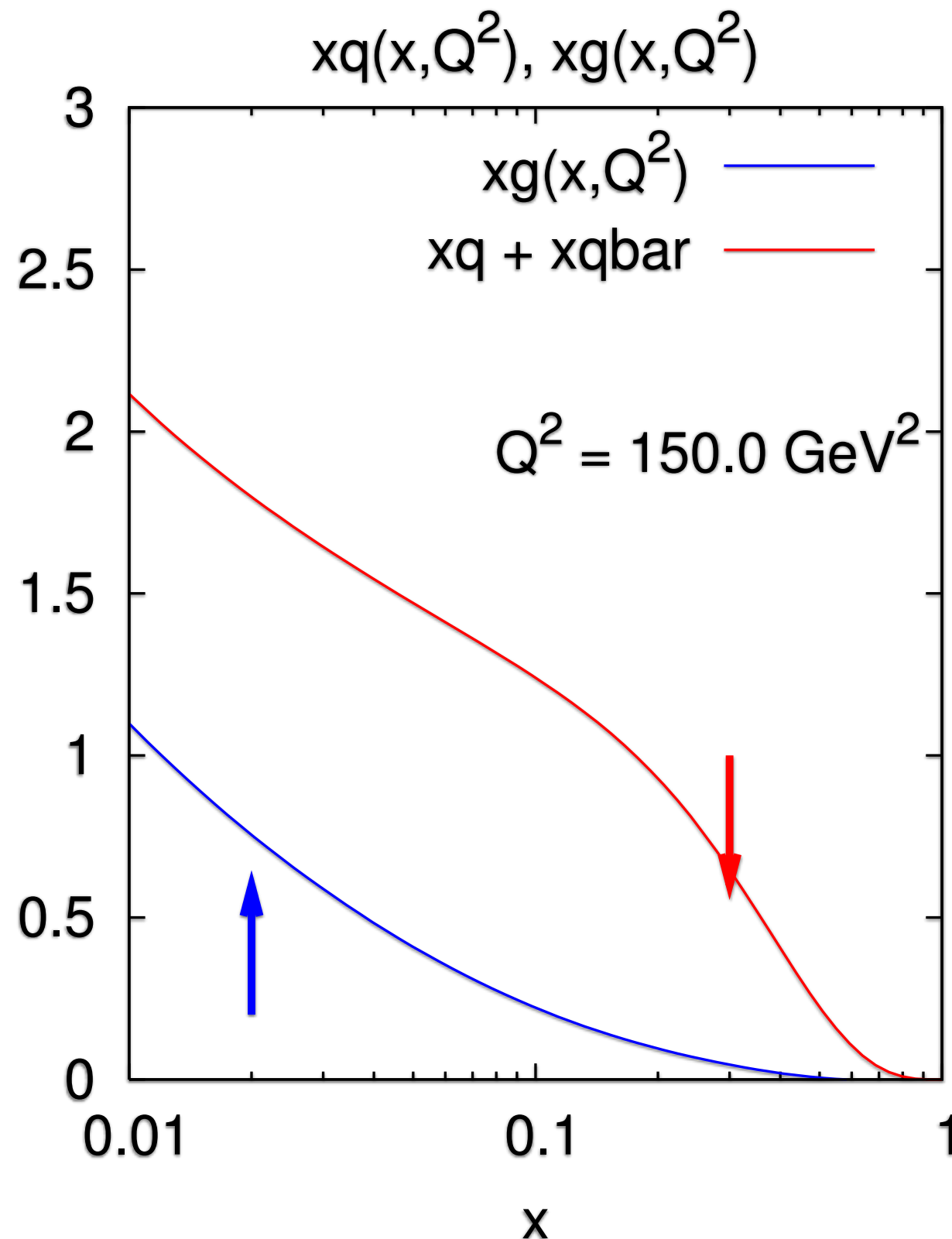


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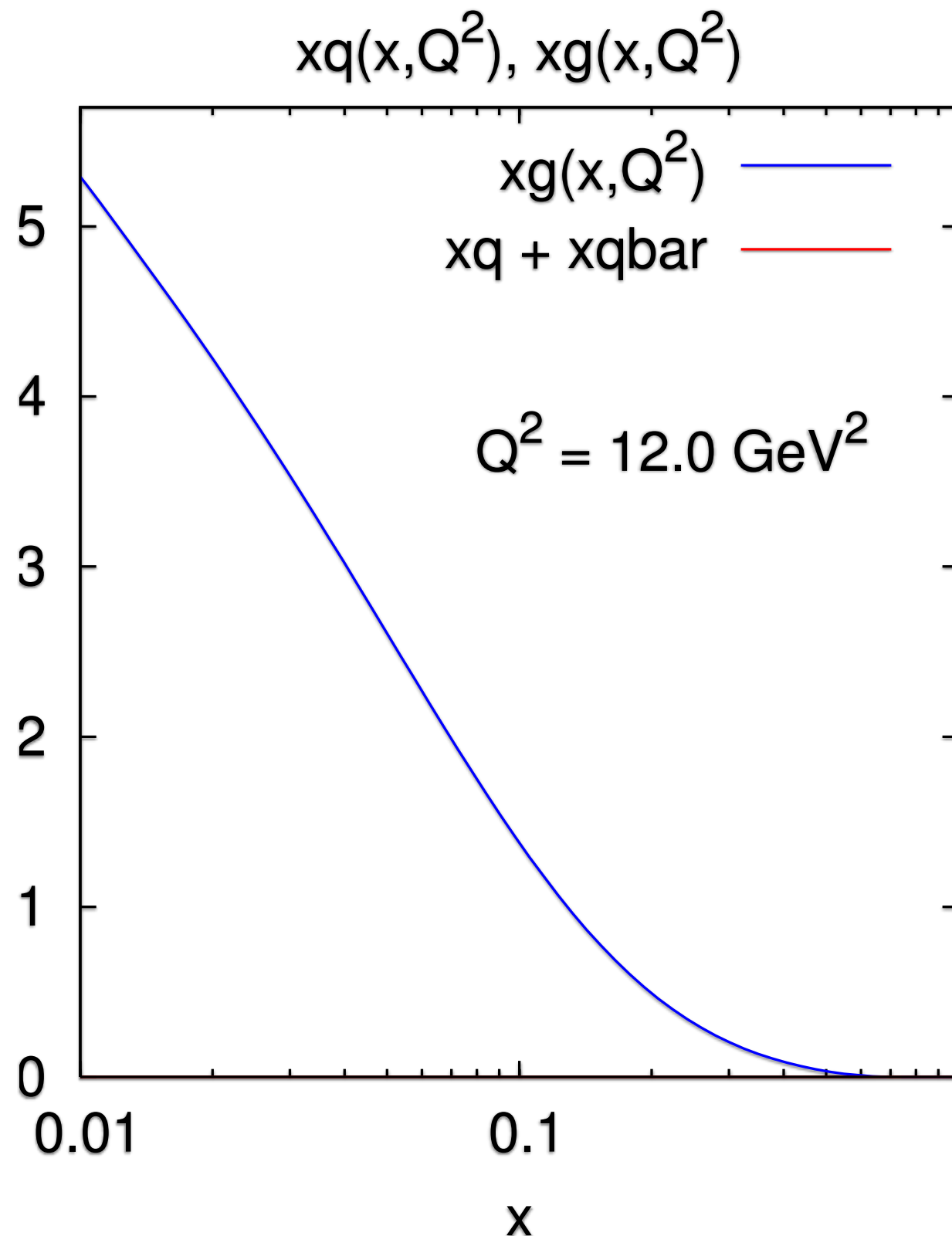


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DGLAP evolution (initial gluons only)



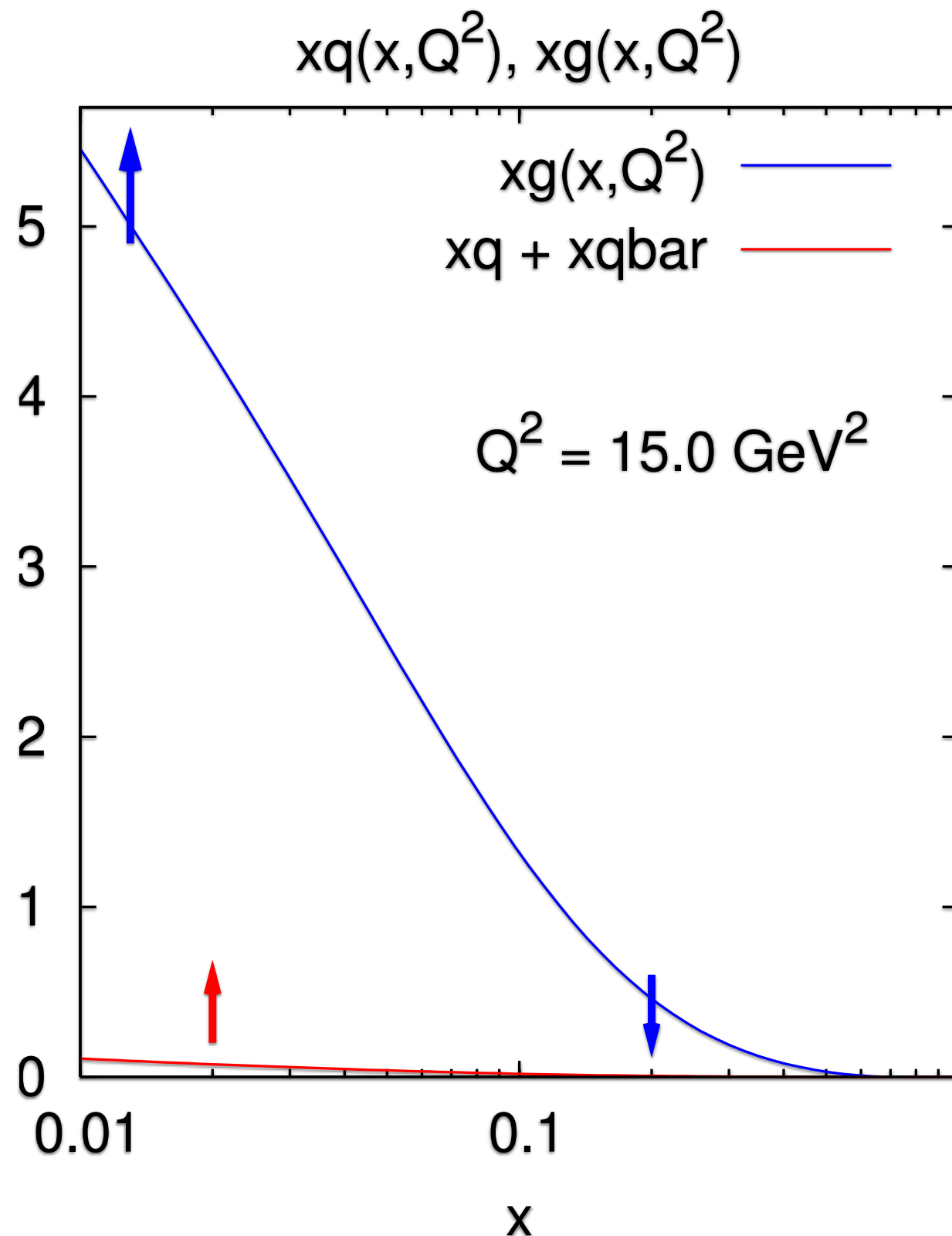
2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$

$$\partial_{\ln Q^2} g = P_{g \leftarrow g} \otimes g$$

- ▶ gluon is depleted at large x .
- ▶ high- x gluon feeds growth of small x gluon & quark.

DGLAP evolution (initial gluons only)



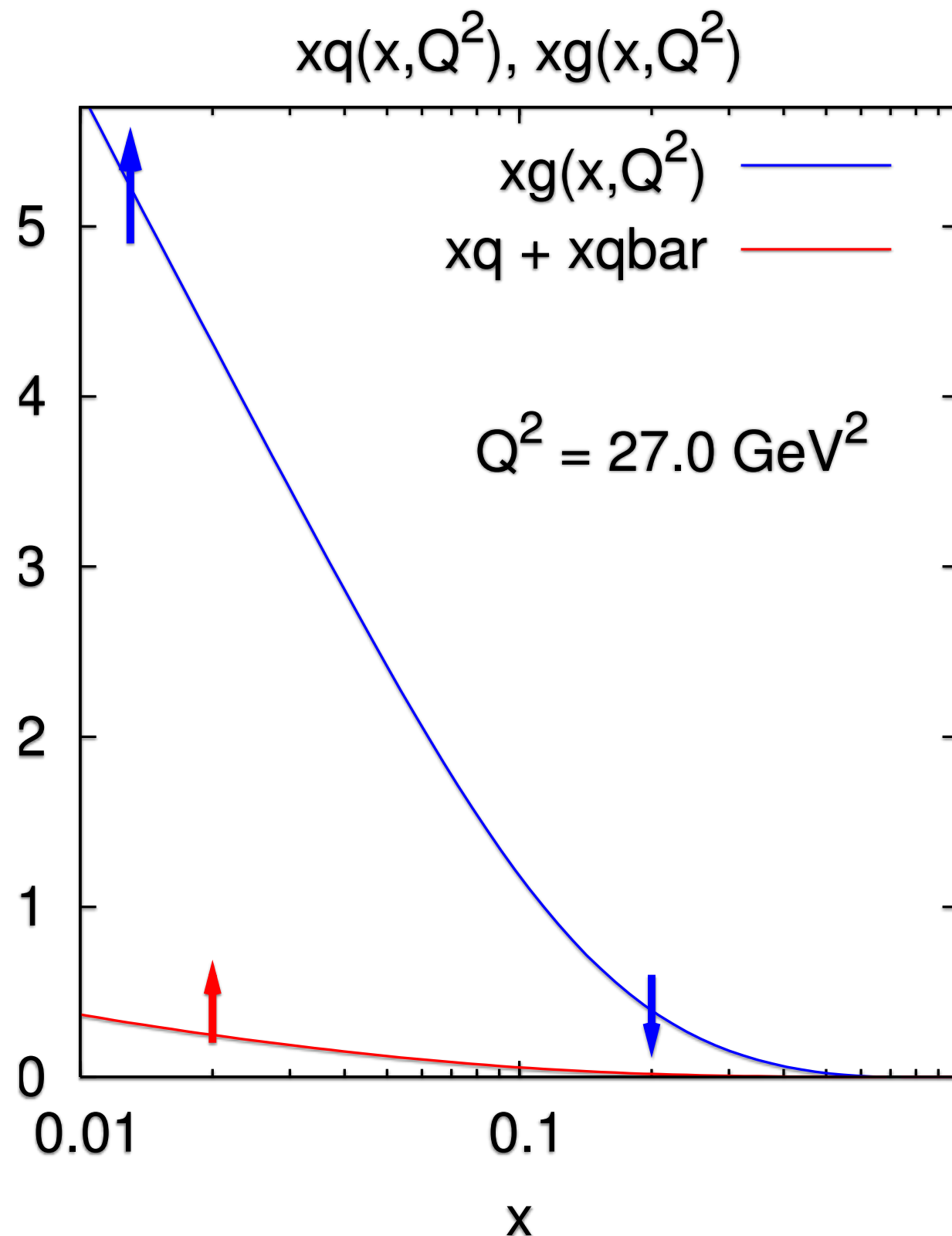
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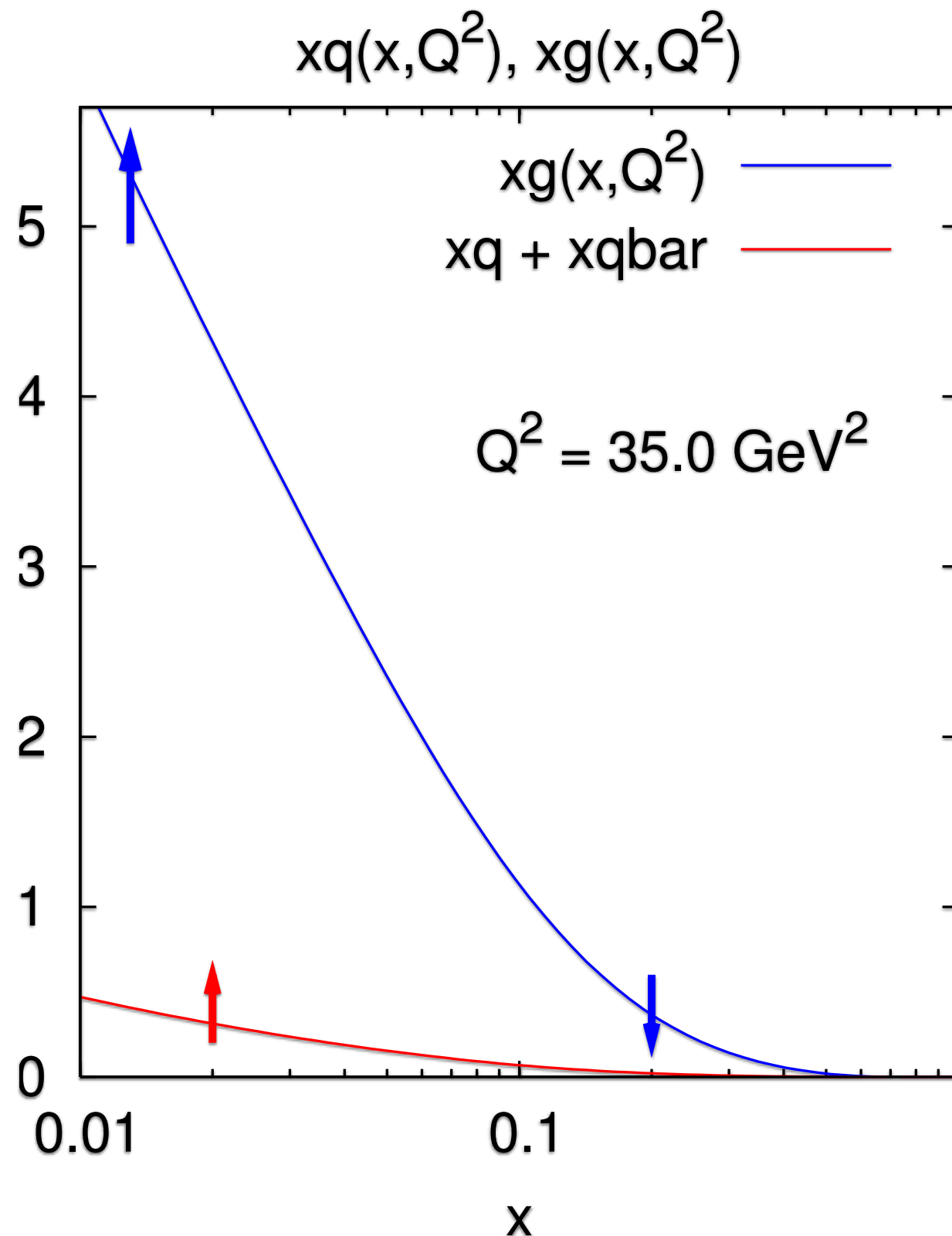
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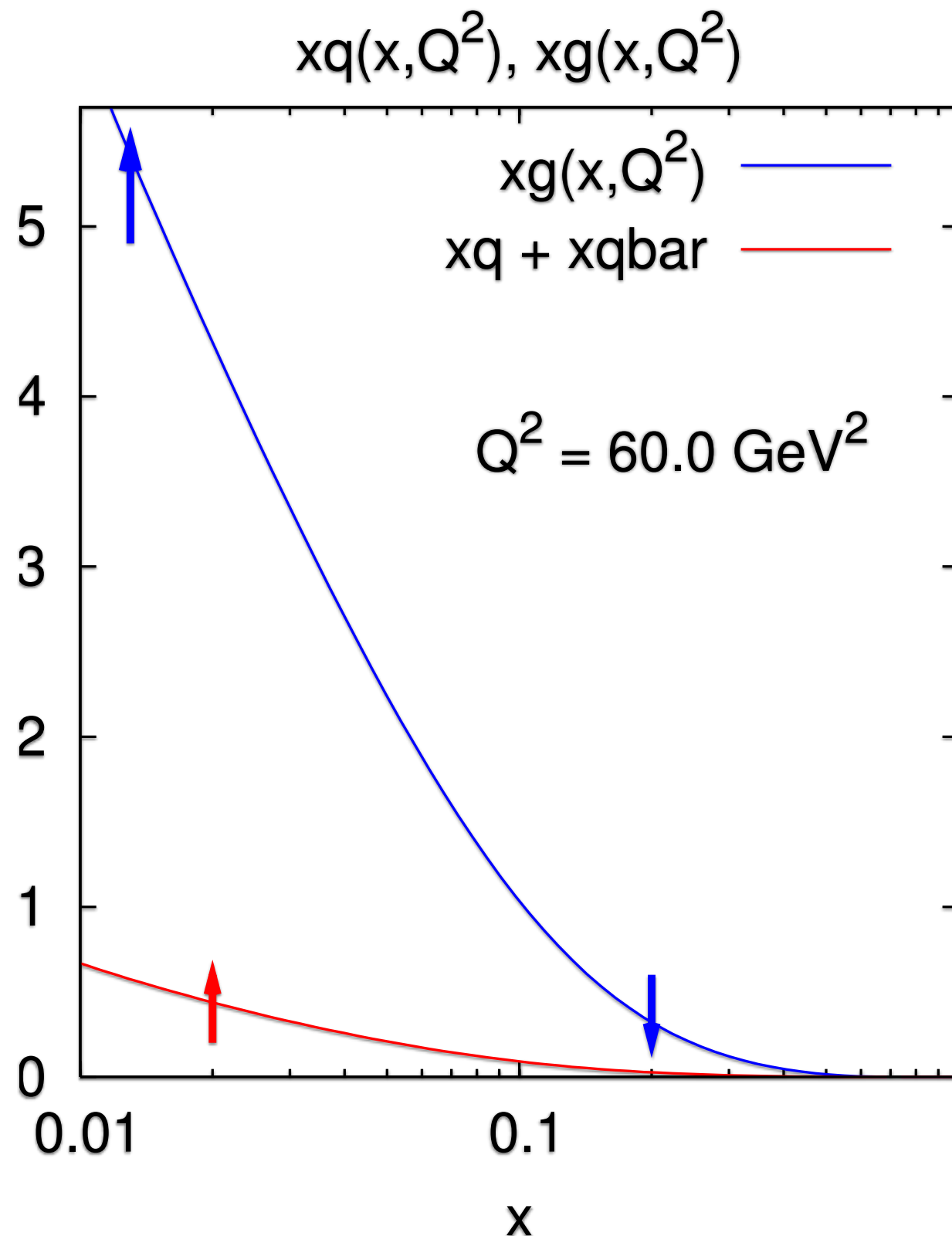
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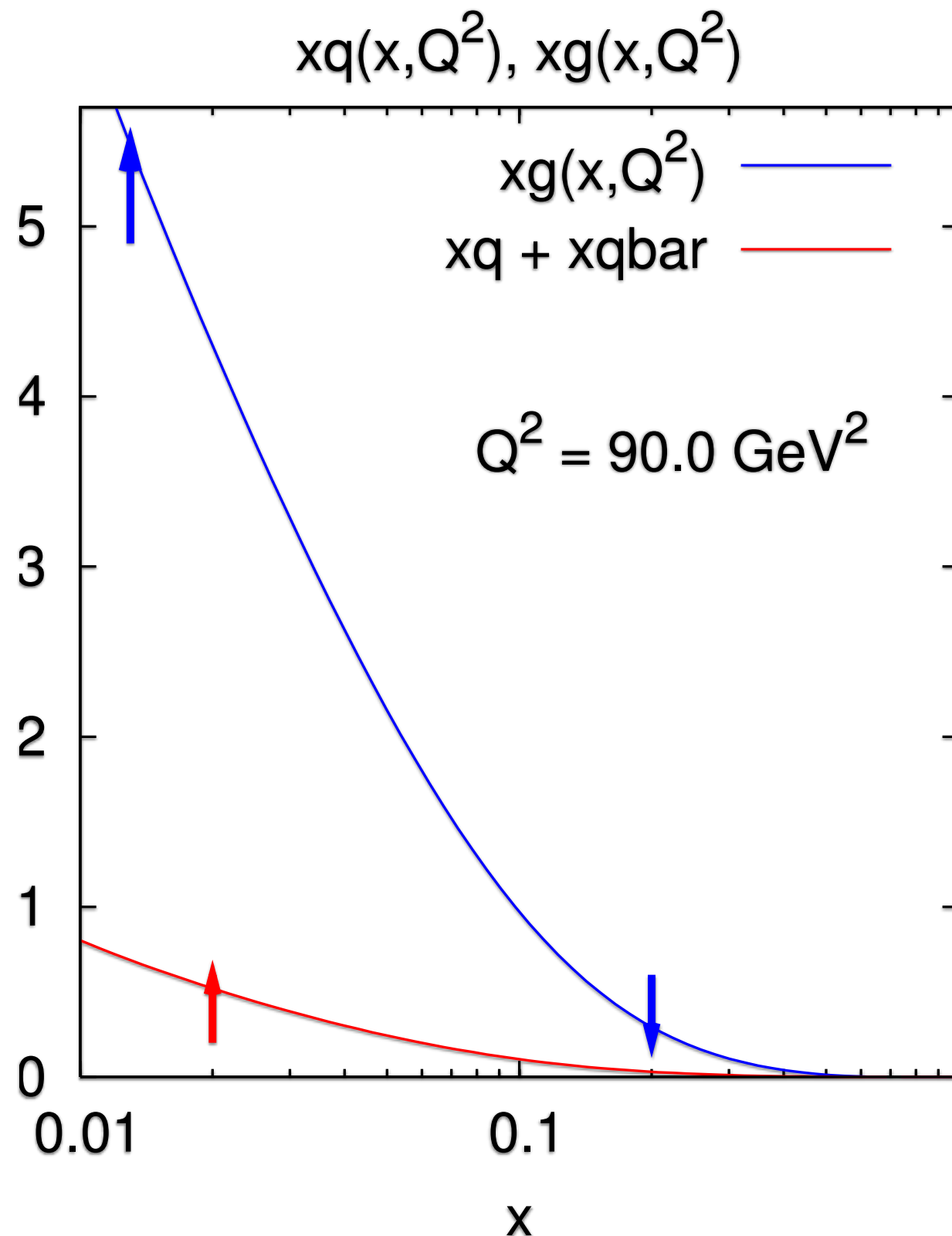
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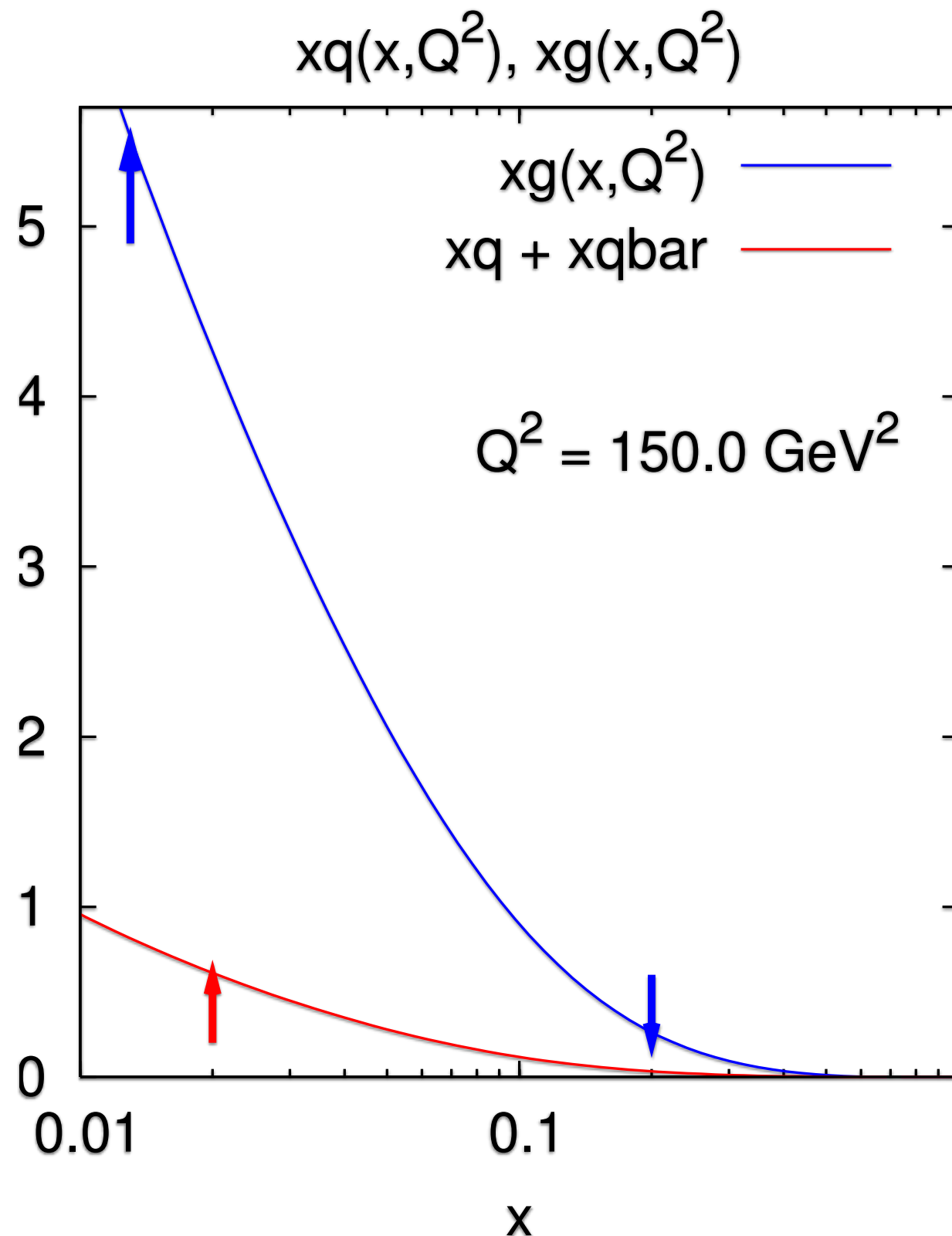
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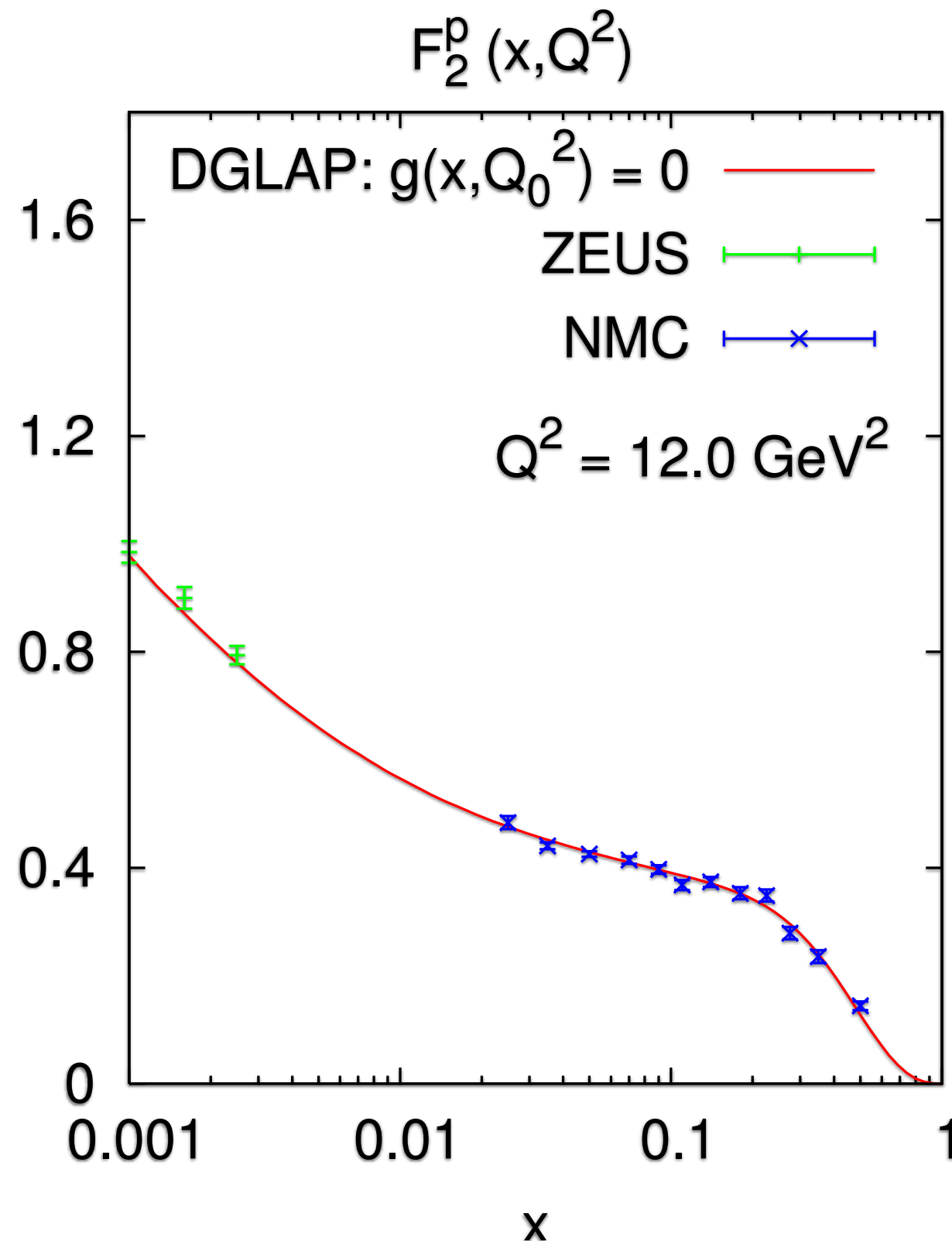
DGLAP evolution:

- partons lose momentum and shift towards smaller x
- high- x partons drive growth of low- x gluon

determining the gluon

which is critical at hadron colliders (e.g. Higgs dominantly produced by gluon-gluon fusion), but not directly probed in Deep-Inelastic-Scattering

Consider DIS data – $F_2(x, Q^2)$ – in a world where the proton just had quarks



Fit quark distributions to $F_2(x, Q_0^2)$,
at *initial scale* $Q_0^2 = 12 \text{ GeV}^2$.

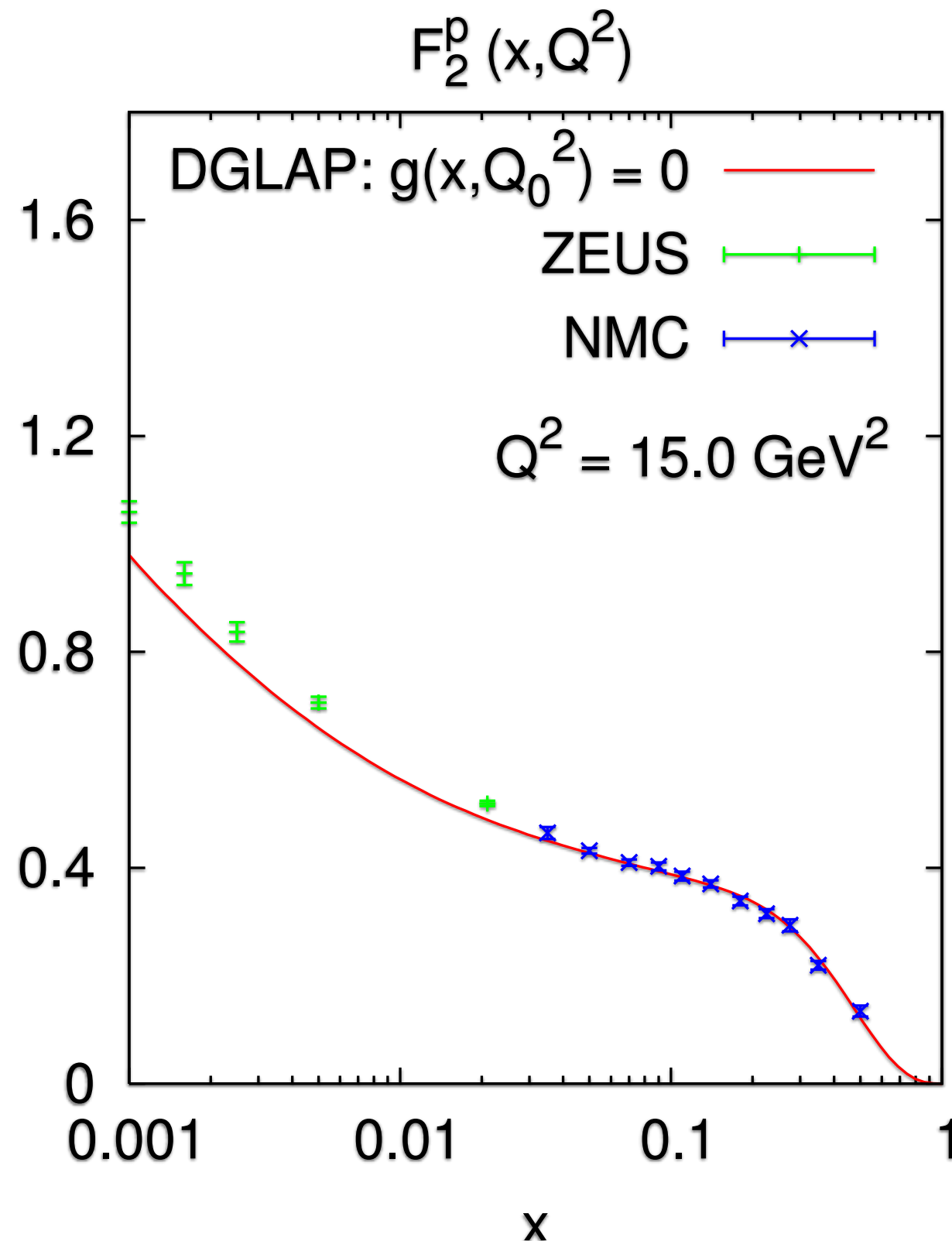
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2 :

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to
higher Q^2 ; compare with data.

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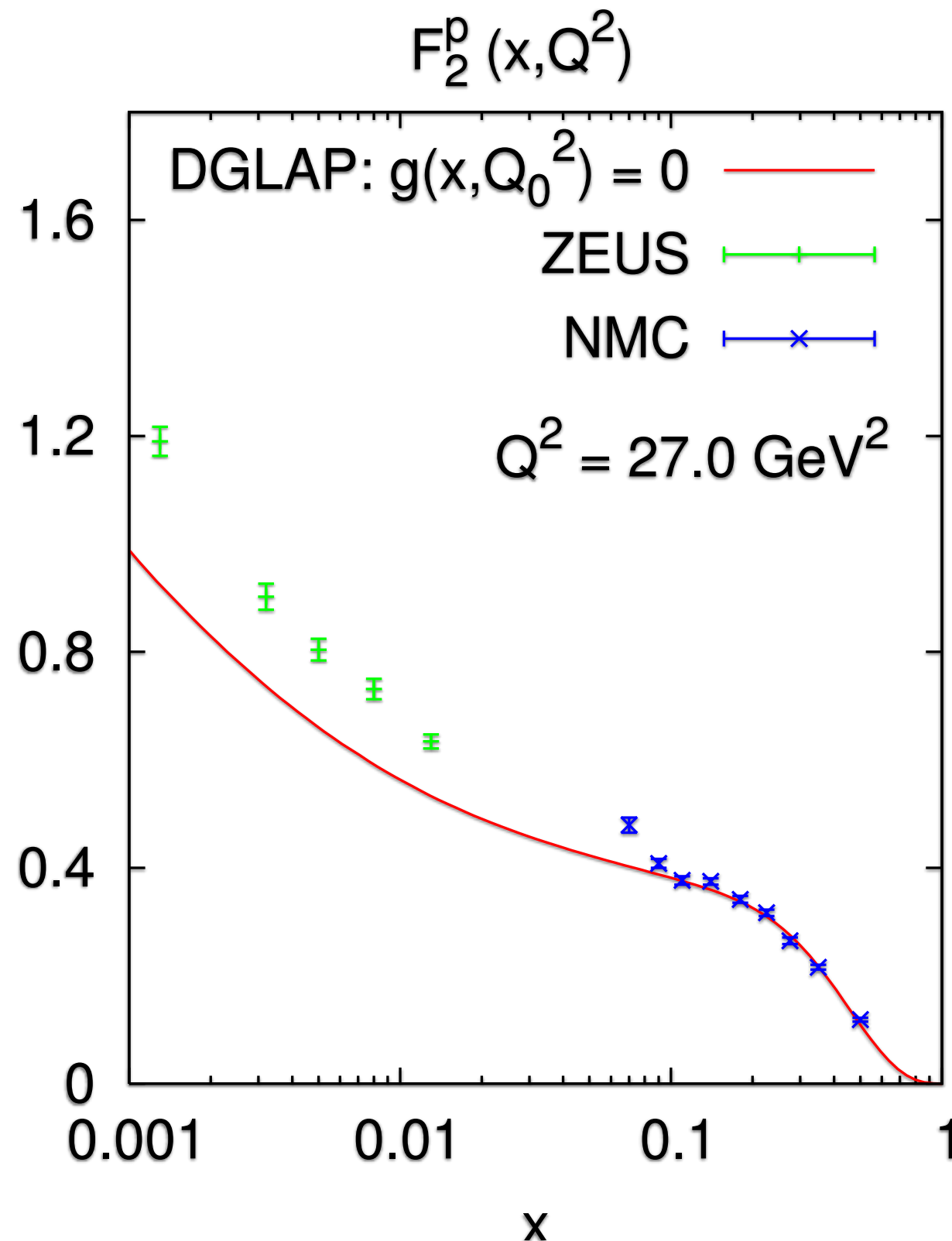
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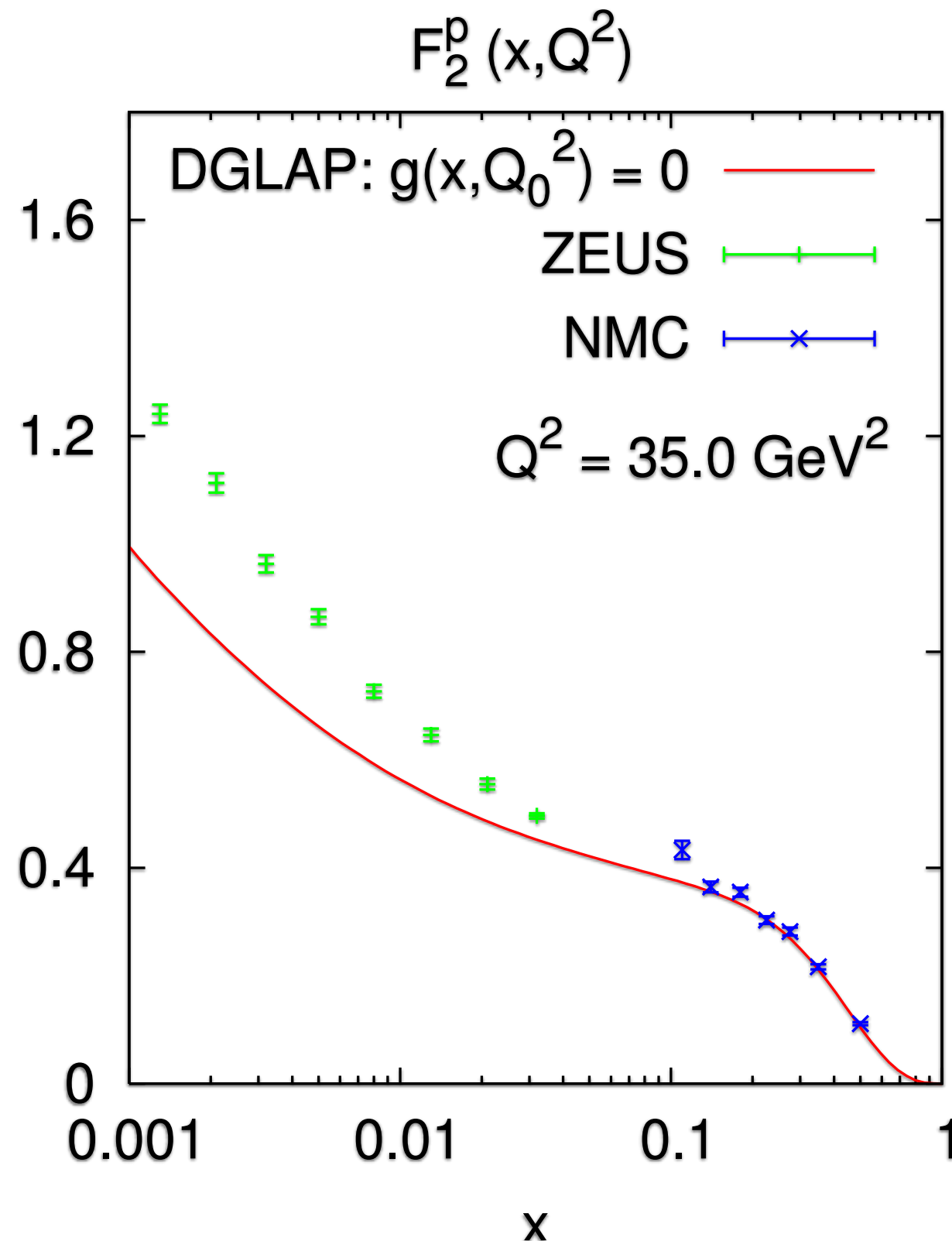
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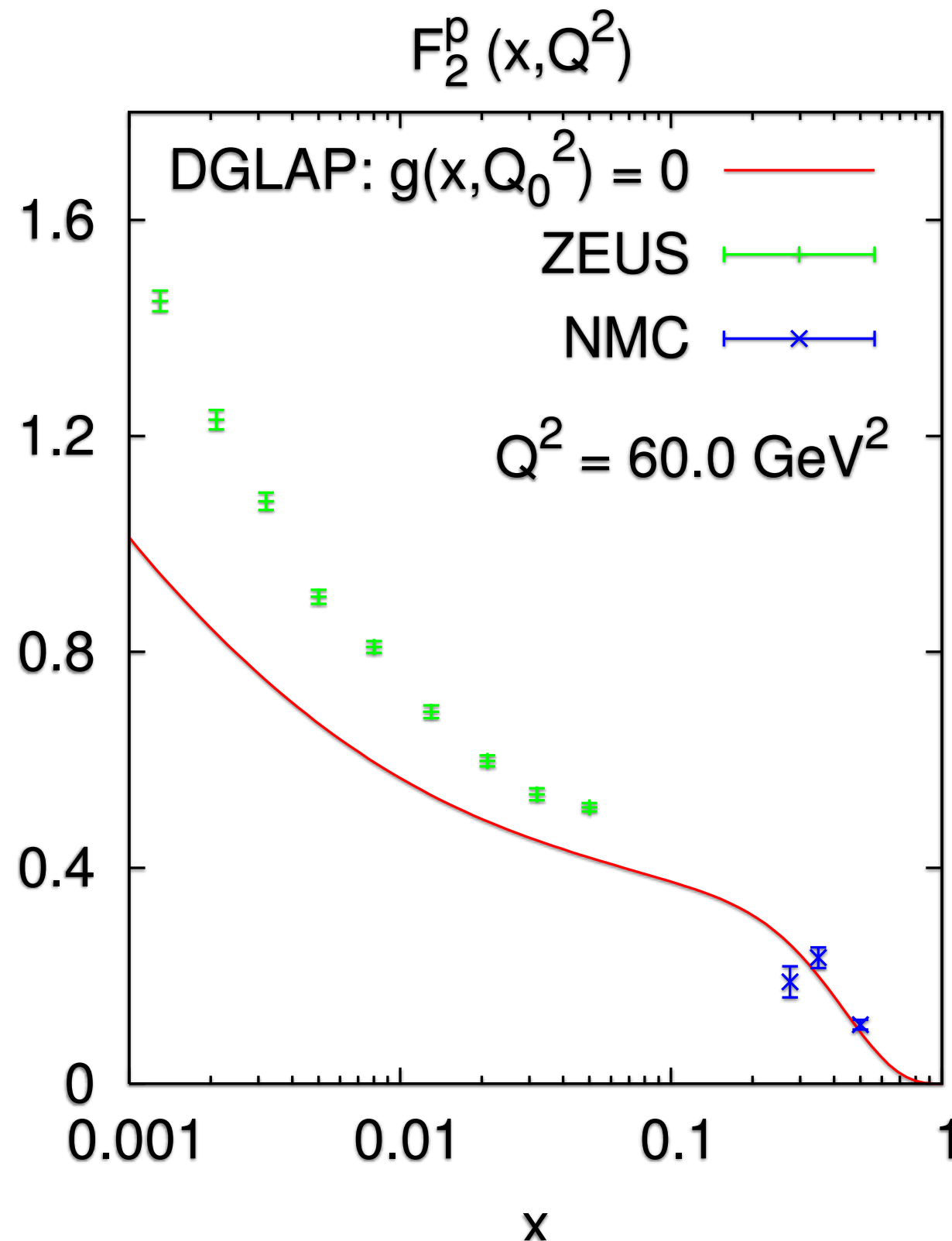
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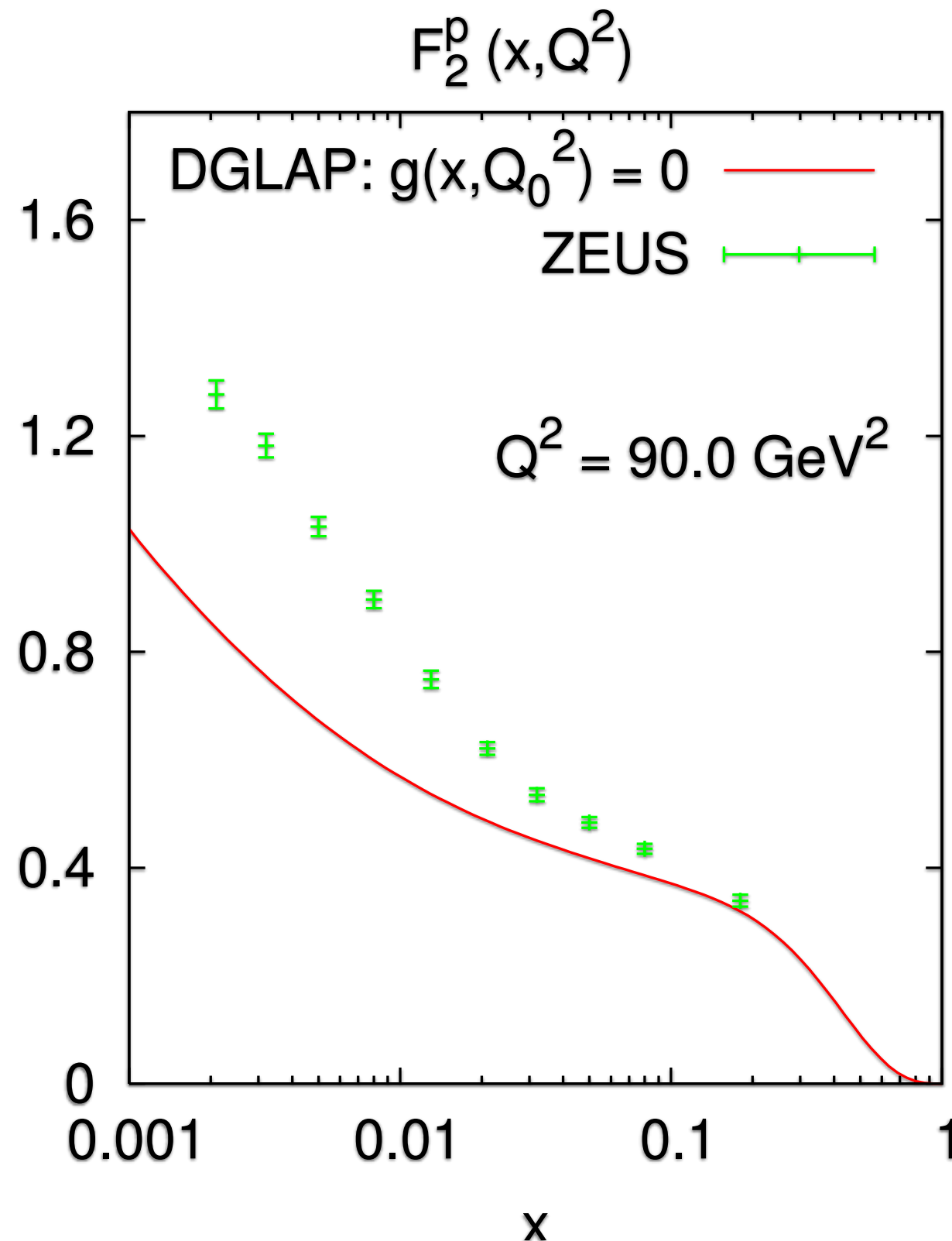
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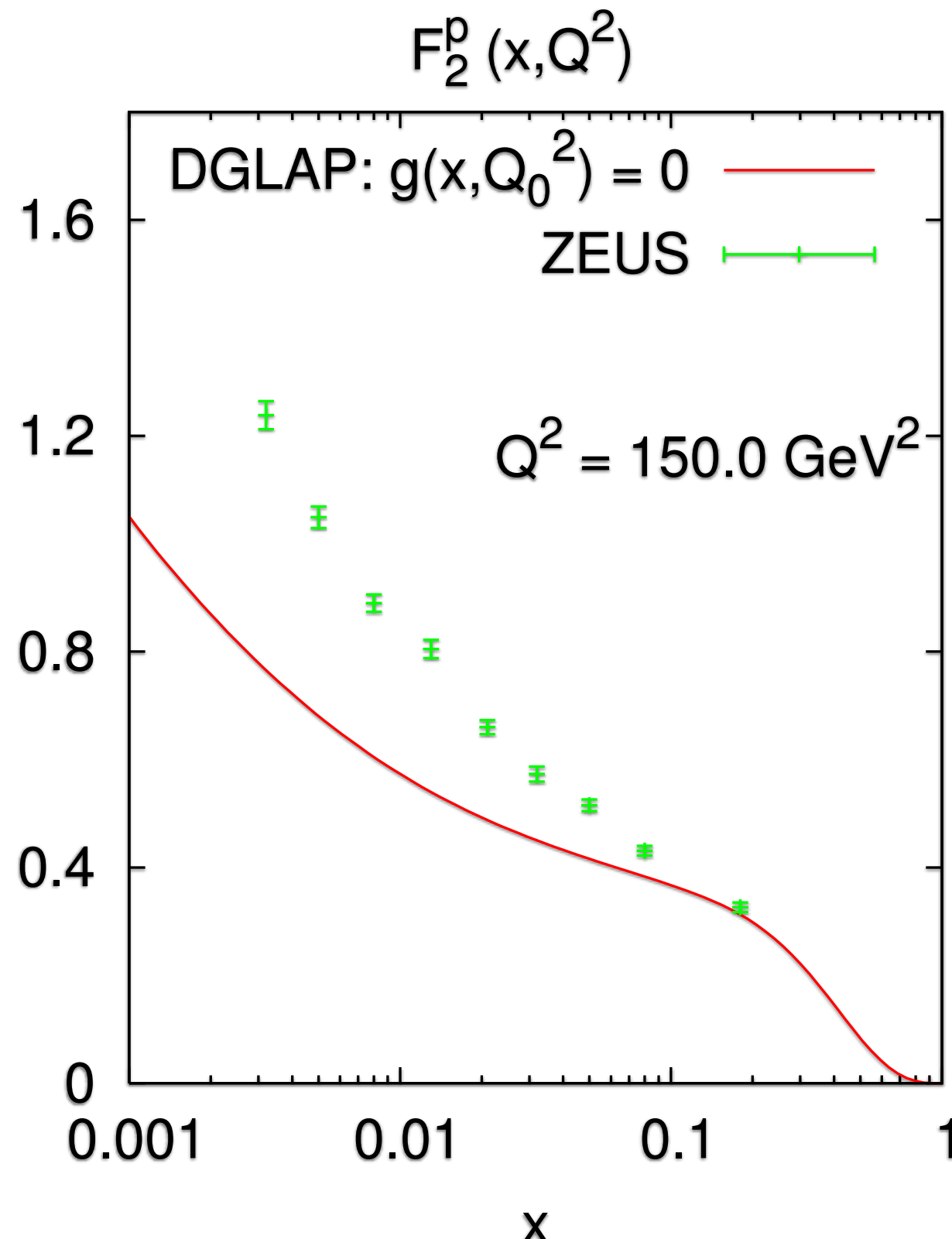
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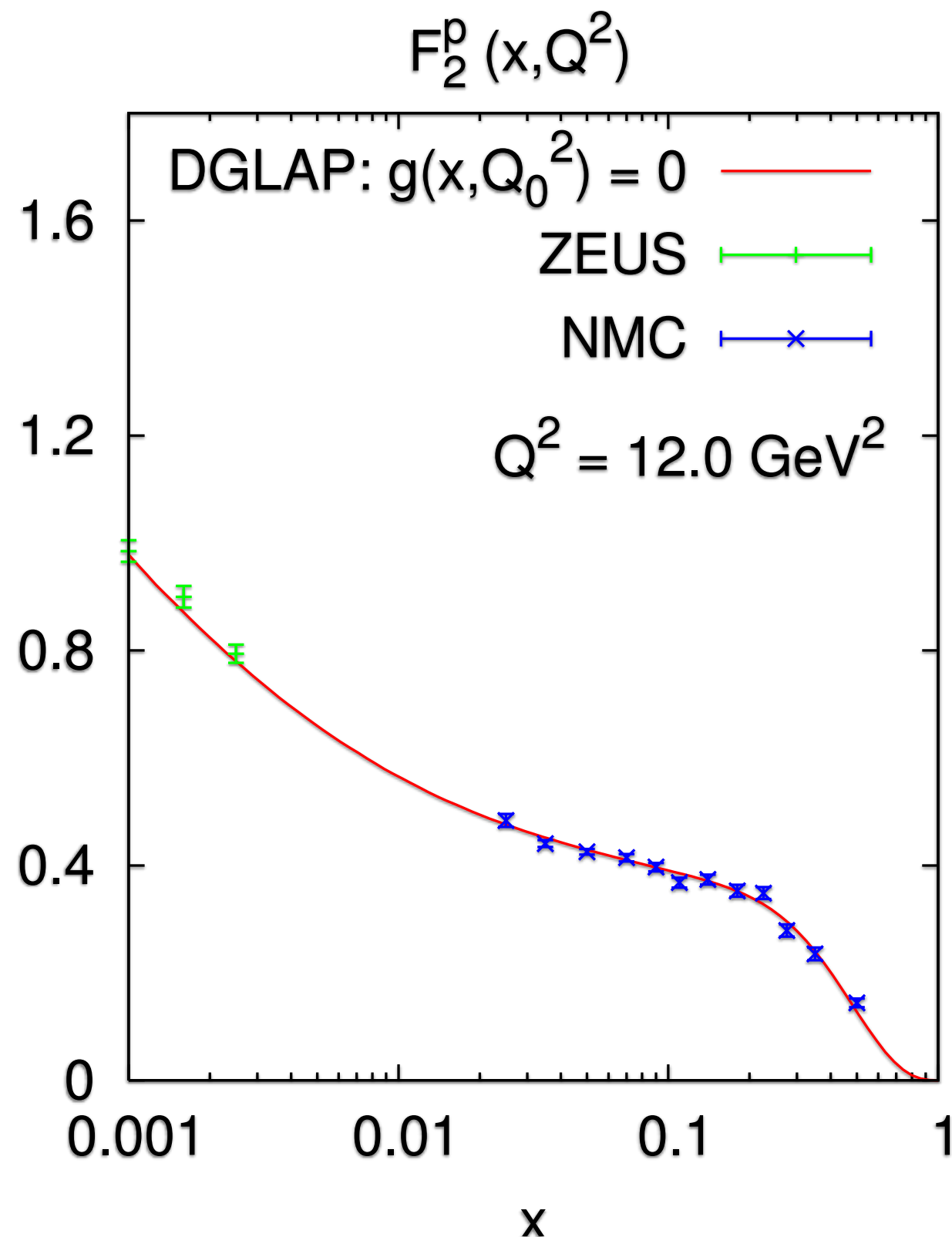
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**COMPLETE FAILURE
to reproduce data evolution**

Consider DIS data – $F_2(x, Q^2)$ – with specially tuned gluon



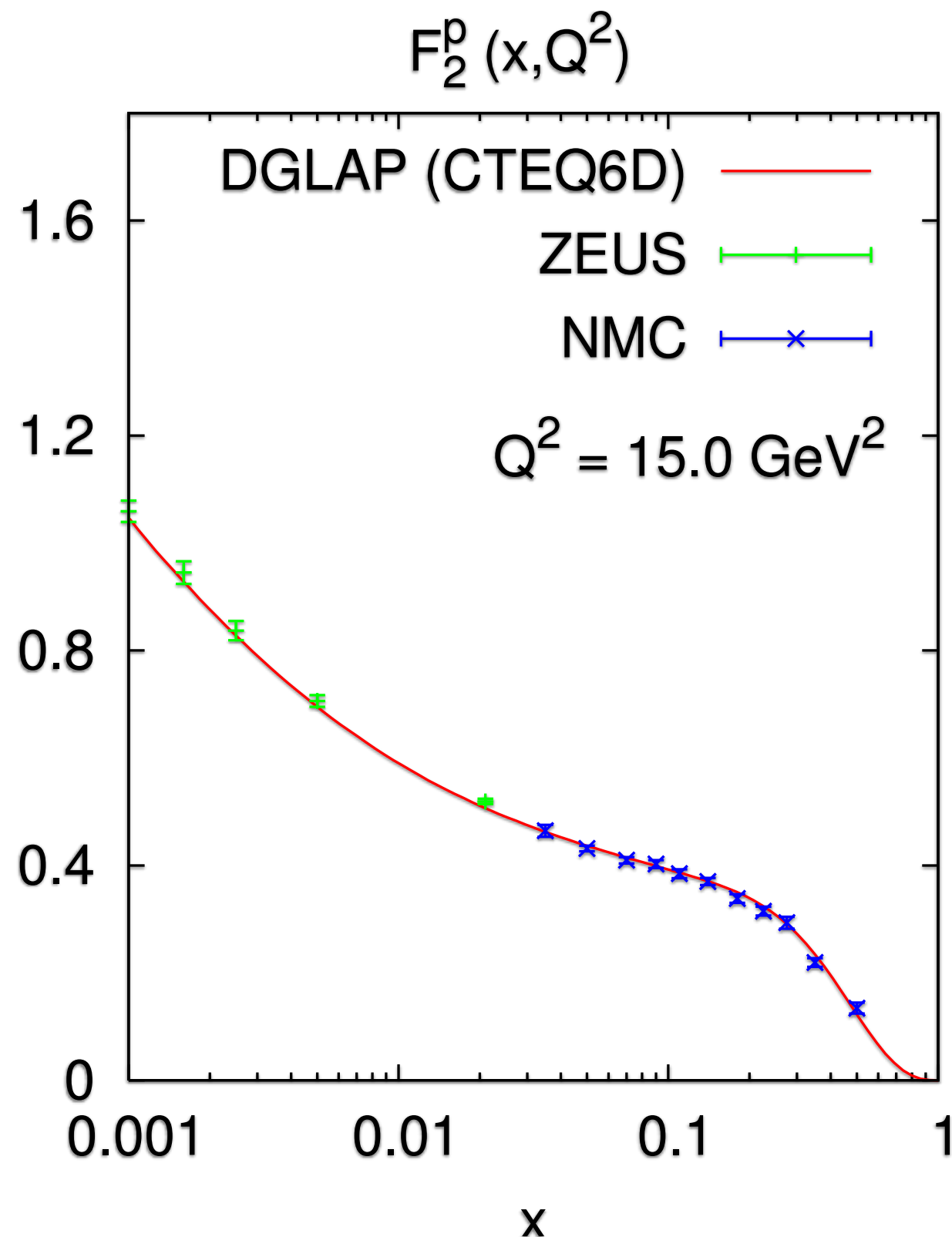
If gluon $\neq 0$, splitting

$$g \rightarrow q\bar{q}$$

generates extra quarks at large Q^2 \Rightarrow faster rise of F_2

Global PDF fits (**CT**, **MMHT**, **NNPDF**, etc.) choose gluon distribution that leads to the correct Q^2 evolution.

Consider DIS data – $F_2(x, Q^2)$ – with specially tuned gluon



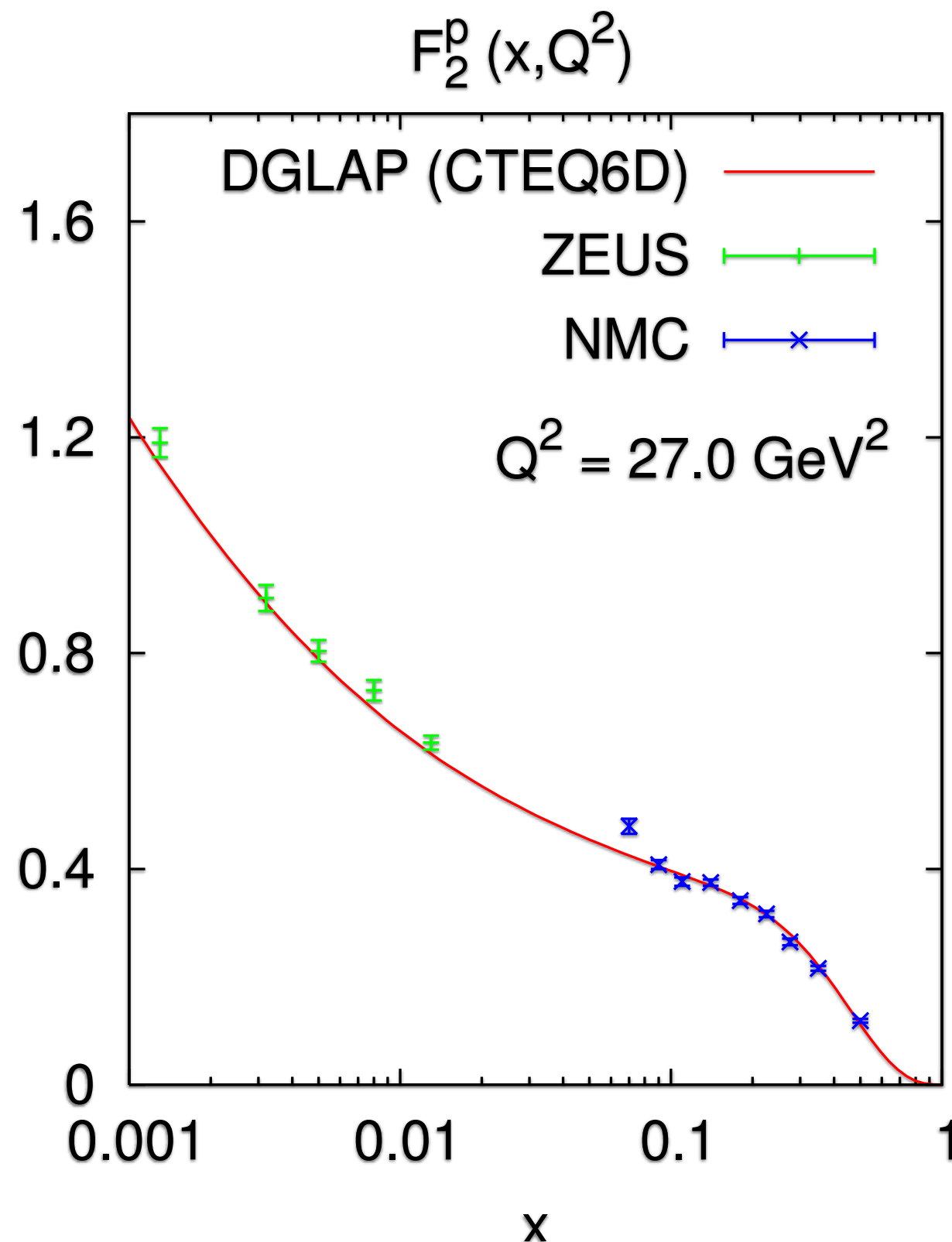
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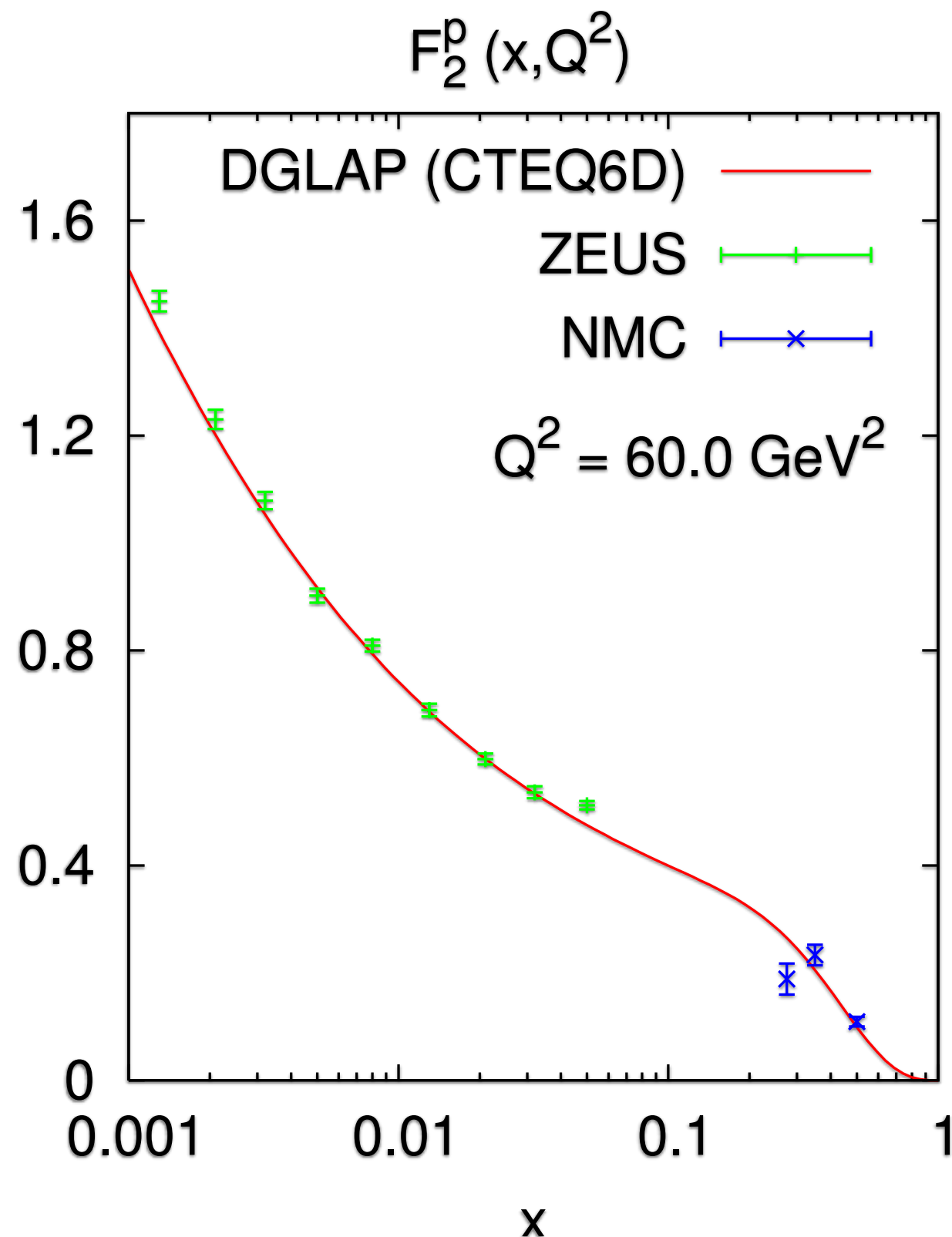
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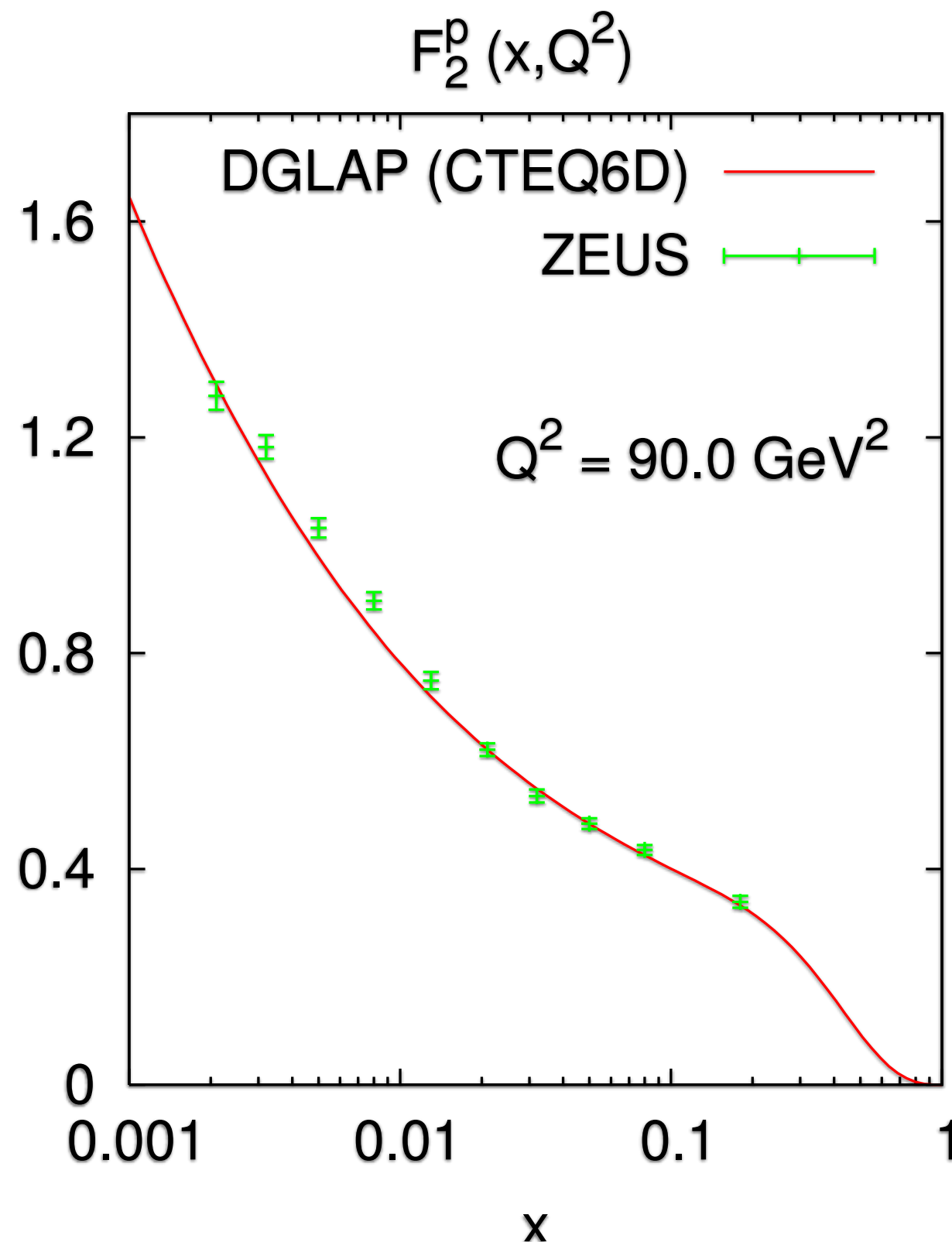
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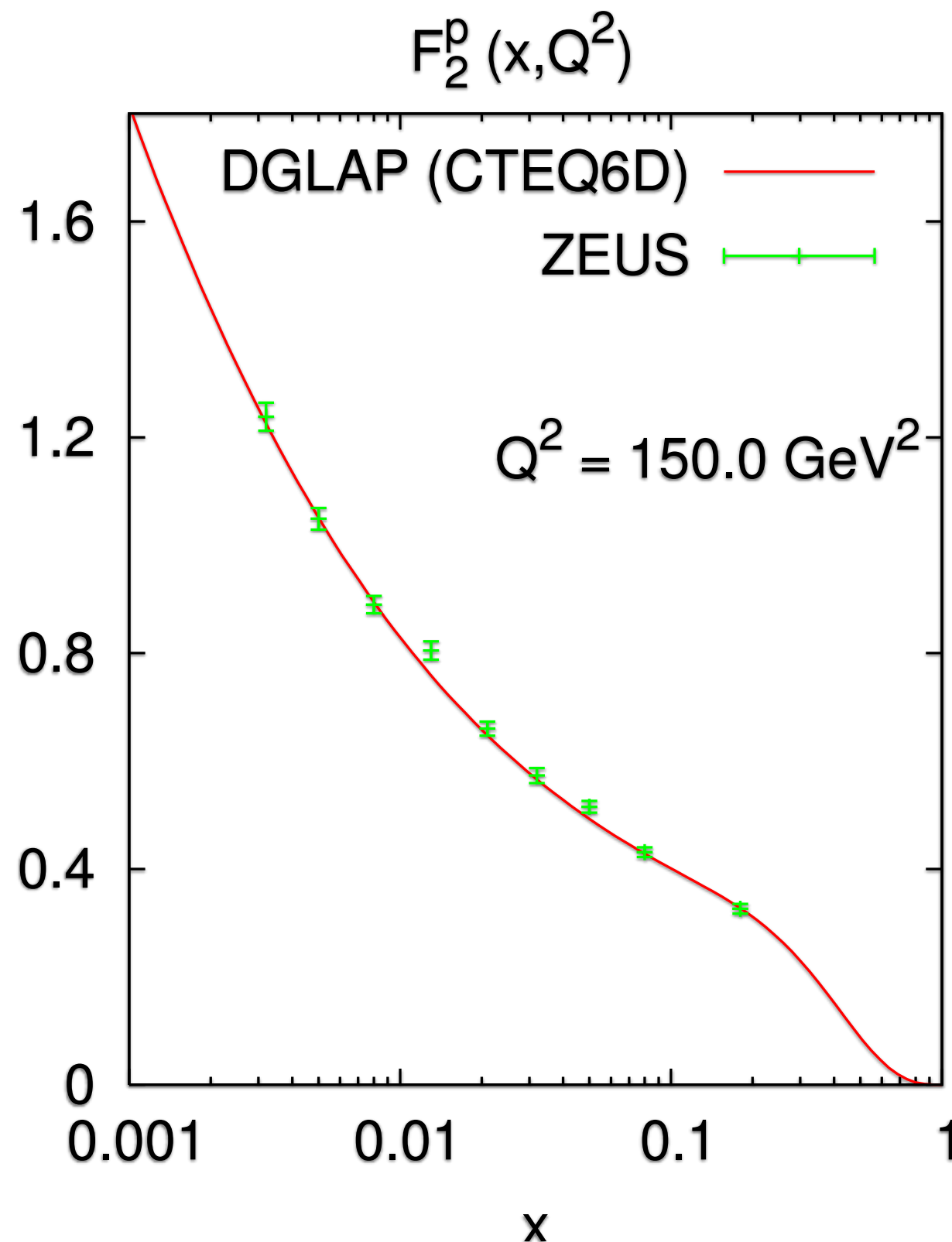
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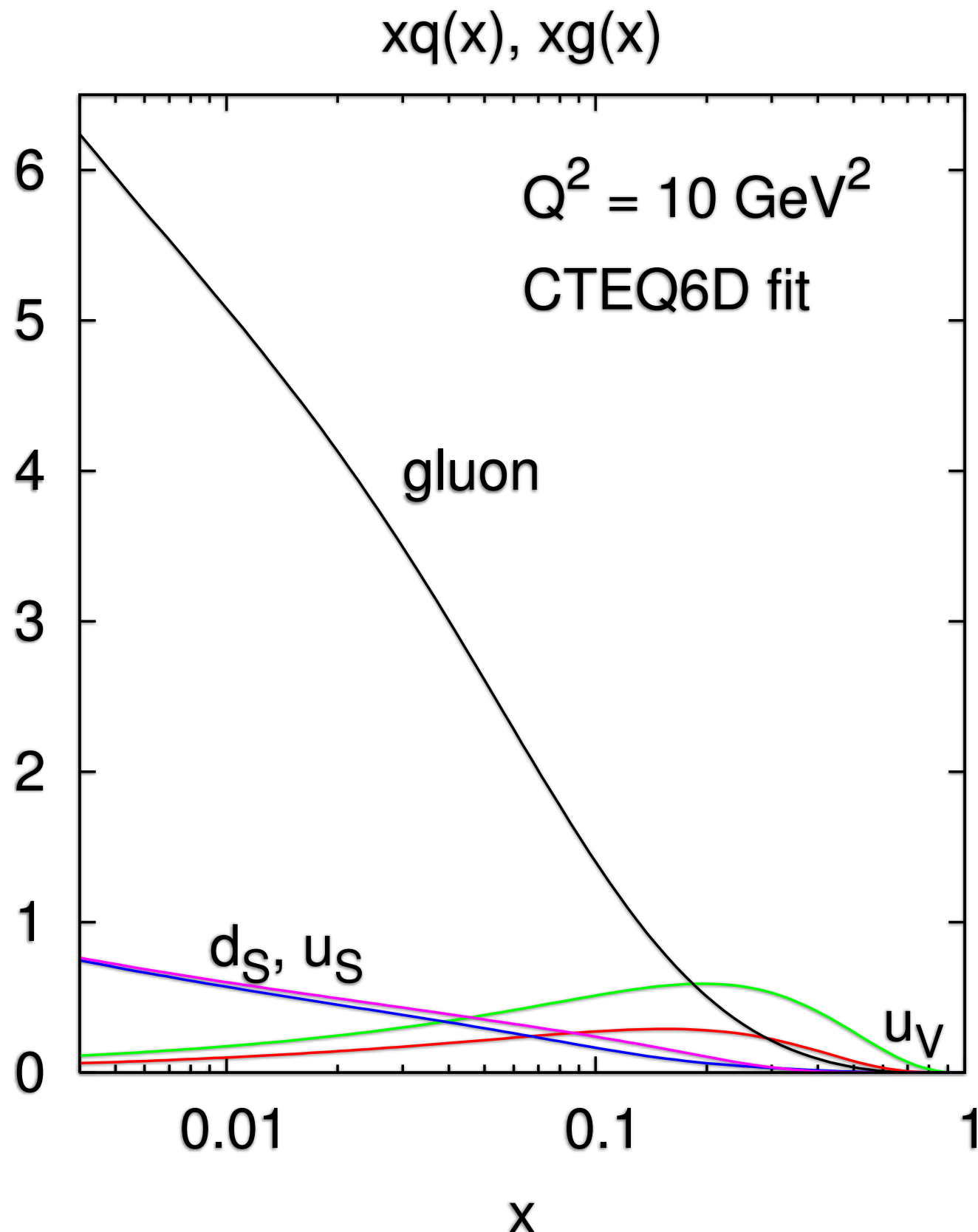
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Global PDF fits (CT, MMHT, NNPDF, etc.) choose gluon distribution that leads to the correct Q^2 evolution.

SUCCESS

Resulting gluon distribution, compared to quarks



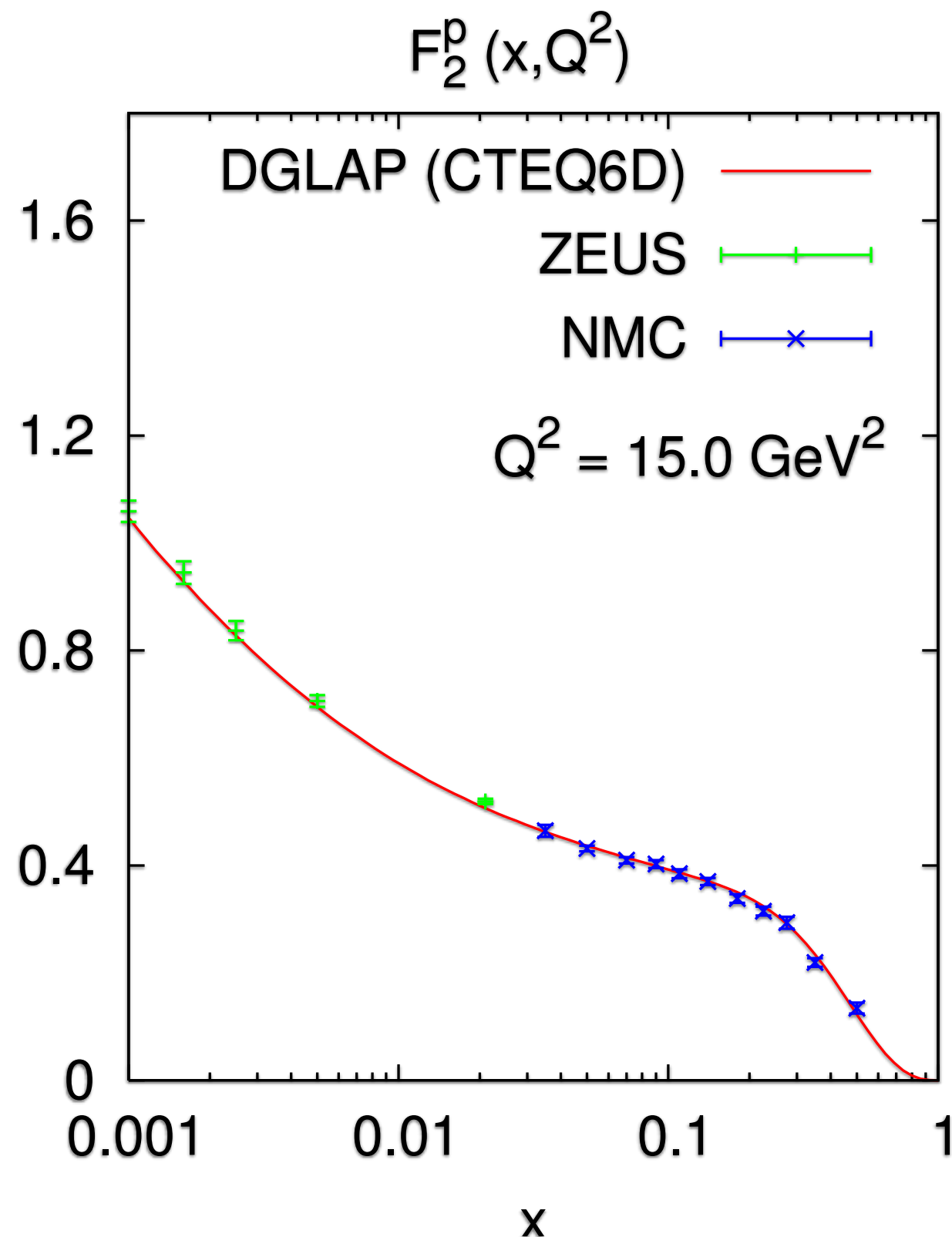
Resulting gluon distribution is **HUGE!**

Carries **47% of proton's momentum**
(at scale of 100 GeV)

Crucial in order to satisfy
momentum sum rule.

Large value of gluon has big
impact on phenomenology

Consider DIS data – $F_2(x, Q^2)$ – with specially tuned gluon



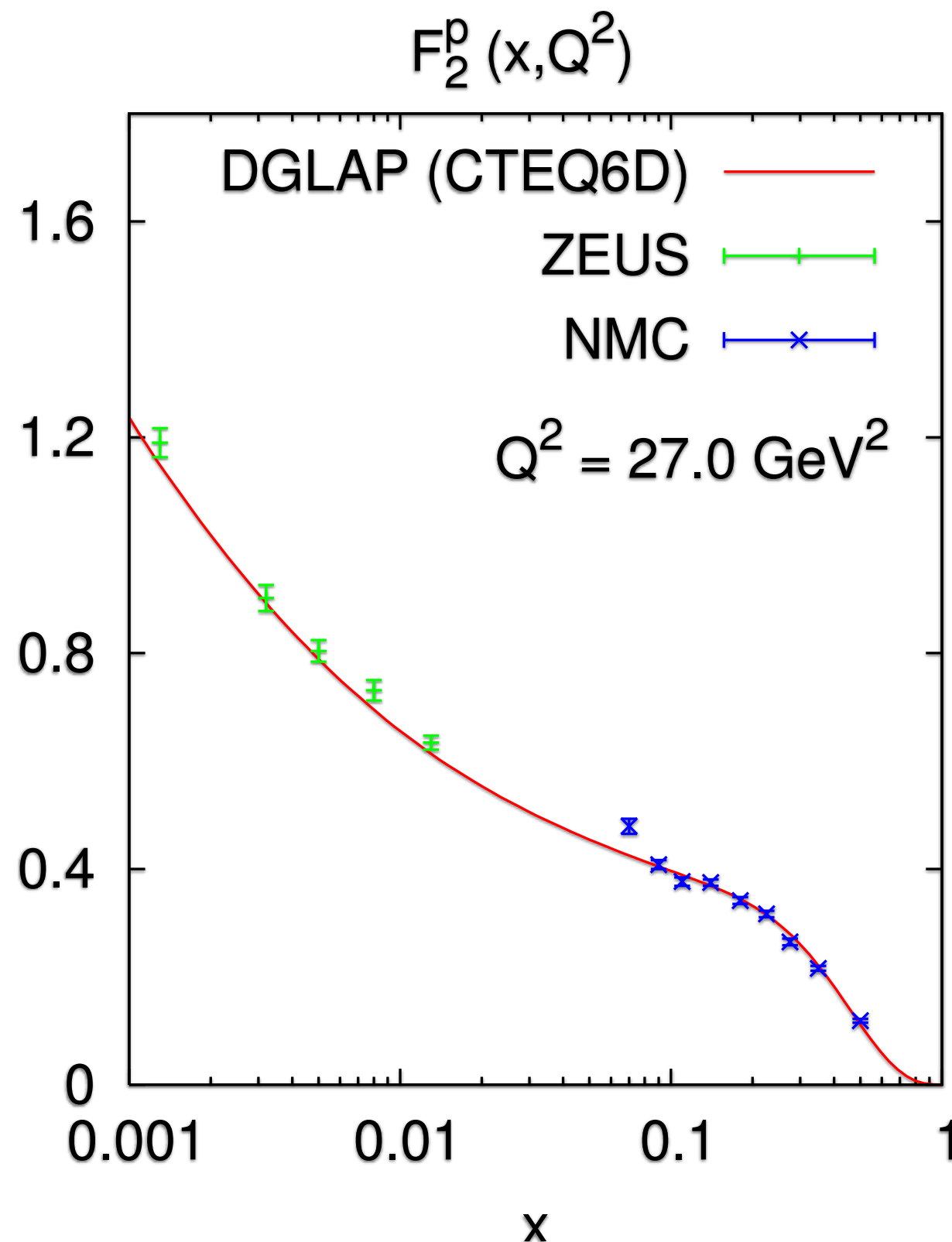
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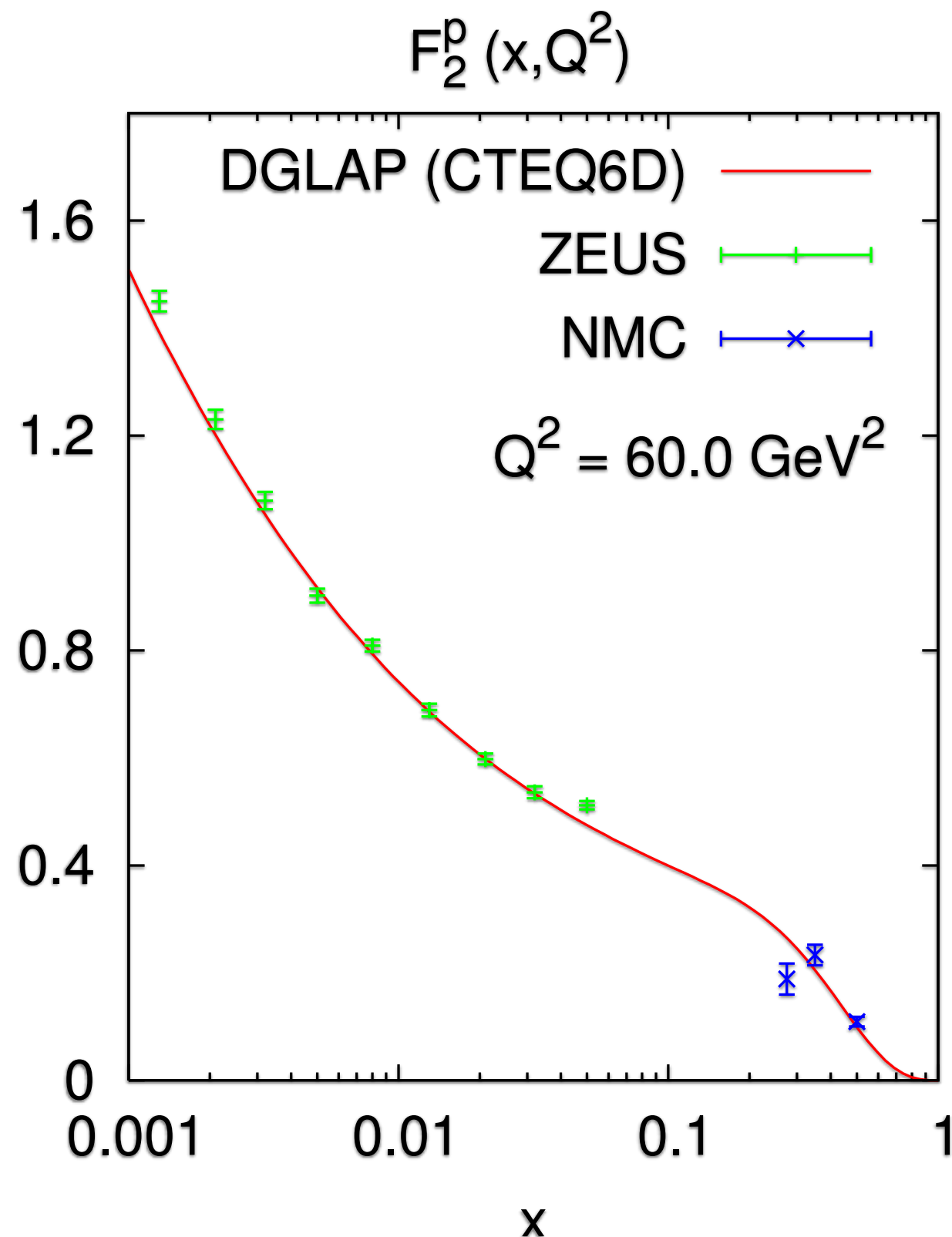
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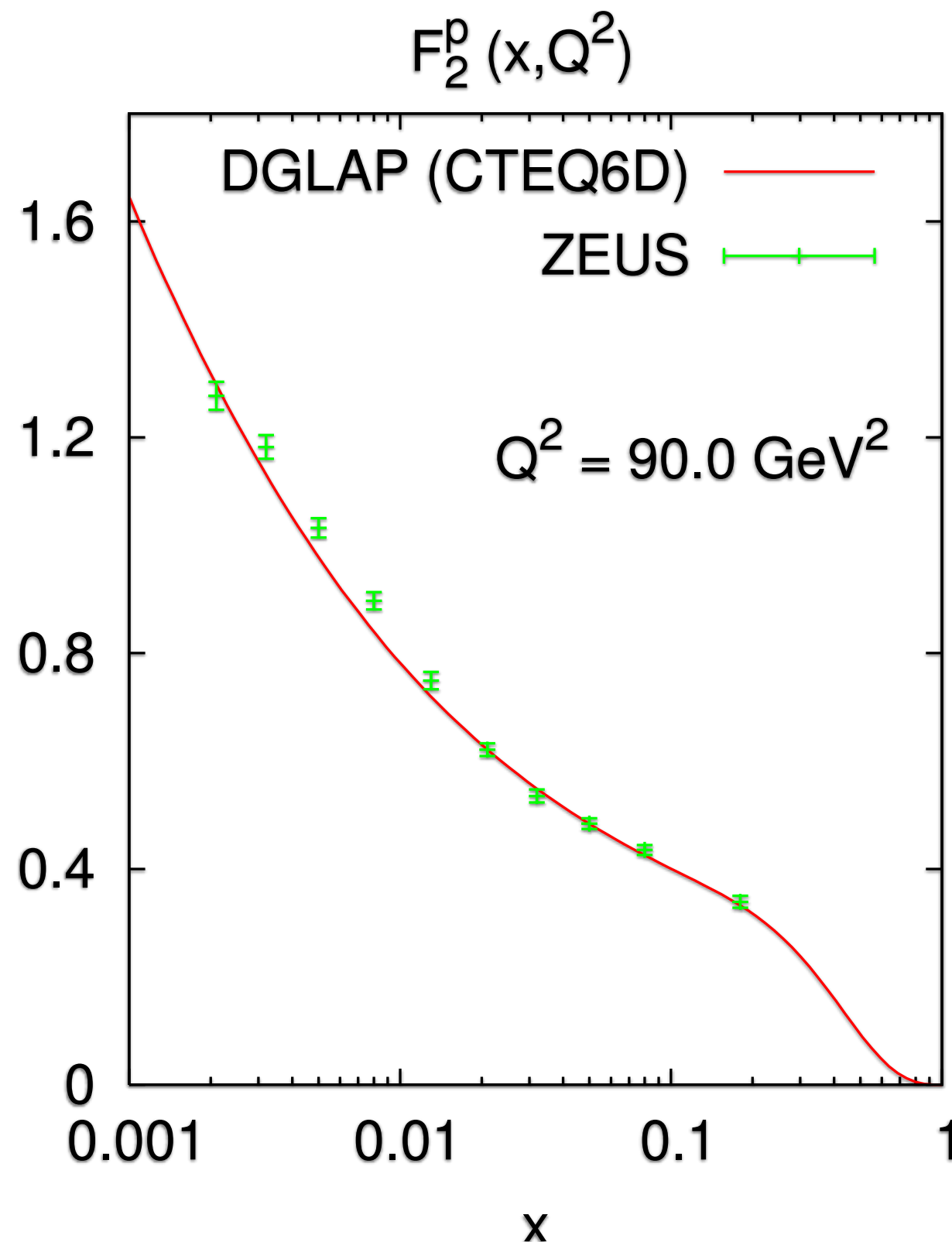
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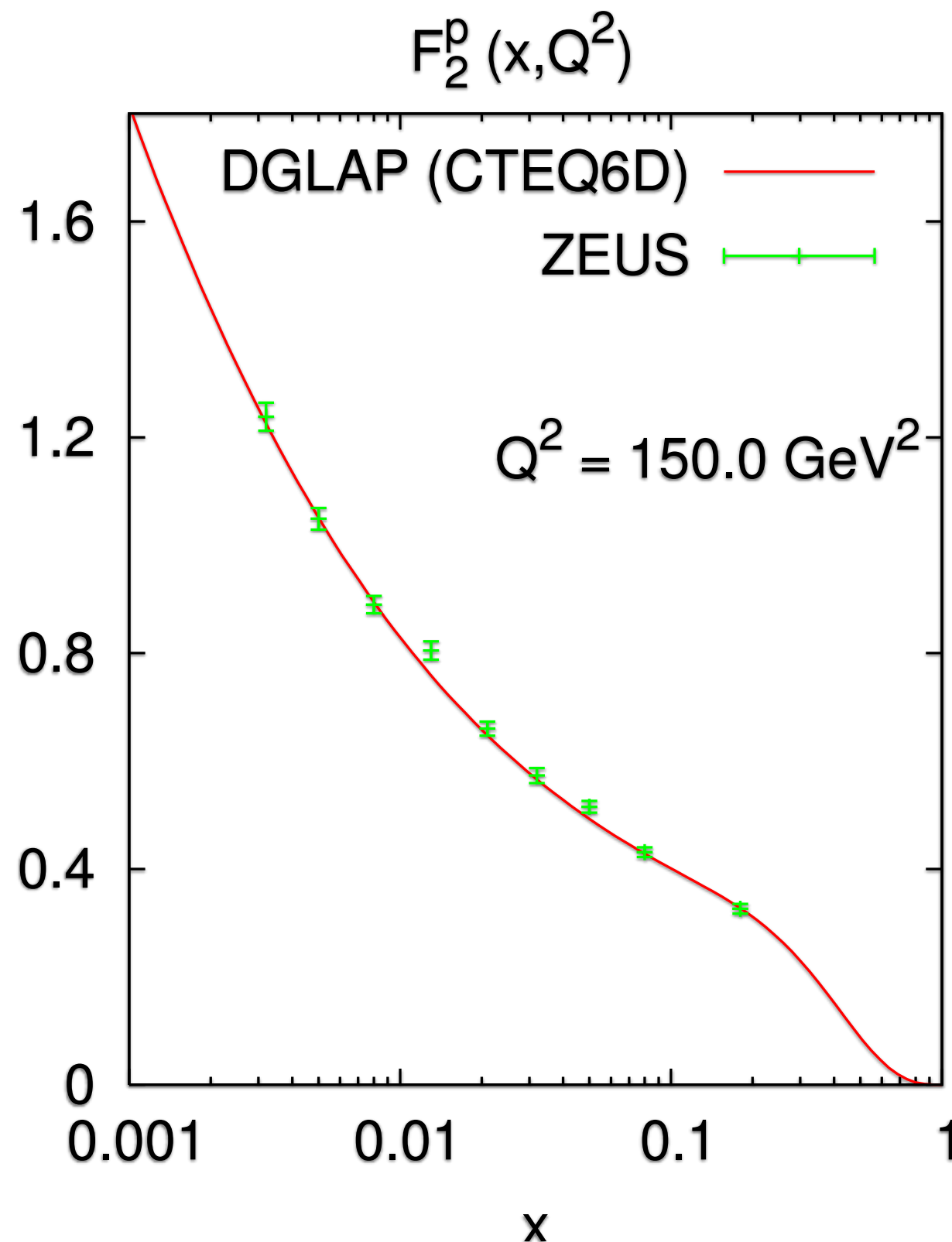
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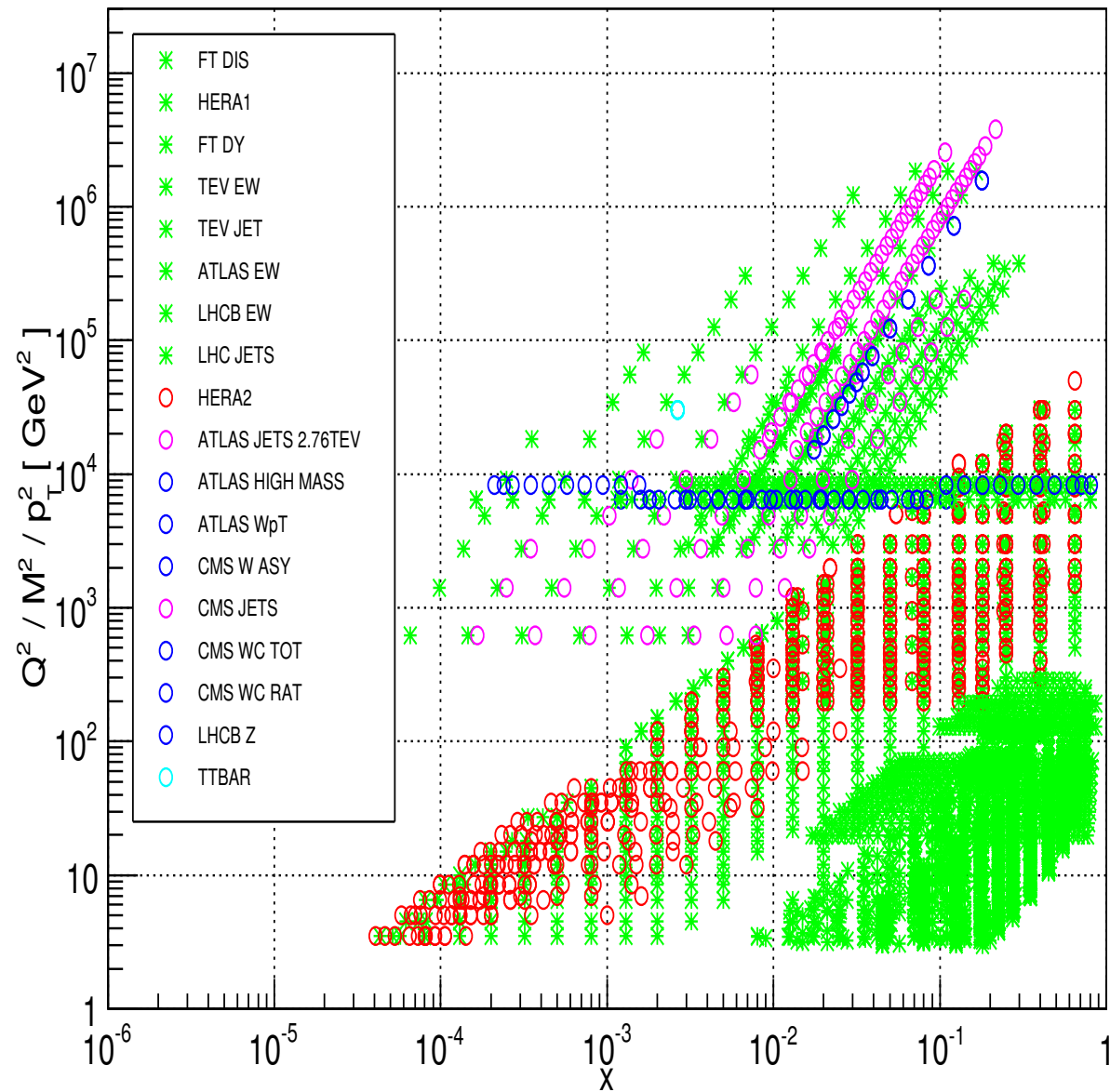
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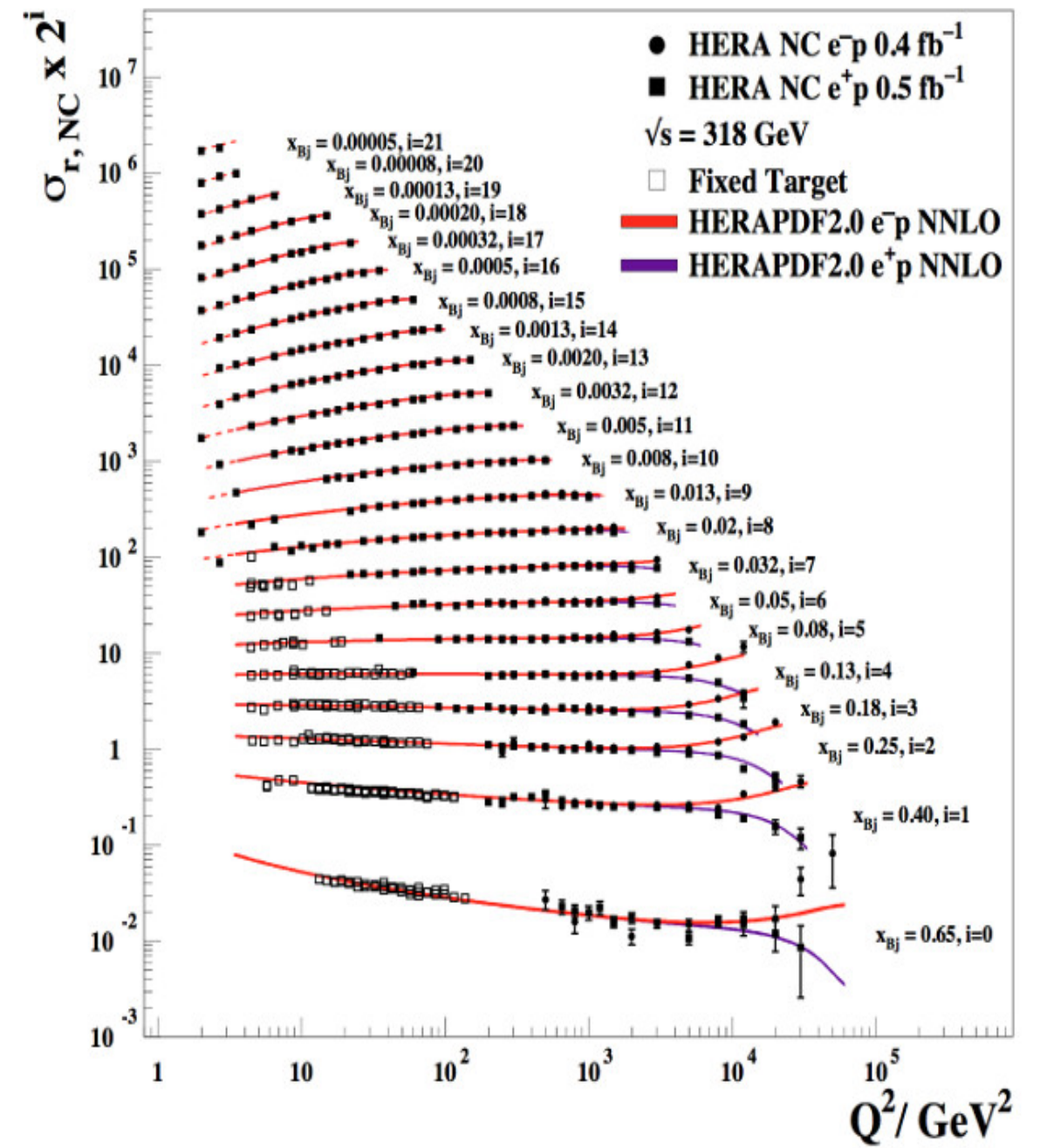
SUCCESS

TODAY'S PDF FITS

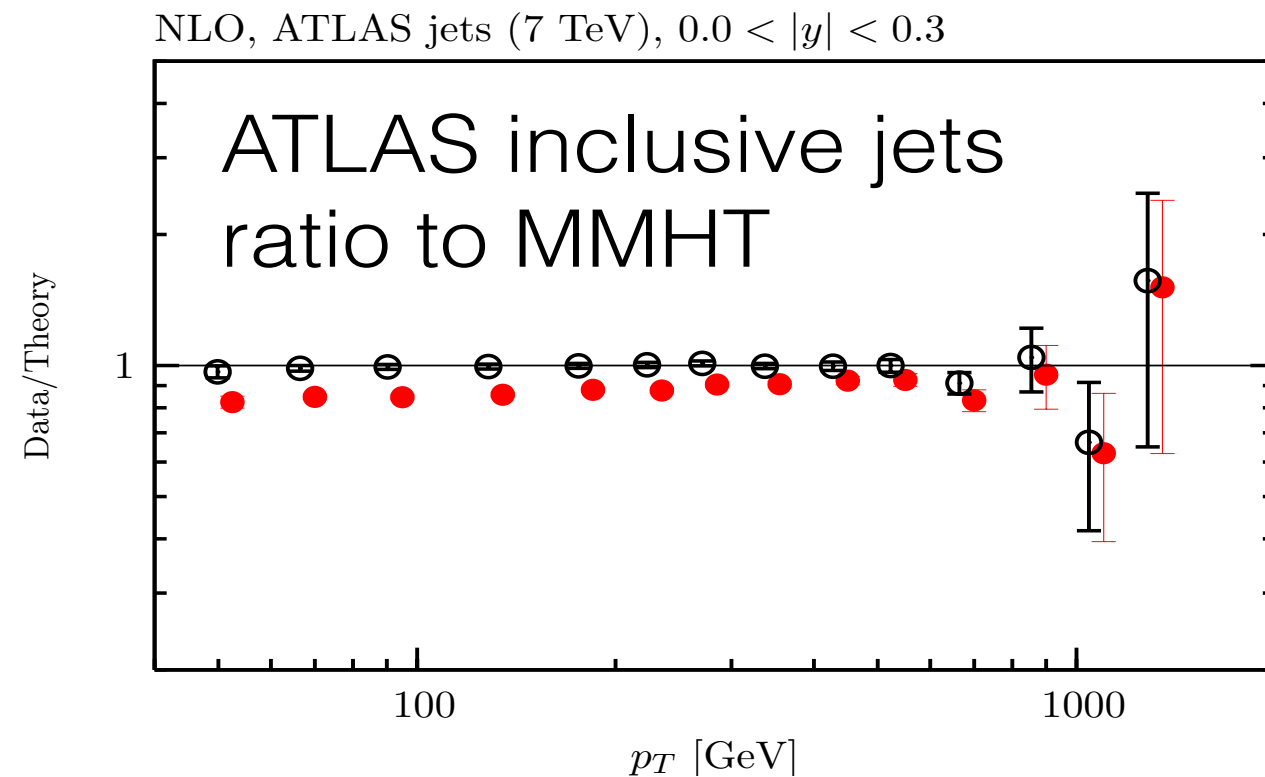
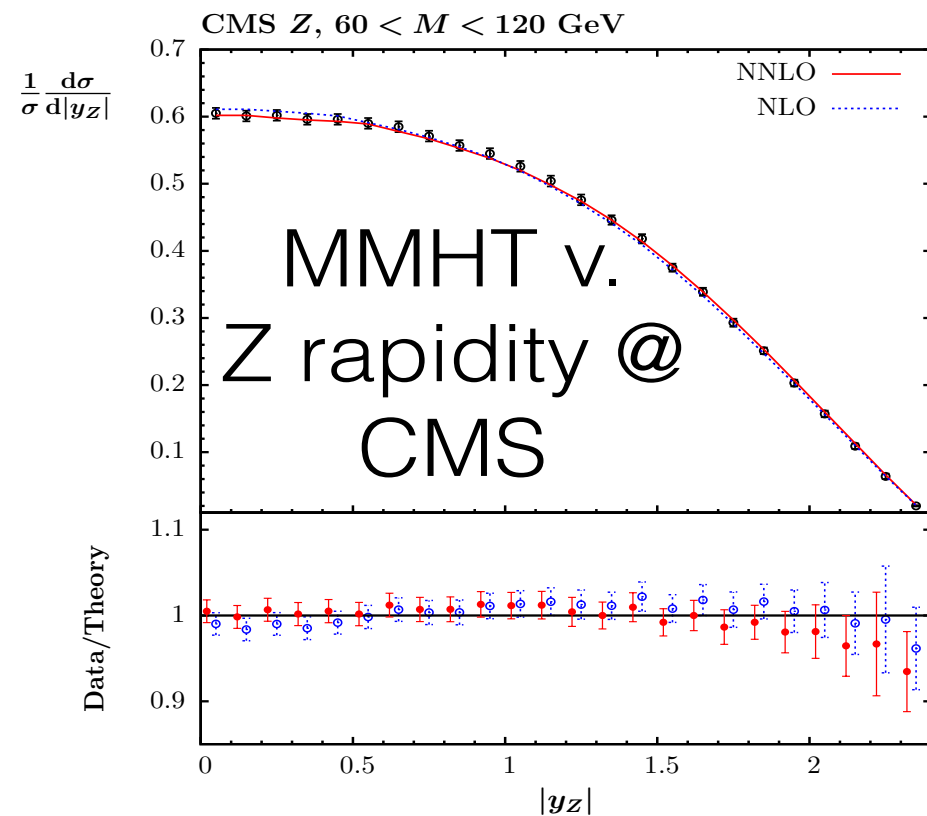
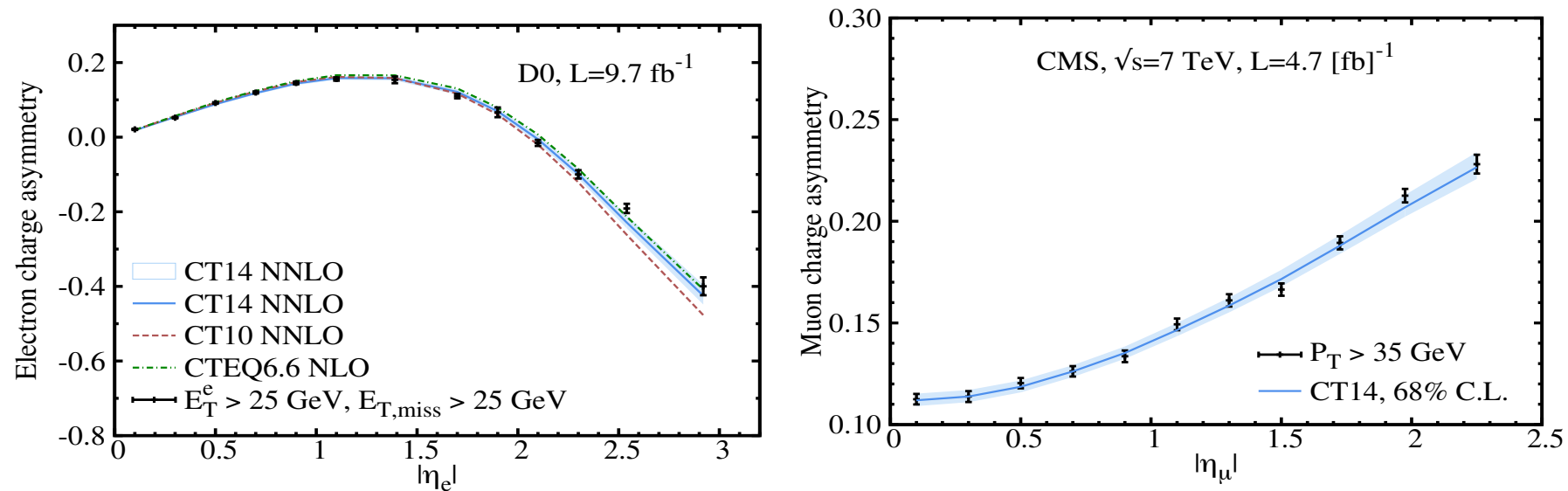
NNPDF3.0 NLO dataset



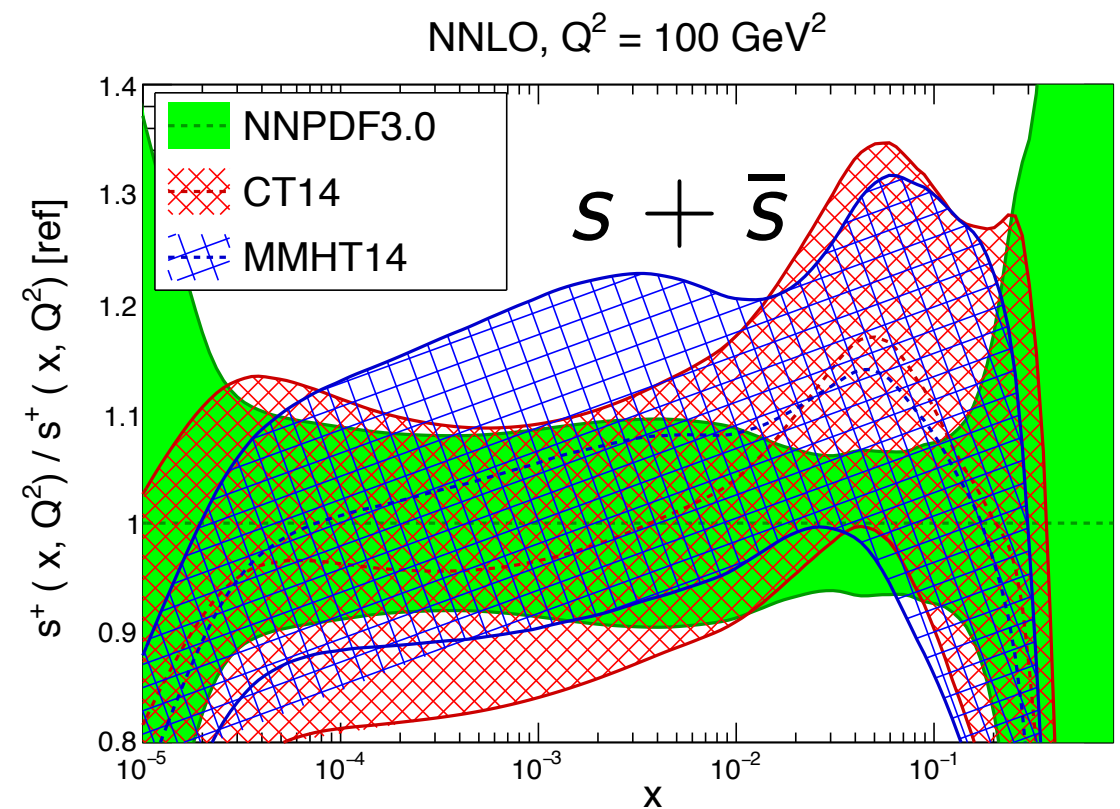
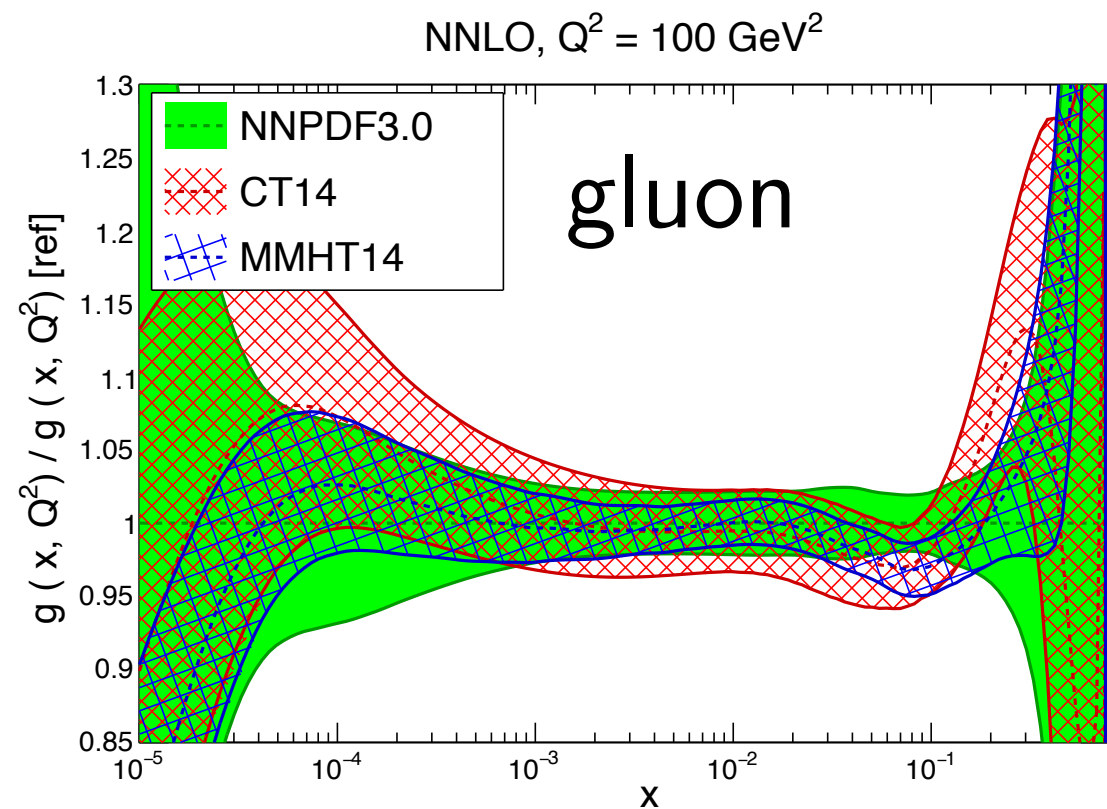
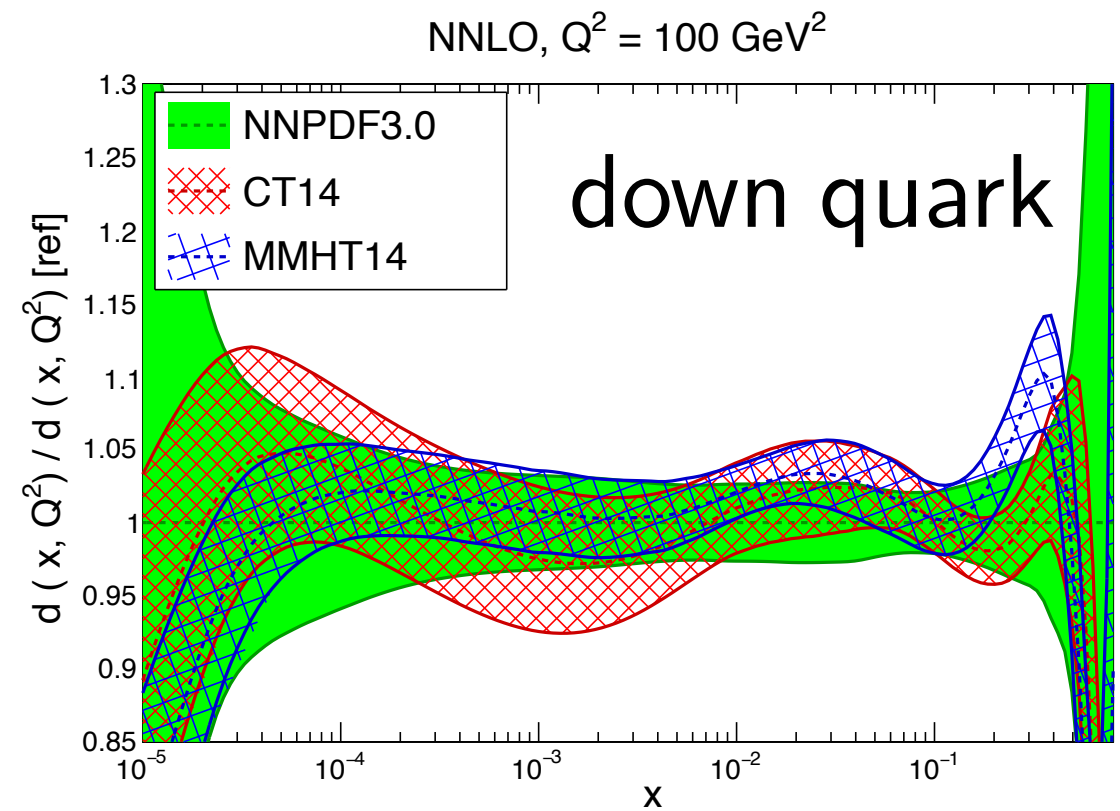
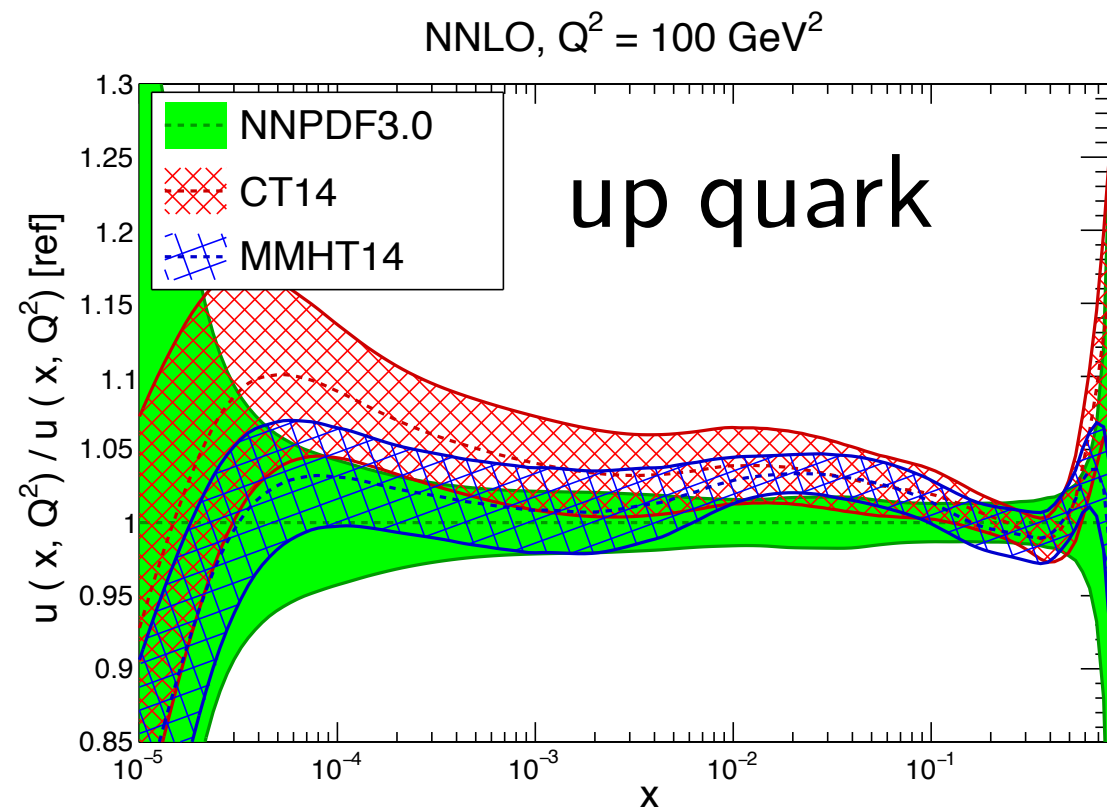
H1 and ZEUS

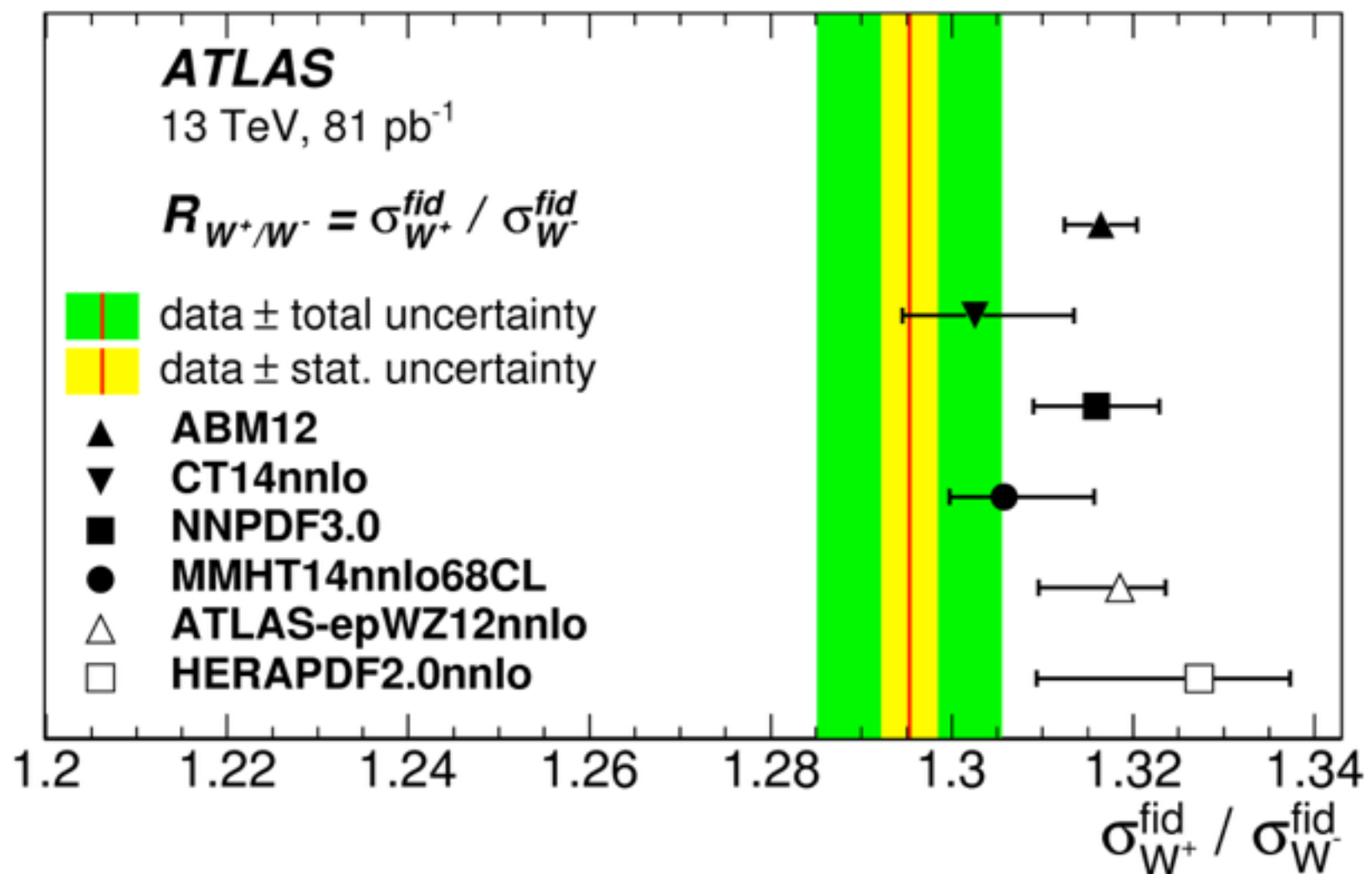


Lepton charge asym. v. CT14 @ D0 & CMS



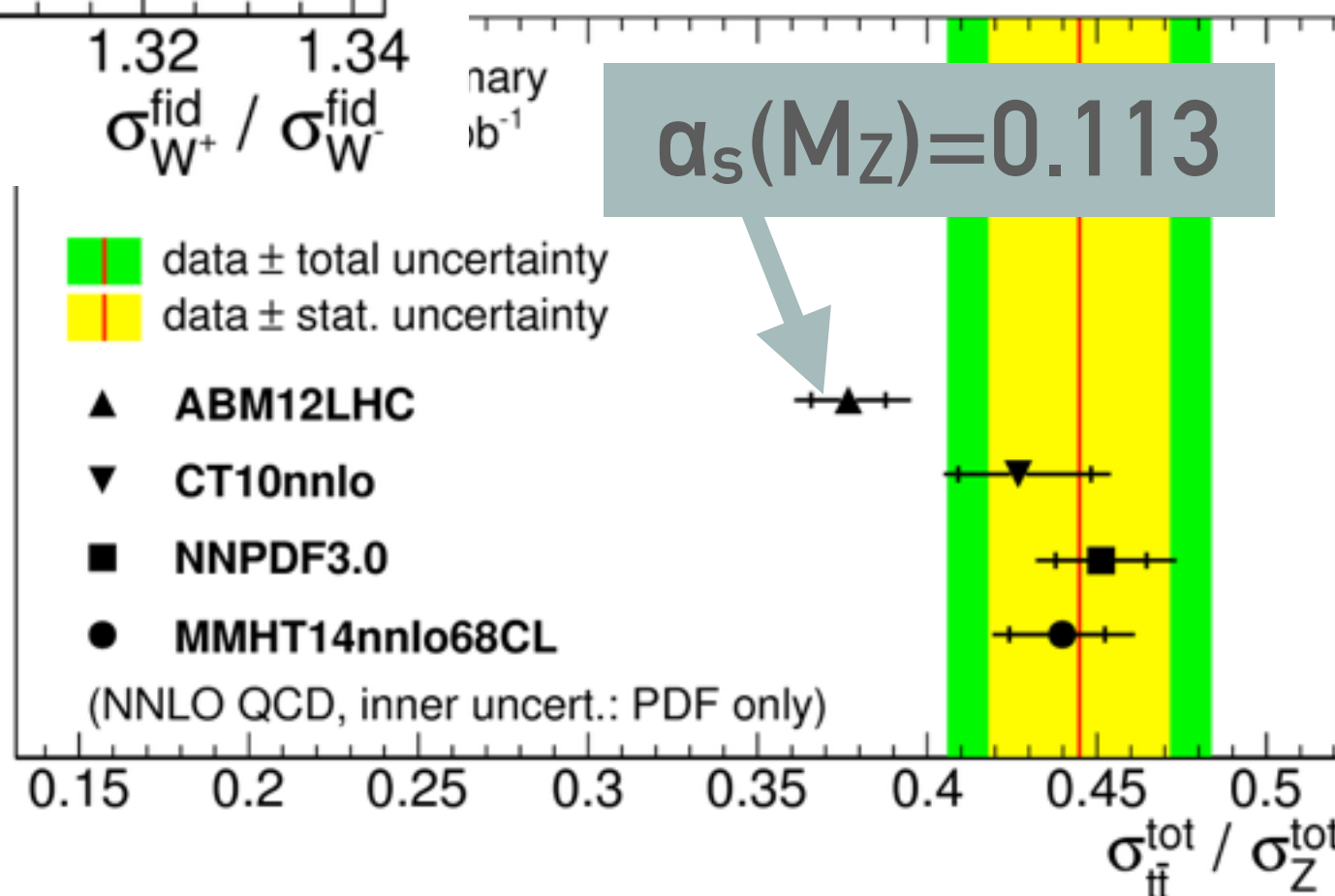
THREE GLOBAL PDF FITS: CT14, MMHT2014, NNPDF3.0





cross-section ratios
(W⁺/W⁻, ttbar/Z)
show tensions with
some PDFs

ATLAS-CONF-2015-049



*NB: top-quark mass
choice affects this plot*

FINAL REMARKS ON PDFS

- In range $10^{-3} < x < 0.1$, core PDFs (up, down, gluon) known to $\sim 1\text{-}2\%$ accuracy
- For many LHC applications, you can use PDF4LHC15 set, which merges CT14, MMHT2014, NNPDF30
- Situation is not full consensus: ABM group claims substantially different gluon distribution

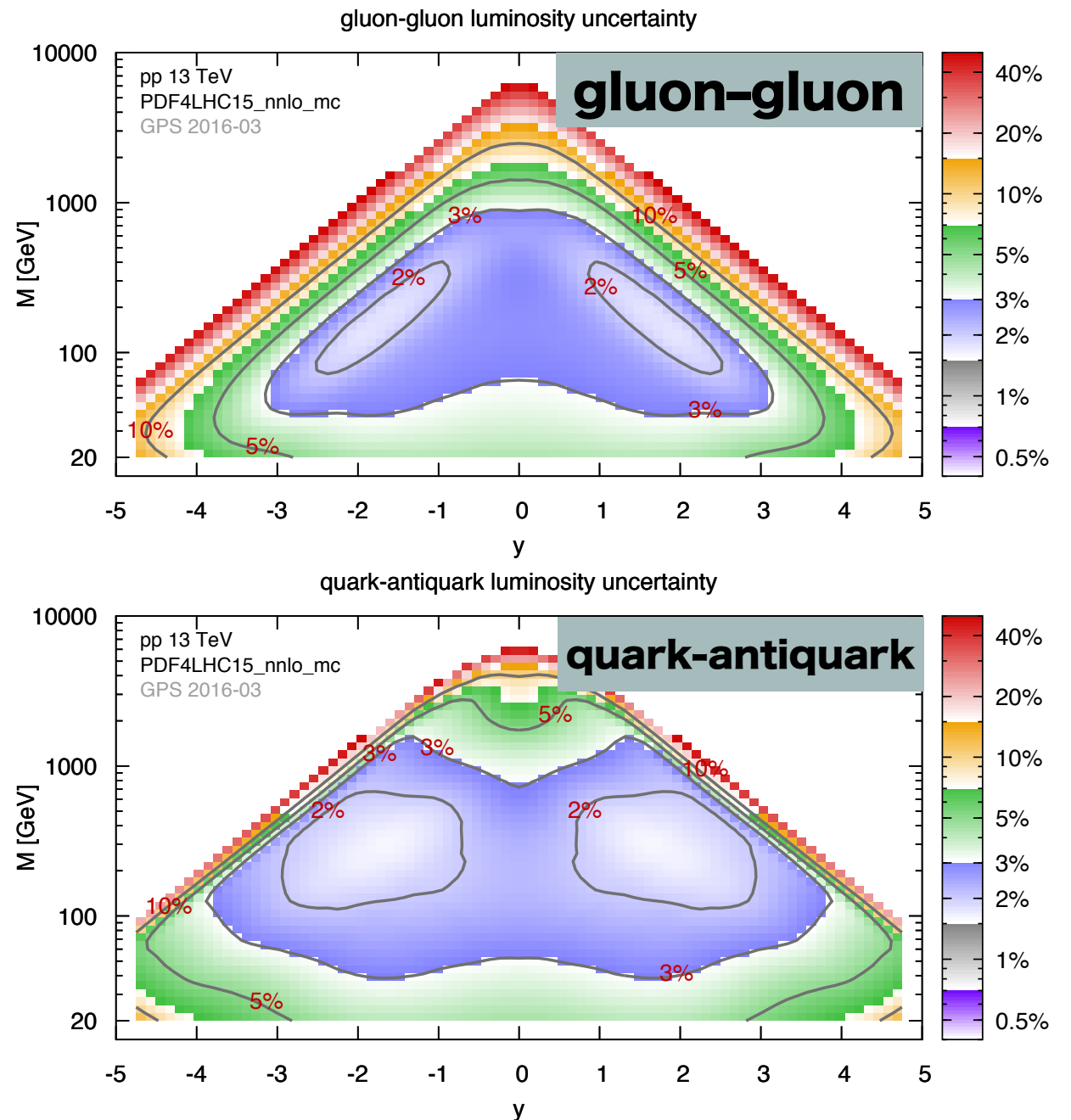
For visualisations of PDFs and related quantities,
a good place to start is

<http://apfel.mi.infn.it/> (ApfelWeb)

EXTRA SLIDES

PDFs: WHAT ROUTE FOR PROGRESS?

- Current status is 2–3% for core “precision” region
- Path to 1% is not clear — e.g. $Z p_T$'s strongest constraint is on qg lumi, which is already best known (why?)
- It'll be interesting to revisit the question once $t\bar{t}$, incl. jets, $Z p_T$, etc. have all been incorporated at NNLO
- **Can expts. get better lumi determination? 0.5%?**



PDF THEORY UNCERTAINTIES

Theory Uncertainties

