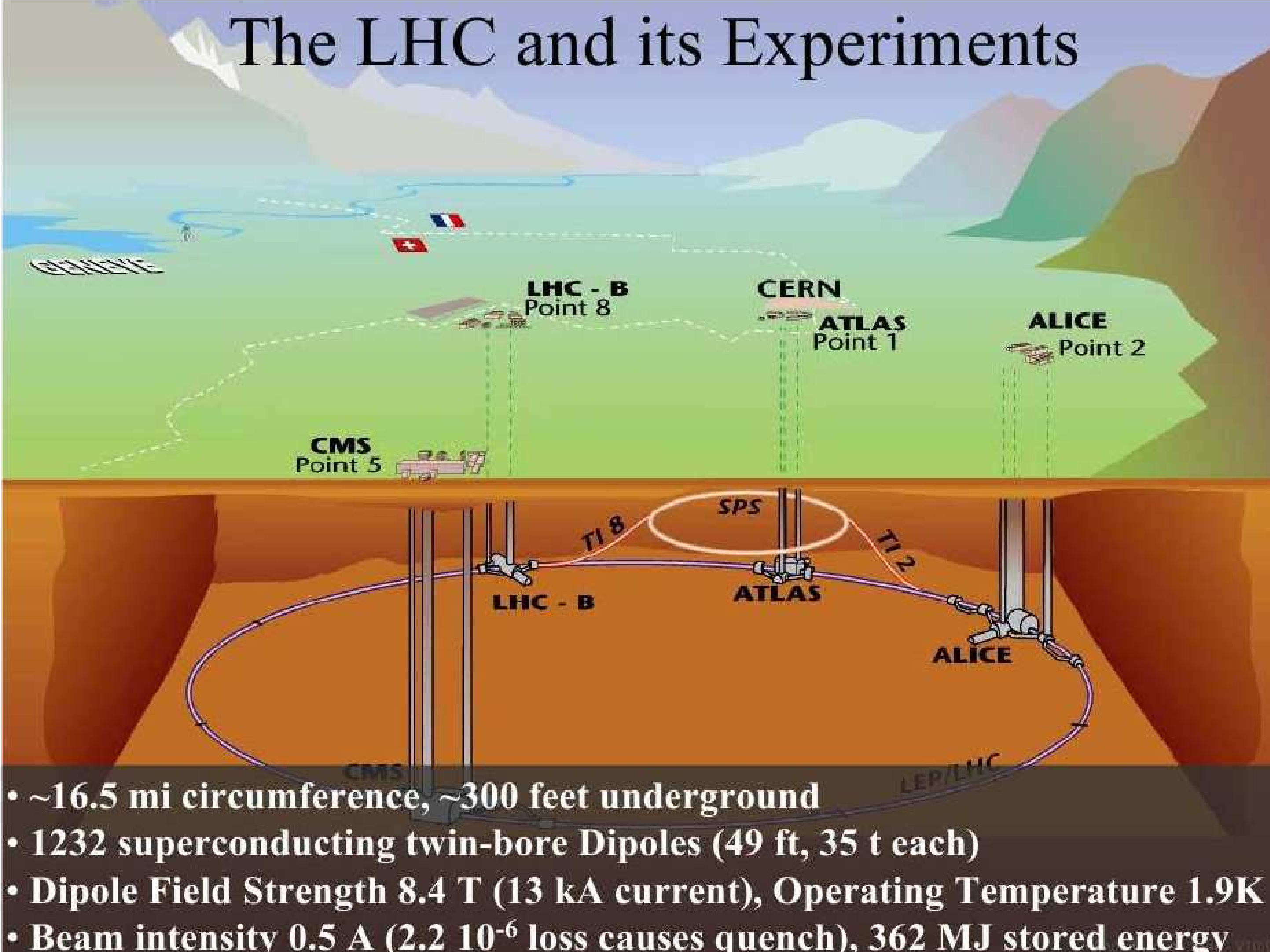
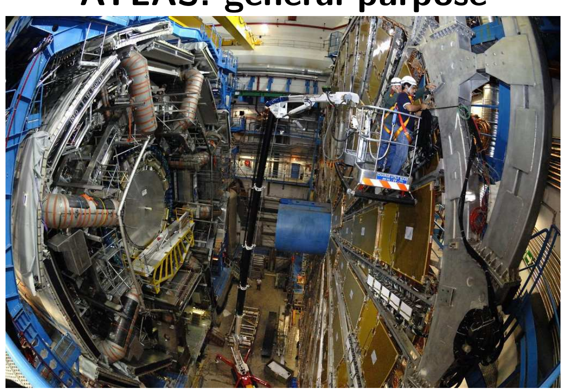
INGREDIENTS FOR ACCURATE COLLIDER PHYSICS

Gavin Salam, CERN

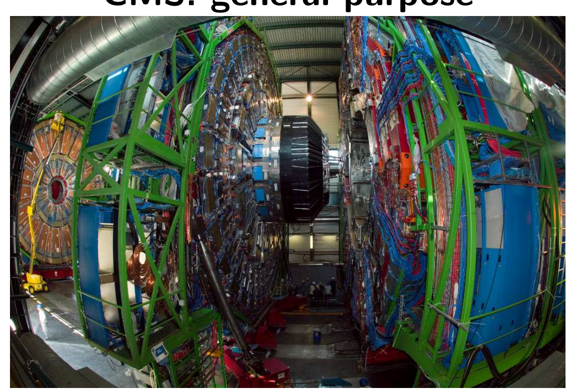
PSI Summer School Exothiggs,
Zuoz, August 2016



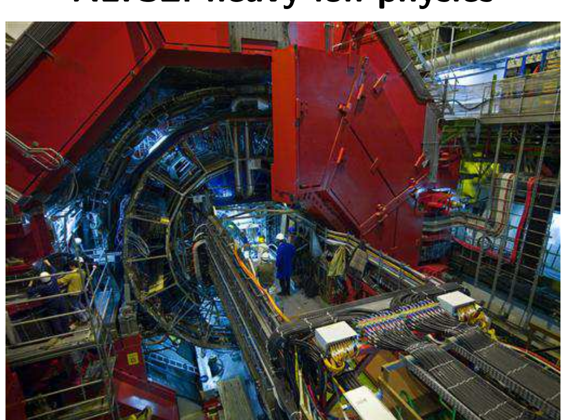
ATLAS: general purpose



CMS: general purpose



ALICE: heavy-ion physics



LHCb: B-physics

+ TOTEM, LHCf

LHC – TWO ROLES – A DISCOVERY MACHINE AND A PRECISION MACHINE

Today

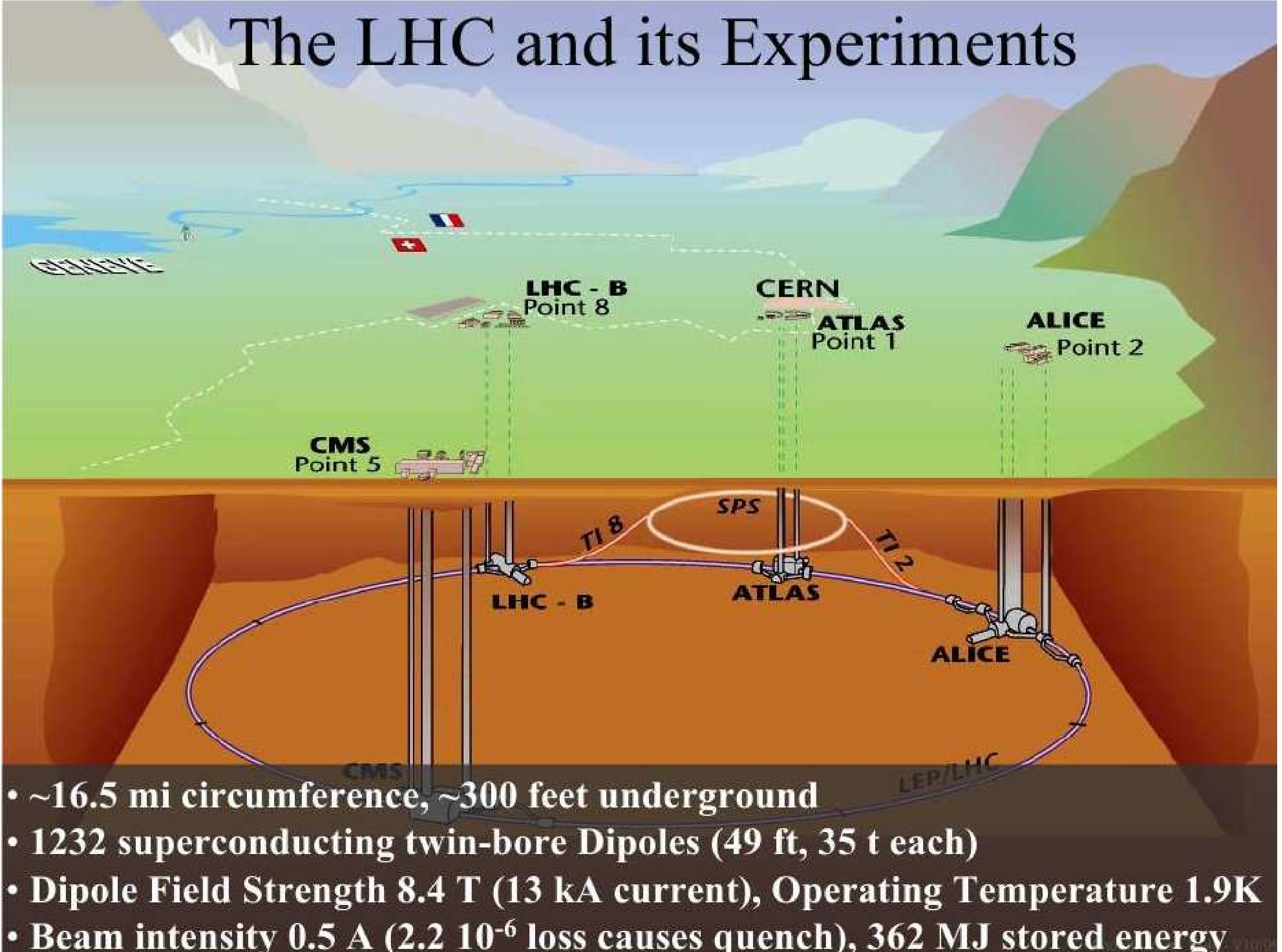
- 20 fb^{-1} at 8 TeV
- 13 fb^{-1} at 13 TeV

Future

- 2018: 100 fb^{-1} @ 13 TeV
- 2023: 300 fb^{-1} @ 1? TeV
- 2035: 3000 fb^{-1} @ 14 TeV

1 fb^{-1} = 10^{14} collisions

Increase in luminosity brings discovery reach and precision



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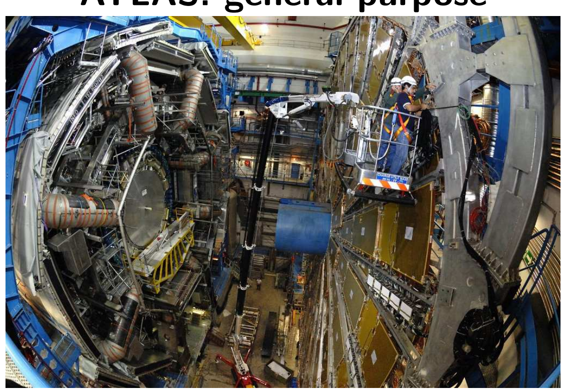

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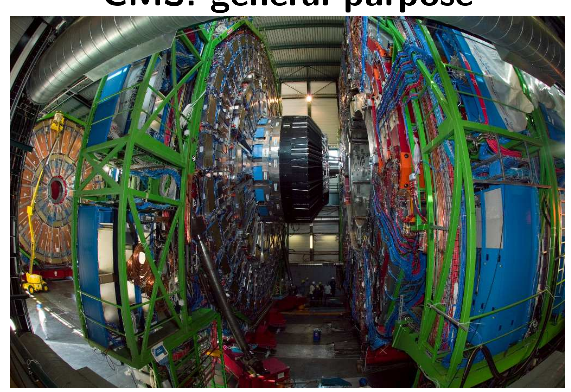

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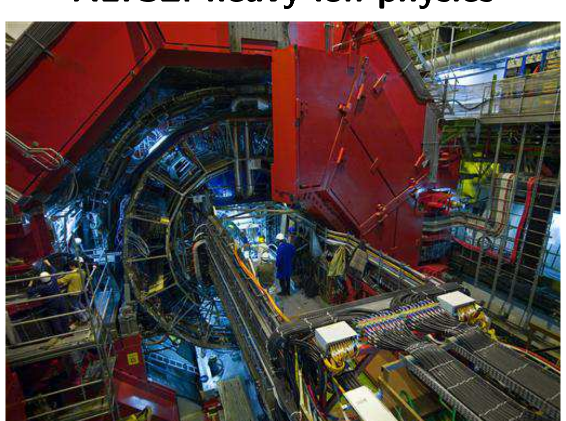

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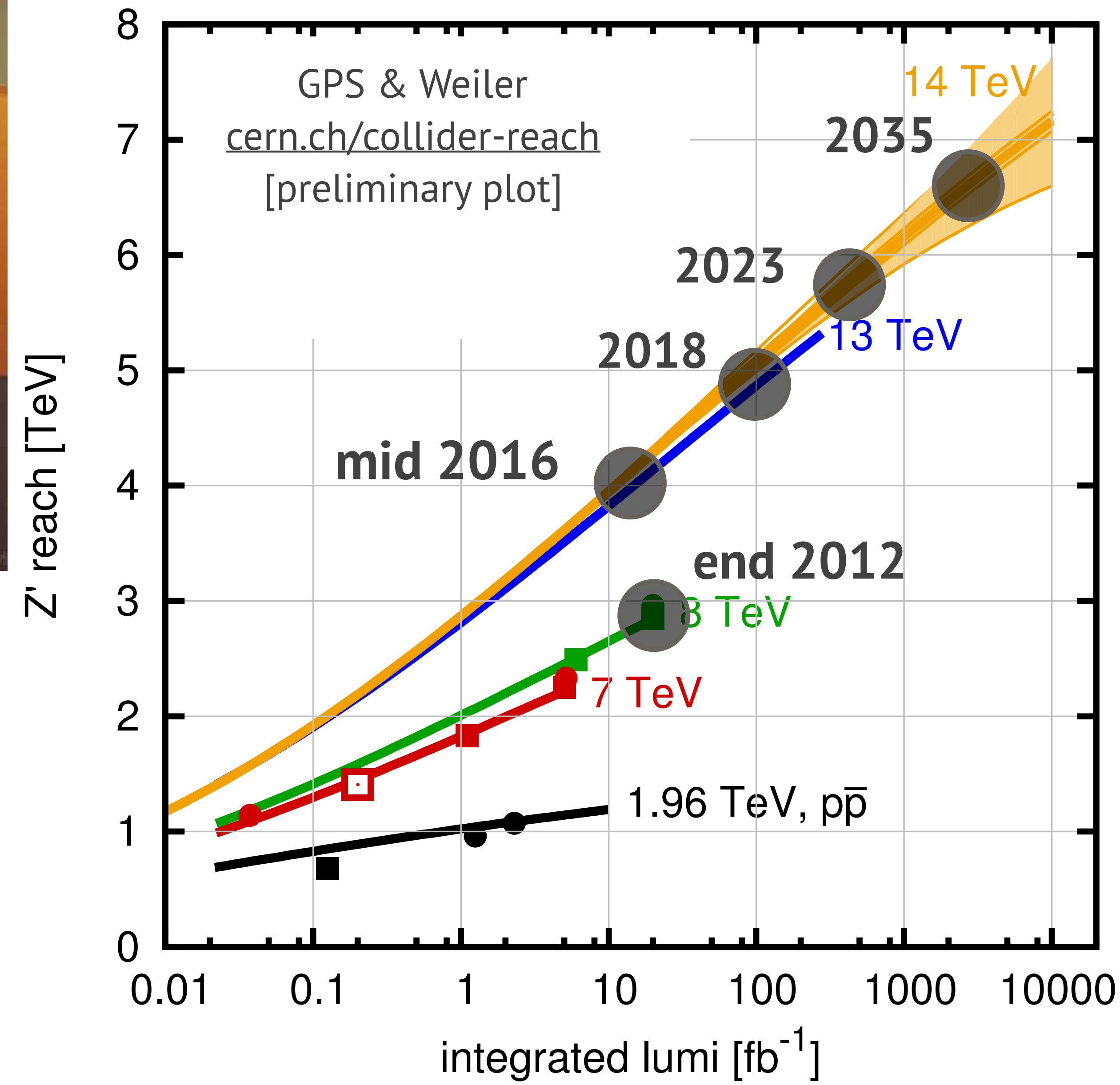
Z' exclusion reach v. lumi



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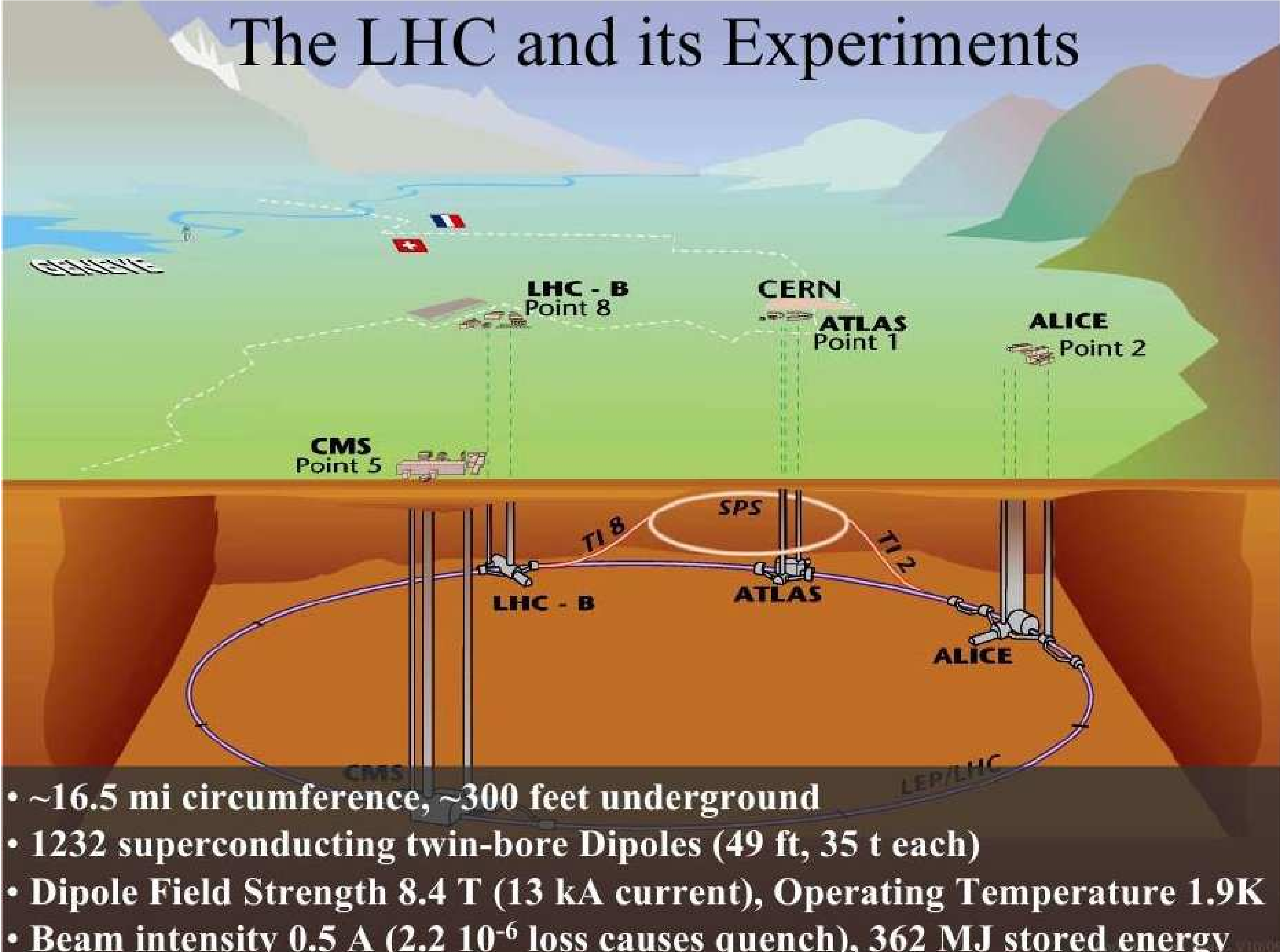

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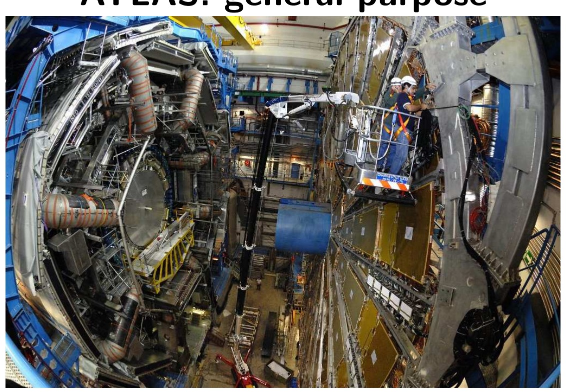

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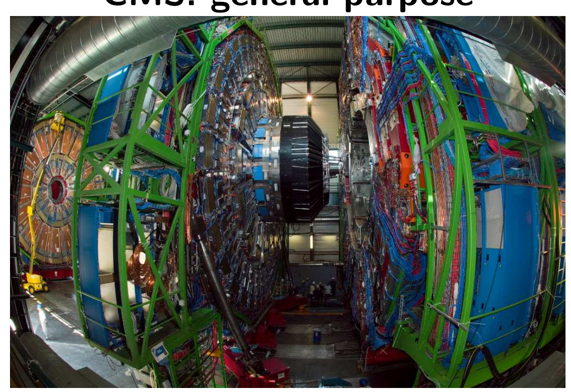

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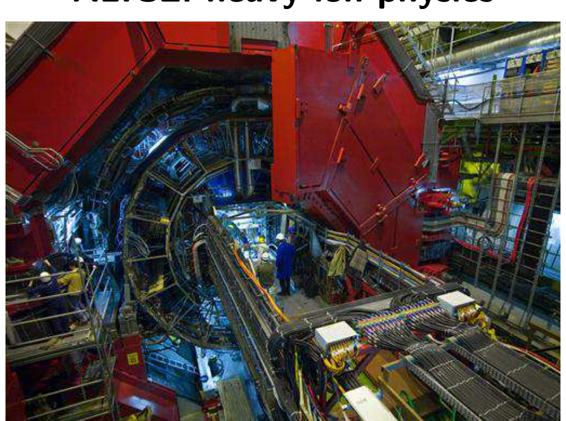

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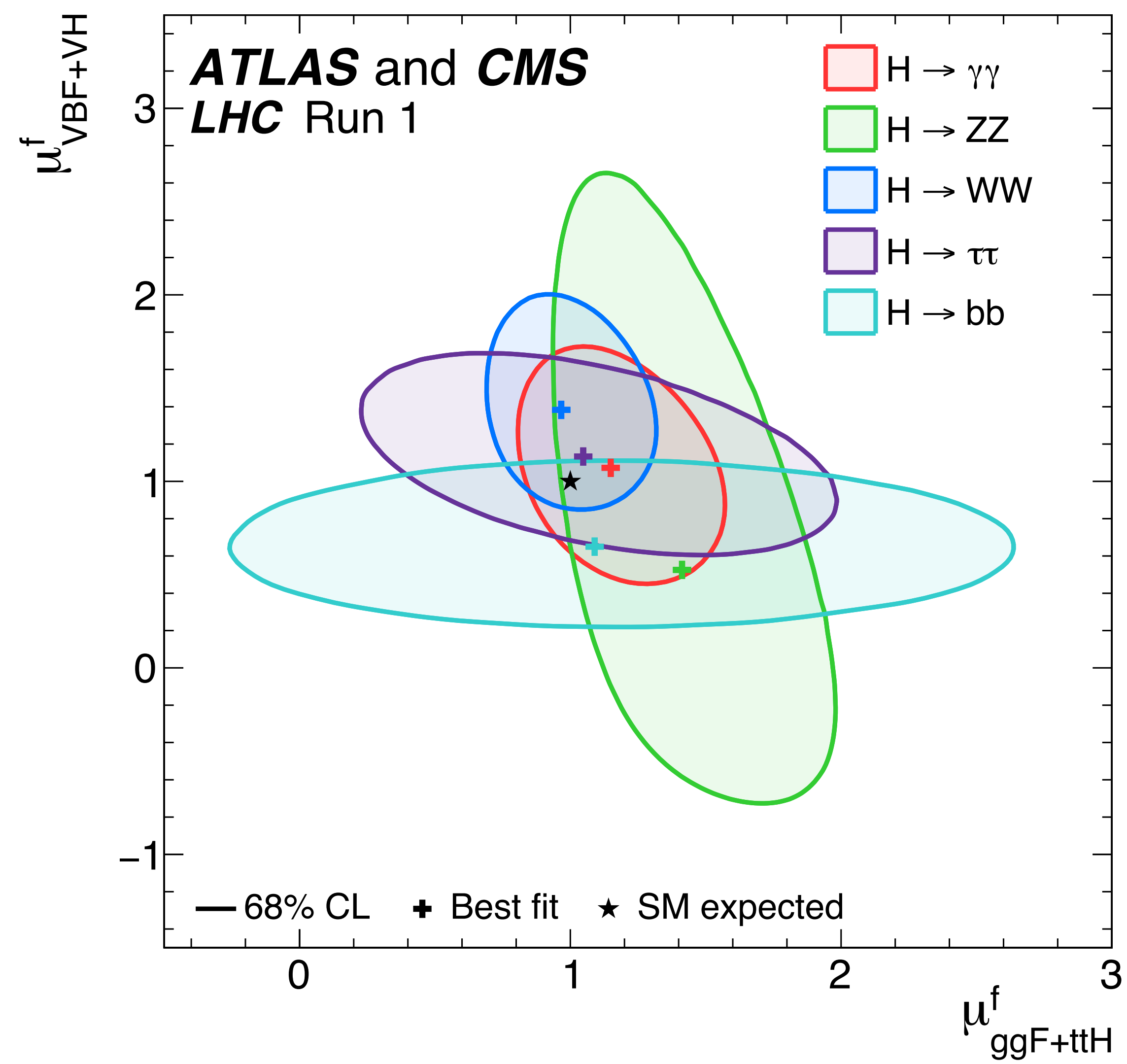
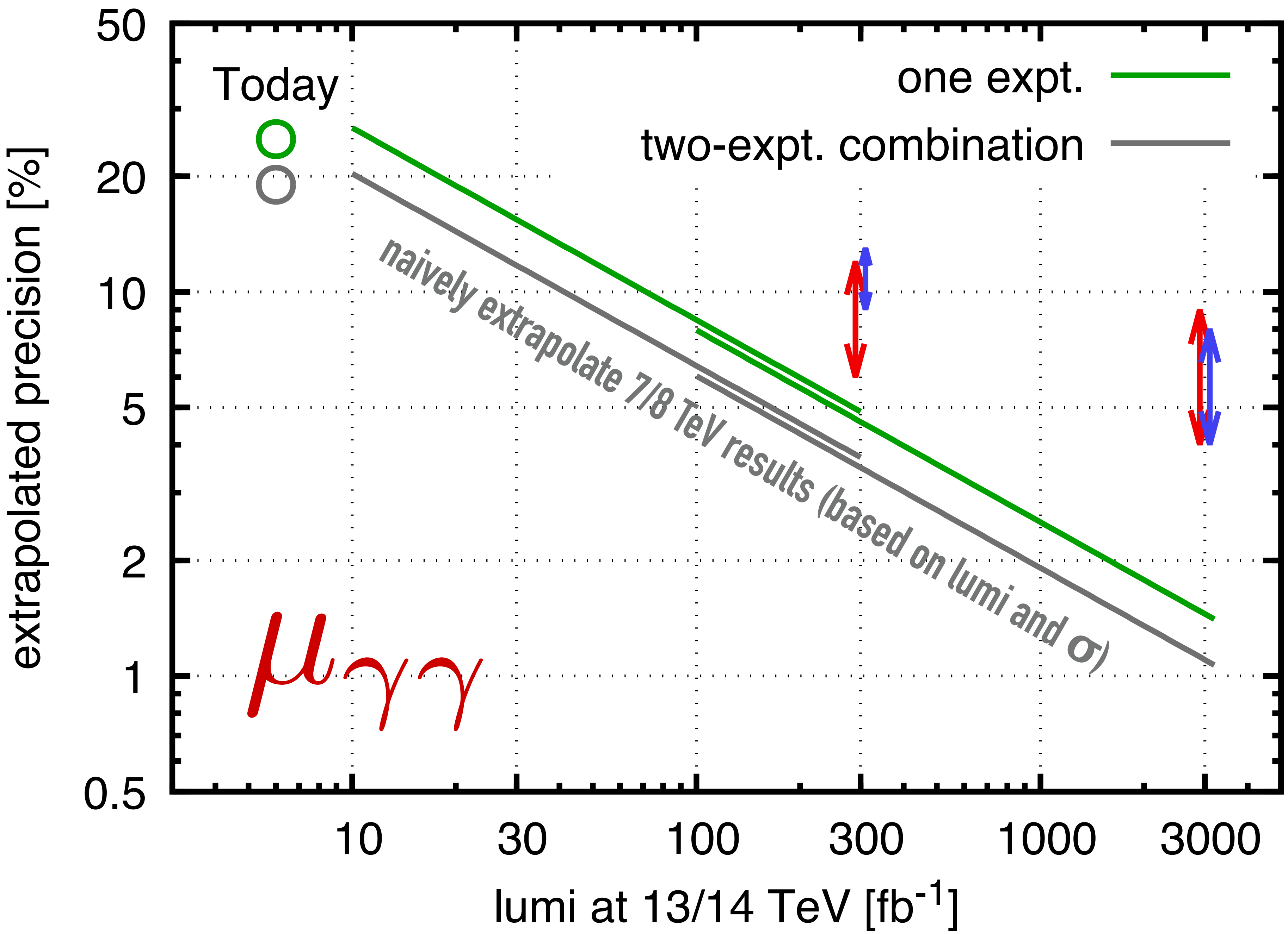
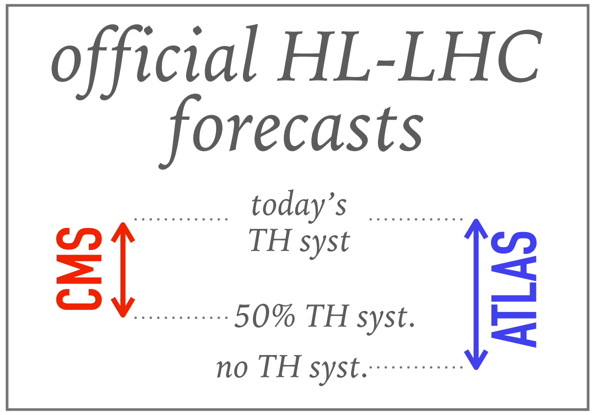
Higgs couplings



Increase in luminosity brings discovery reach and precision

LONG-TERM HIGGS PRECISION?





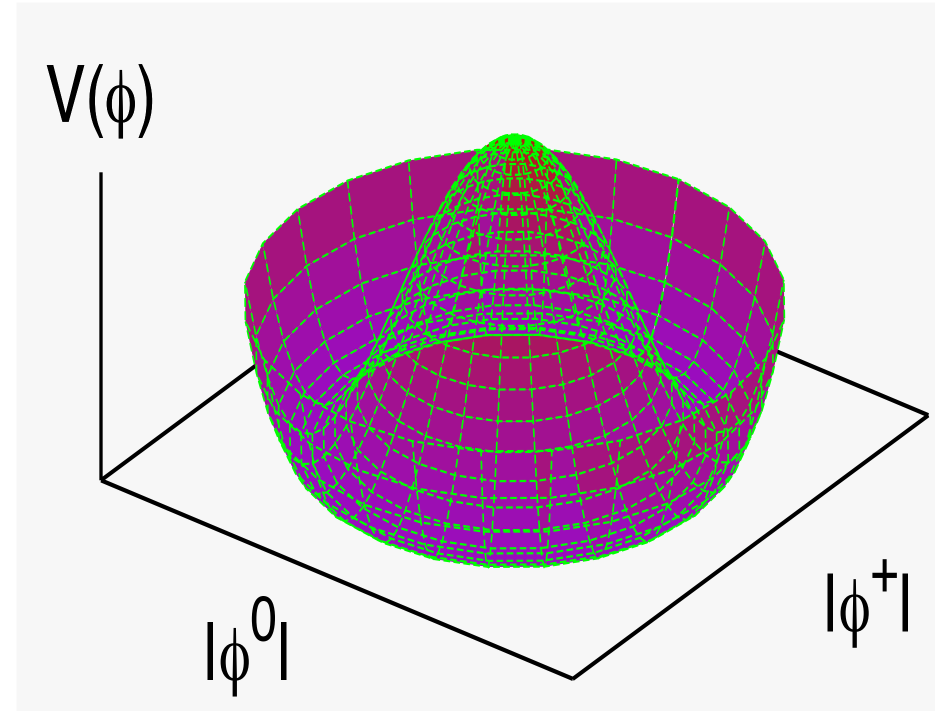
Naive extrapolation suggests LHC has long-term potential to do Higgs physics at **1% accuracy**

THE HIGGS SECTOR

The theory is old (1960s-70s).

But the particle and it's theory are unlike anything we've seen in nature.

- A fundamental scalar φ, i.e. spin 0
  (all other particles are spin 1 or 1/2)
- A potential $V(\varphi) \sim -\mu^2(\varphi\varphi^\dagger) + \lambda(\varphi\varphi^\dagger)^2$, which until now was limited to being theorists' "toy model" (φ^4)
- "Yukawa" interactions responsible for fermion masses, $y_i \phi \bar{\psi}\psi$, with couplings (y_i) spanning 5 orders of magnitude



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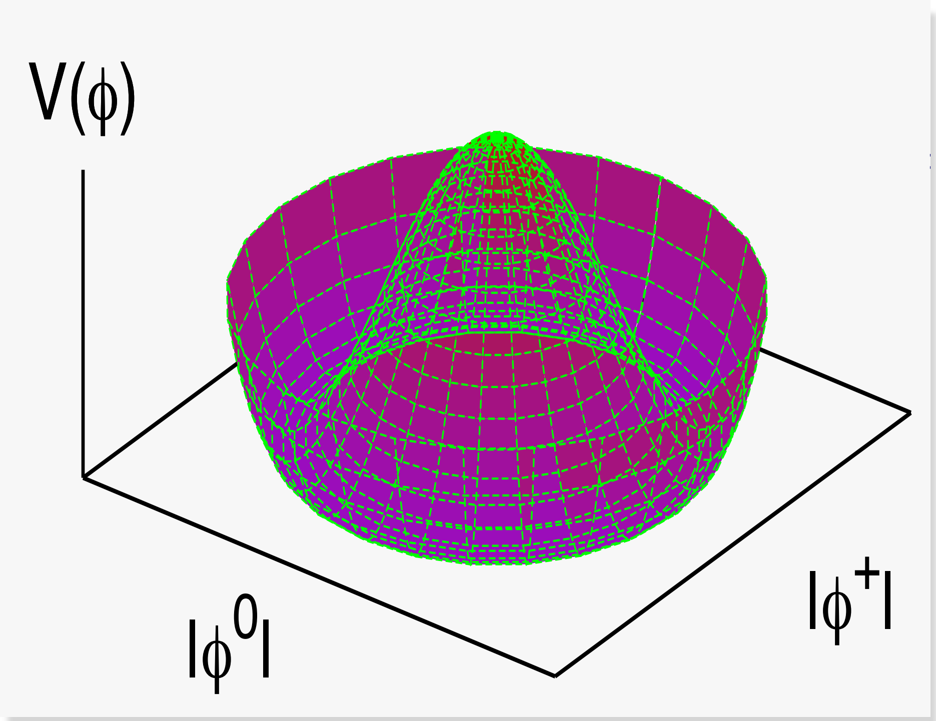


Higgs sector needs stress-testing

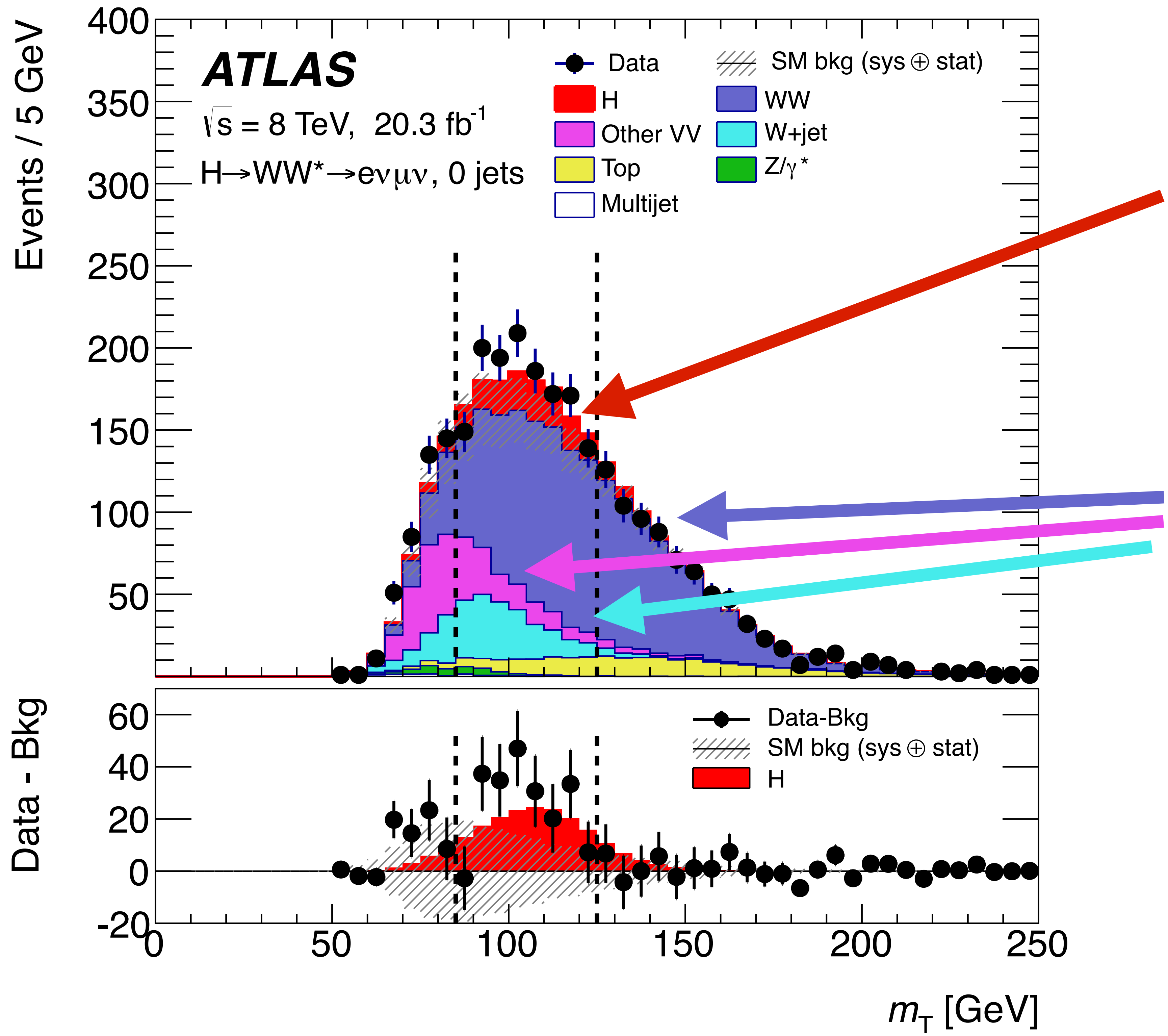
Is Higgs fundamental or composite?
If fundamental, is it "minimal"?
Is it really φ^4?
Are Yukawa couplings responsible for all fermion masses?

3 Signal and background models

The ggF and VBF production modes for $H \to WW^*$ are modelled at next-to-leading order (NLO) in the strong coupling α_S with the POWHEG MC generator [22–25], interfaced with PYTHIA8 [26] (version 8.165) for the parton shower, hadronisation, and underlying event. The CT10 [27] PDF set is used and the parameters of the PYTHIA8 generator controlling the modelling of the parton shower and the underlying event are those corresponding to the AU2 set [28]. The Higgs boson mass set in the generation is 125.0 GeV, which is close to the measured value. The POWHEG ggF model takes into account finite quark masses and a running-width Breit–Wigner distribution that includes electroweak corrections at NLO [29]. To improve the modelling of the Higgs boson p_T distribution, a reweighting scheme is applied to reproduce the prediction of the next-to-next-to-leading-order (NNLO) and next-to-next-to-leading-logarithm (NNLL) dynamic-scale calculation given by the HRES 2.1 program [30]. Events with ≥ 2 jets are further reweighted to reproduce the p_T^H spectrum predicted by the NLO POWHEG simulation of Higgs boson production in association with two jets (H + 2 jets) [31]. Interference with continuum WW production [32, 33] has a negligible impact on this analysis due to the transverse-mass selection criteria described in Section 4 and is not included in the signal model.

Jets are reconstructed from topological clusters of calorimeter cells [50–52] using the anti-k_t algorithm with a radius parameter of $R = 0.4$ [53]. Jet energies are corrected for the effects of calorimeter non-

ATLAS H →WW* ANALYSIS [1604.02997]



(a) $N_{\text{jet}} = 0$

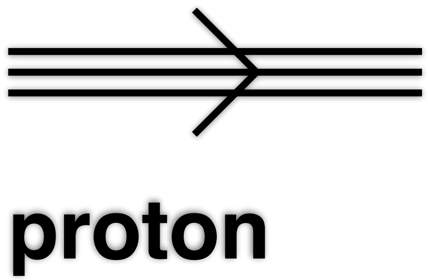
That whole paragraph was just for the red part of this distribution (the Higgs signal).

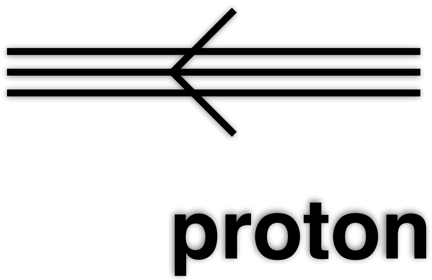
Complexity of modelling each of the backgrounds is comparable

AIMS OF THESE LECTURES

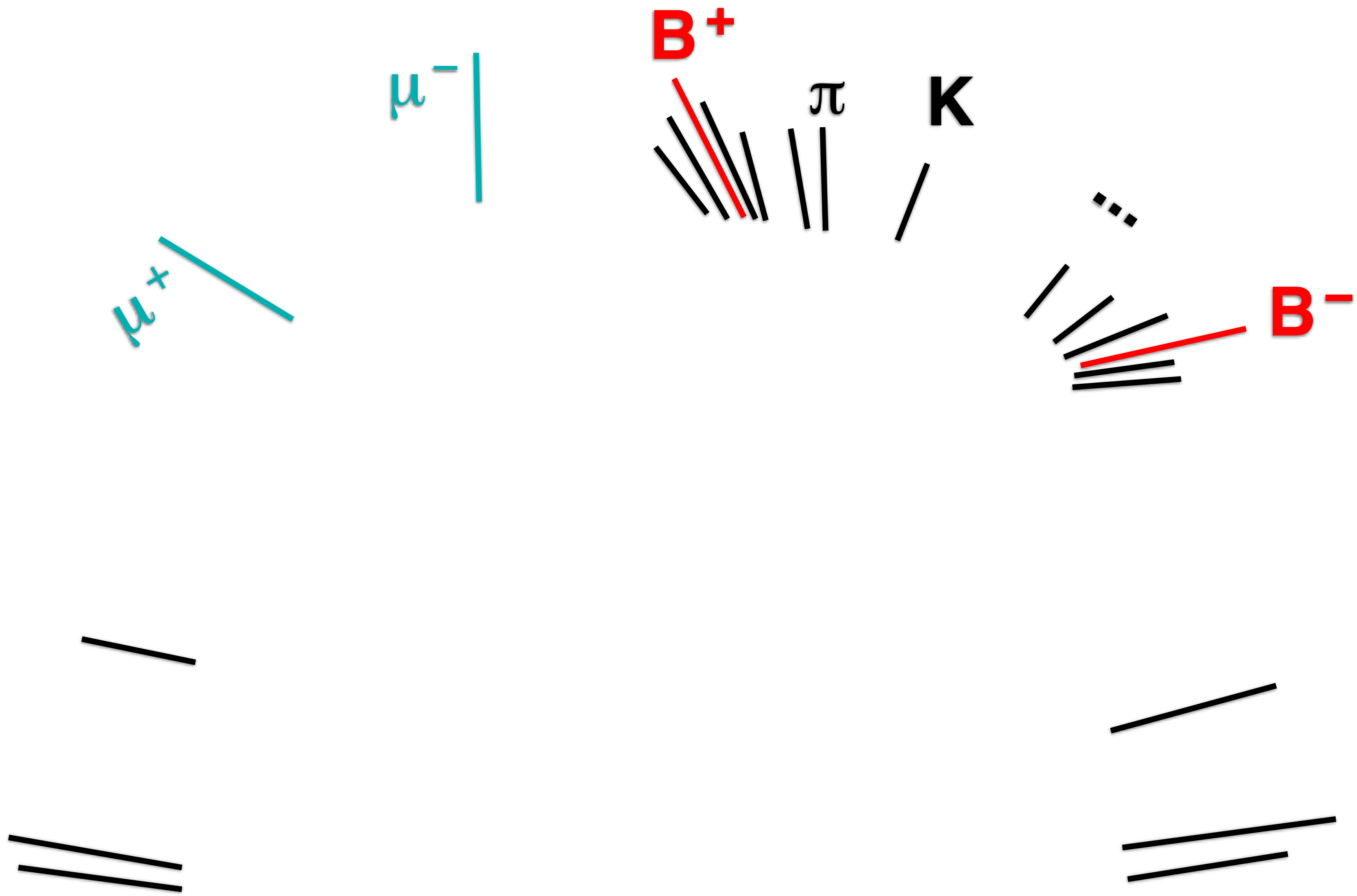
- Give you basic understanding of the **"jargon"** of theoretical collider prediction methods and inputs
- Give you insight into the **power & limitations of different techniques** for making collider predictions

A proton-proton collision: INITIAL STATE



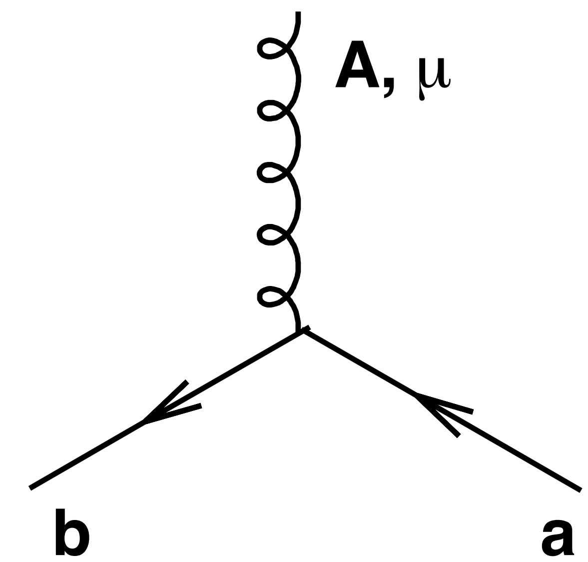


A proton-proton collision: FINAL STATE



(actual final-state multiplicity ~ several hundred hadrons)

IT'S MOSTLY QUANTUM CHROMODYNAMICS (QCD)



$\bar{\psi}_b(-ig_s t^A_{ba}\gamma^\mu)\psi_a$

Quarks — 3 colours: $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Quark part of Lagrangian:

$$\mathcal{L}_q = \bar{\psi}_a(i\gamma^\mu\partial_\mu\delta_{ab} - g_s\gamma^\mu t^C_{ab}\mathcal{A}^C_\mu - m)\psi_b$$

$SU(3)$ local gauge symmetry $\leftrightarrow$ 8 $(= 3^2 - 1)$ generators $t^1_{ab} \ldots t^8_{ab}$ corresponding to 8 gluons $\mathcal{A}^1_\mu \ldots \mathcal{A}^8_\mu$.

A representation is: $t^A = \frac{1}{2}\lambda^A$,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

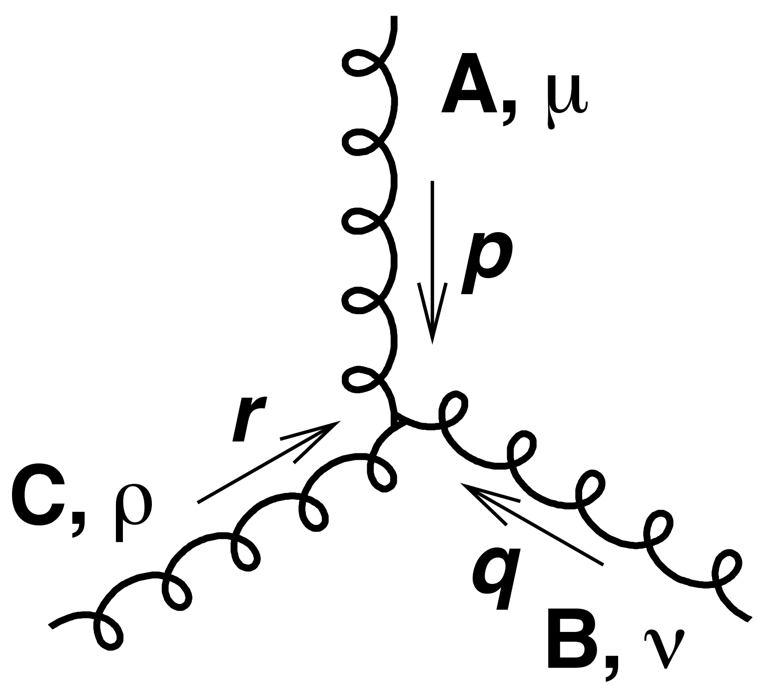
$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix},$$

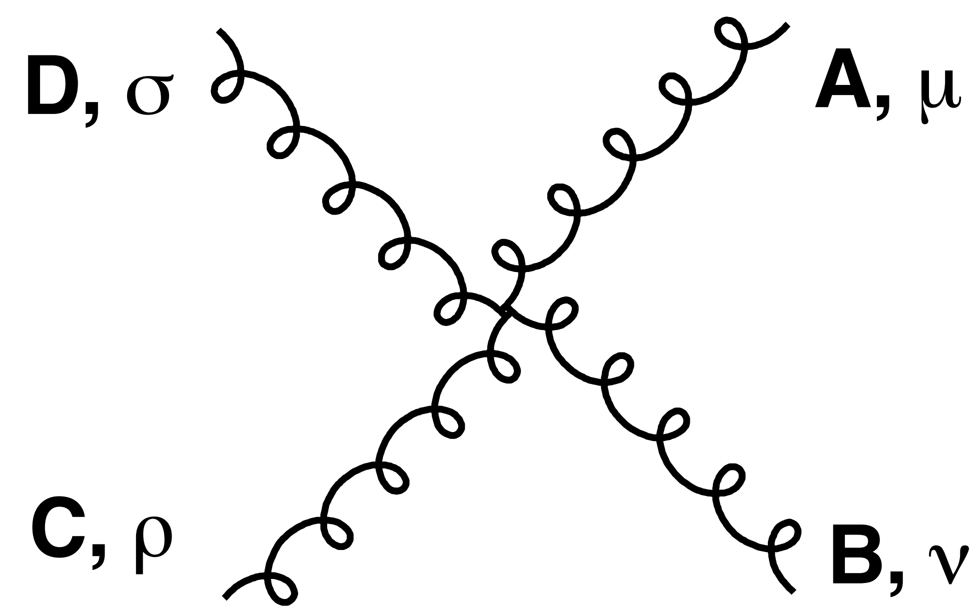
IT'S MOSTLY QUANTUM CHROMODYNAMICS (QCD)

Field tensor: $F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\nu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu$ $\qquad [t^A, t^B] = i f_{ABC} t^C$

f_{ABC} are structure constants of $SU(3)$ (antisymmetric in all indices — $SU(2)$ equivalent was ϵ^{ABC}). Needed for gauge invariance of gluon part of Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4} F^{\mu\nu}_A F^{A\,\mu\nu}$$





IT'S MOSTLY QUANTUM CHROMODYNAMICS (QCD)

The only complete solution uses **lattice QCD**

- put all quark & gluon fields on a 4d lattice
  (NB: imaginary time)
- Figure out most likely configurations
  (Monte Carlo sampling)

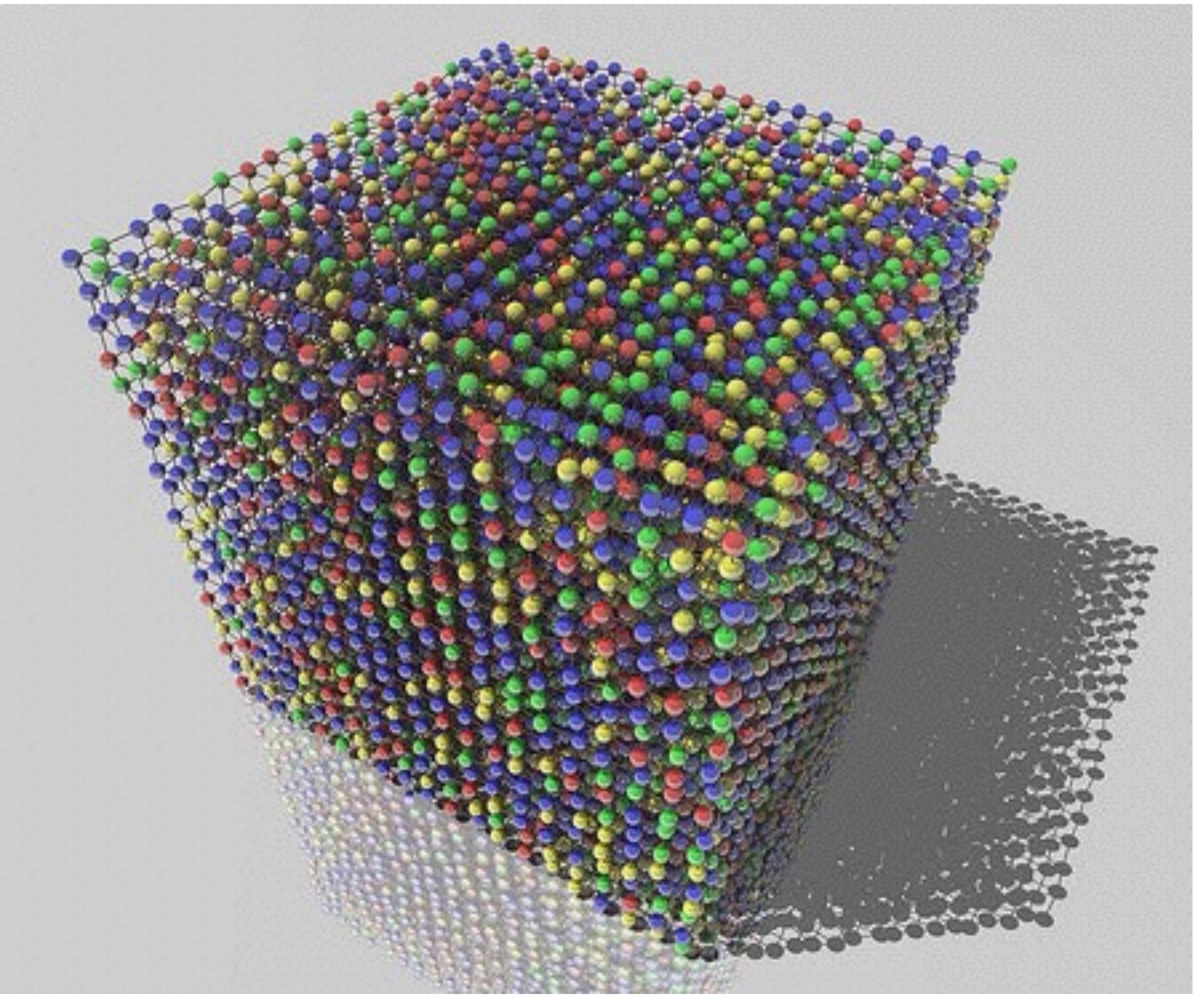


image credit fdecomite [flickr]

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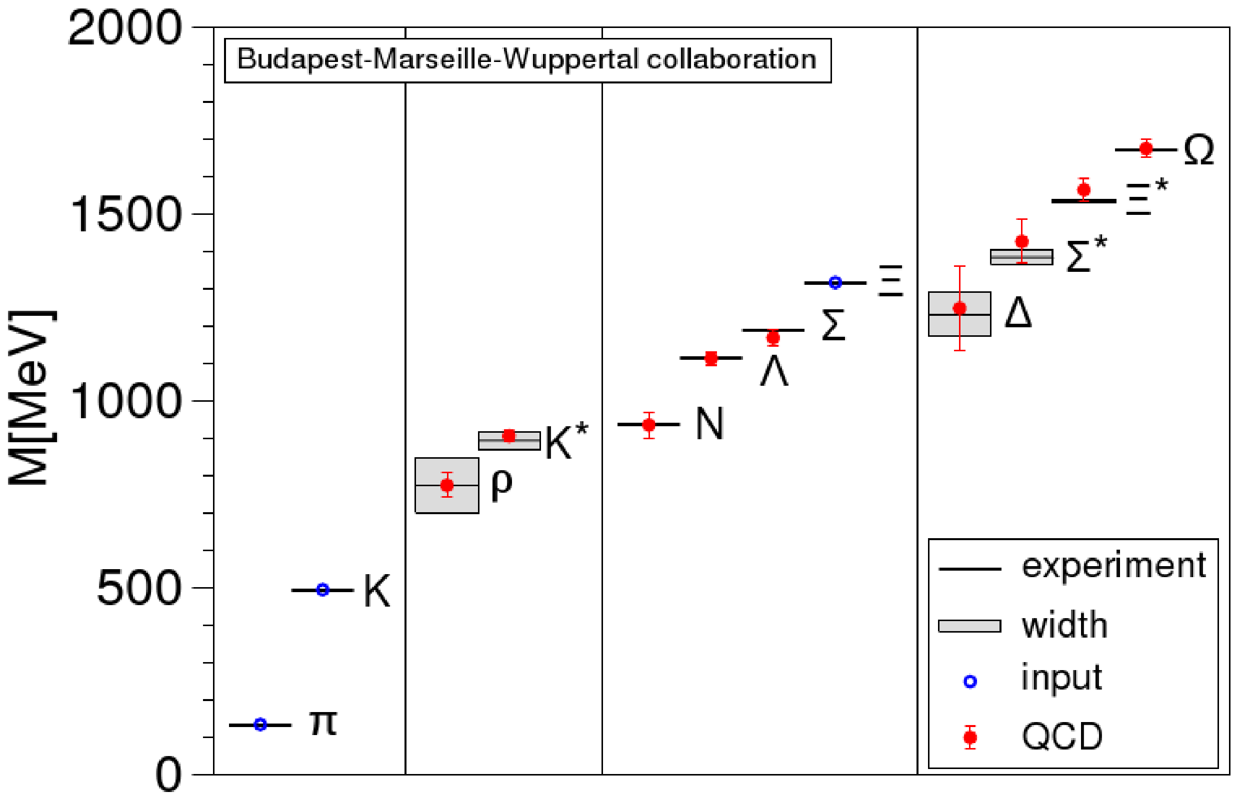
hadron spectrum from lattice QCD



Durr et al, arXiv:0906.3599

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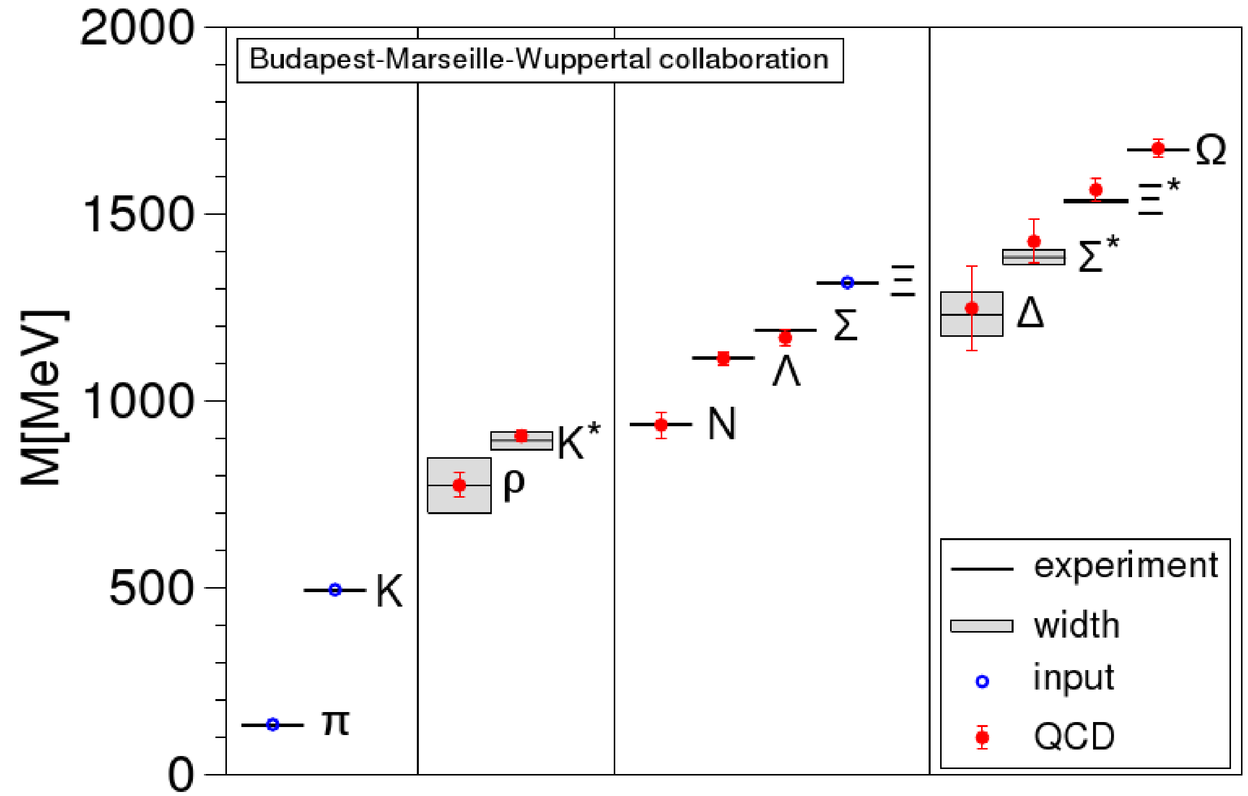

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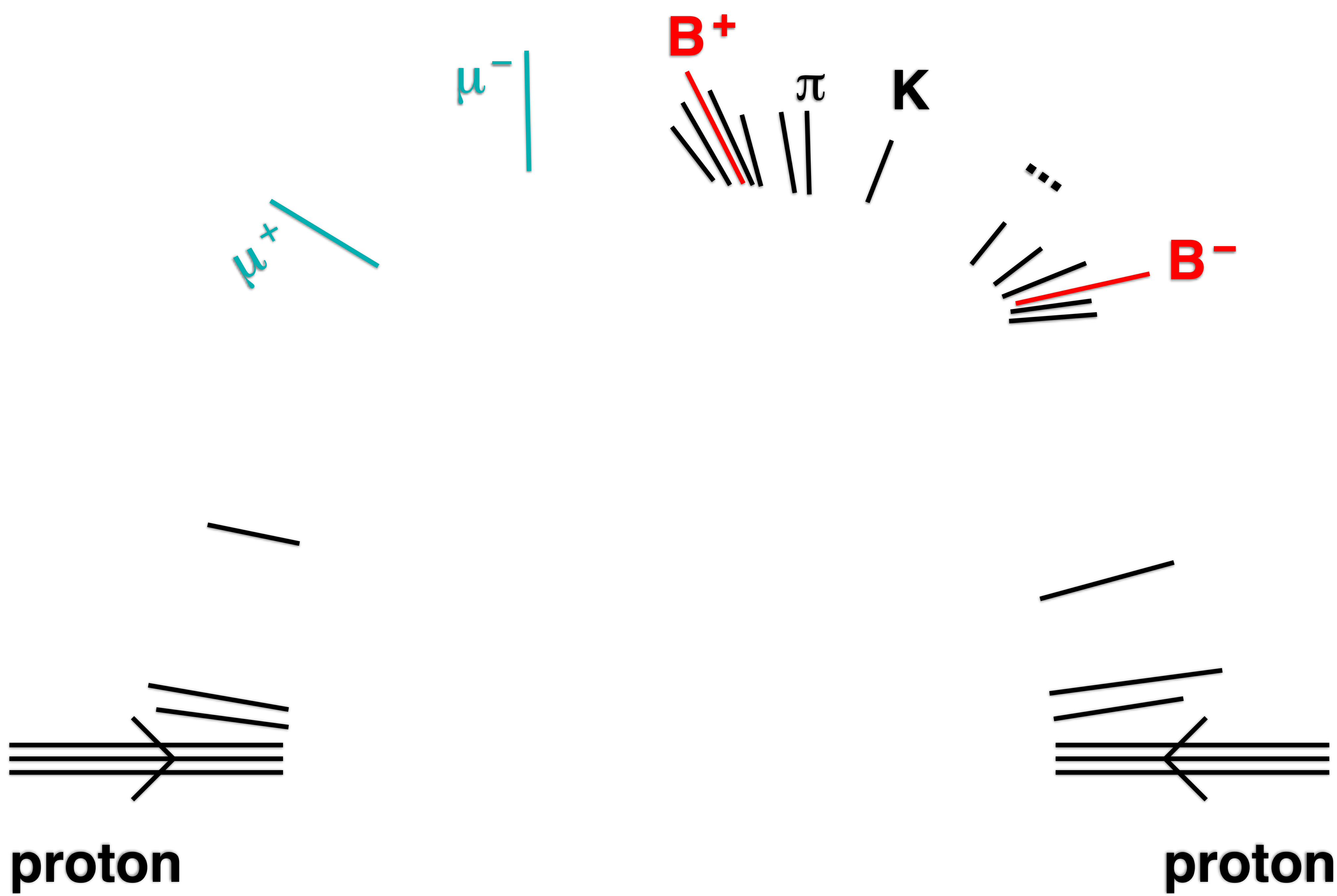
For LHC reactions, lattice would have to

- Resolve smallest length scales (2 TeV ~ 10^{-4} fm)
- Contain whole reaction (pion formed on timescale of 1fm, with boost of 10000 — i.e. 10^4 fm)

That implies 10^8 nodes in each dimension, i.e. 10^{32} nodes — **unrealistic**

A proton-proton collision: FILLING IN THE PICTURE



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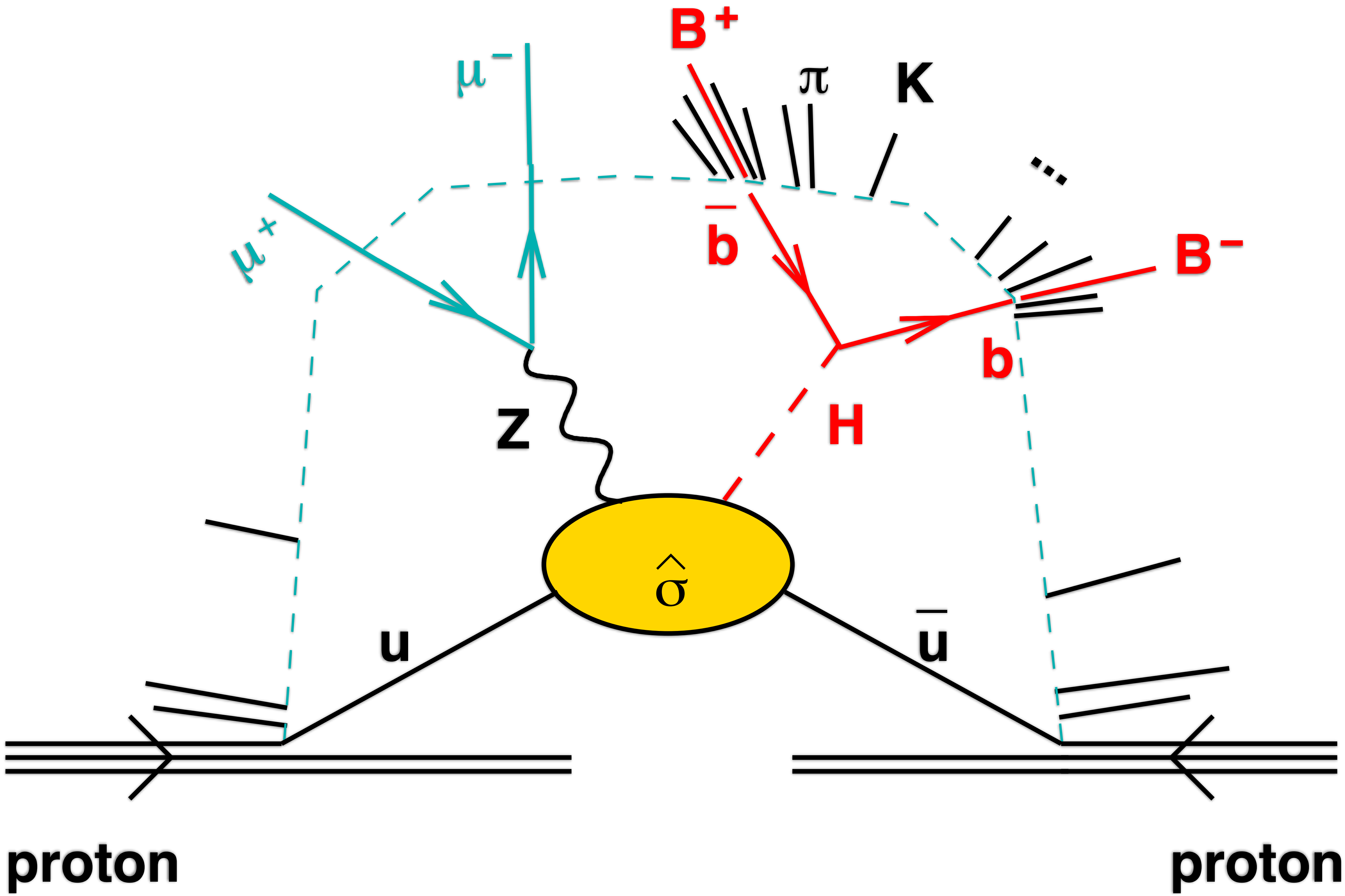
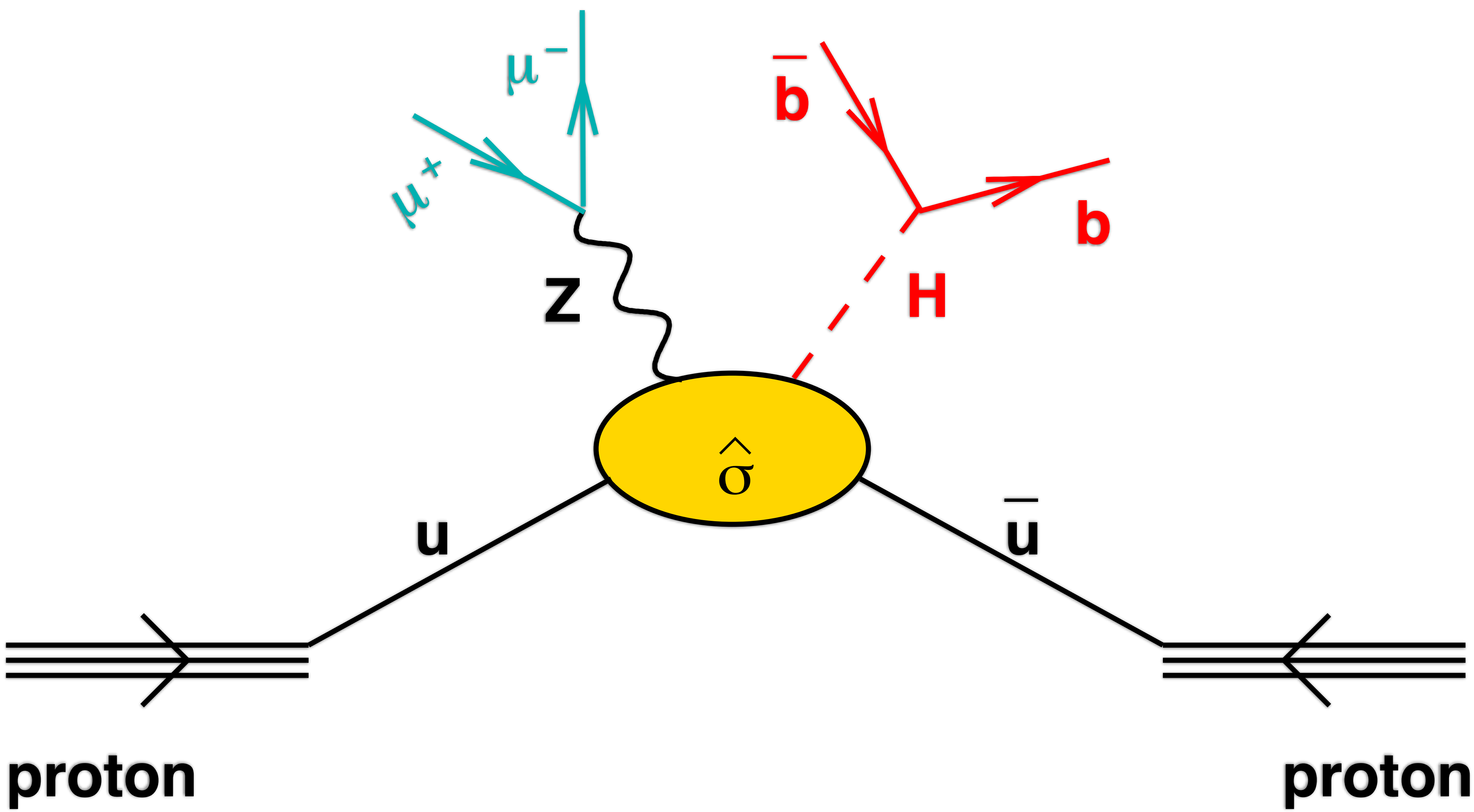
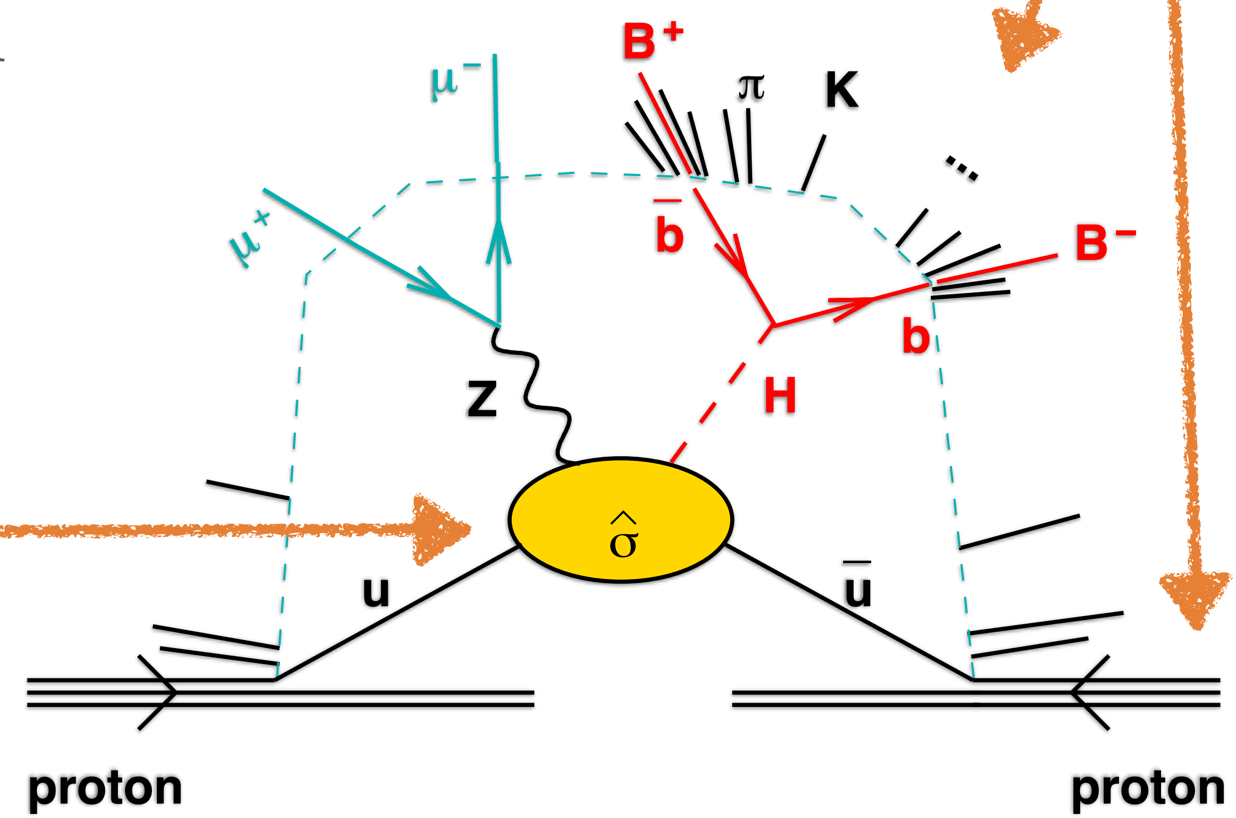


A proton-proton collision: SIMPLIFYING IN THE PICTURE



- Proton's dynamics occurs on timescale O(1 fm)
  Final-state hadron dynamics occurs on timescale O(1fm)
- Production of Higgs, Z (and other "**hard processes**") occurs on timescale $1/M_H \sim 1/125$ GeV ~ 0.002 fm



That means we can separate — "**factorise**" — the hard process, i.e. treat it as independent from all the hadronic dynamics

WHY IS SIMPLIFICATION "ALLOWED"? KEY IDEA #2

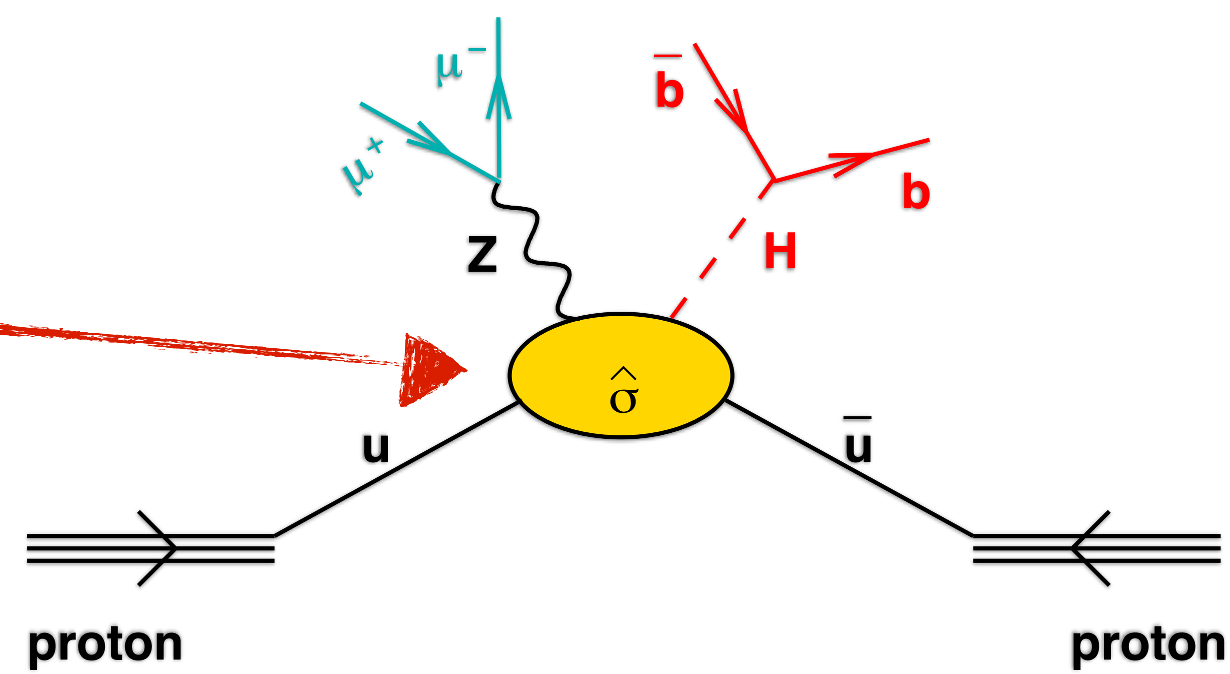
SHORT-DISTANCE QCD CORRECTIONS ARE PERTURBATIVE

- On timescales $1/M_H \sim 1/125$ GeV ~ 0.002 fm you can take advantage of **asymptotic freedom**
- i.e. you can write results in terms of an expansion in the (*not so*) strong coupling constant $\alpha_s(125\text{ GeV}) \sim 0.11$

$$\hat{\sigma} = \hat{\sigma}_0 (1 + c_1 \alpha_s + c_2 \alpha_s^2 + \cdots)$$

LO

(Leading Order)



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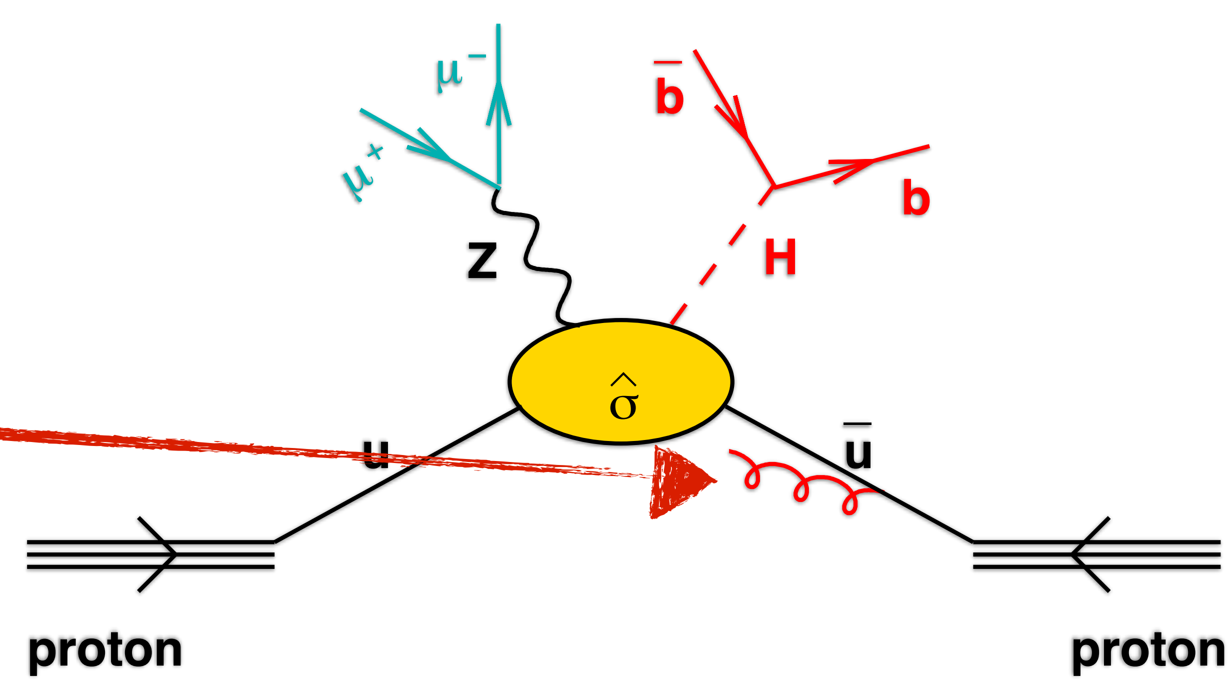
NLO
(Next-to-Leading Order)



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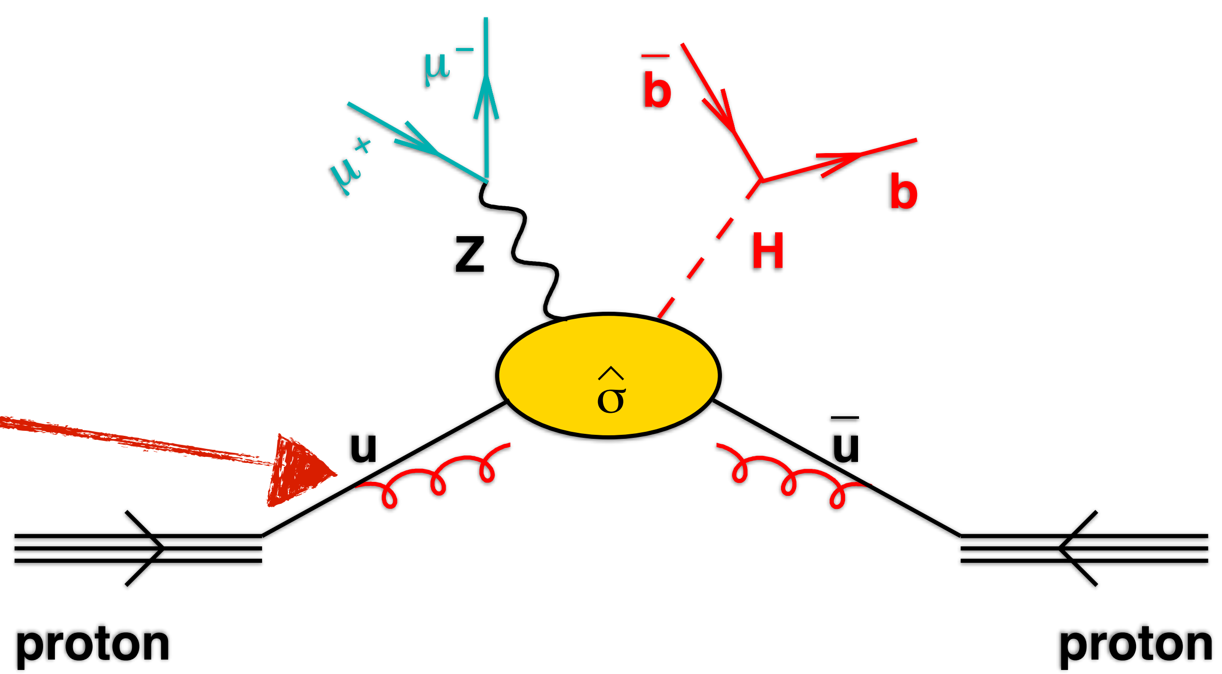
NNLO

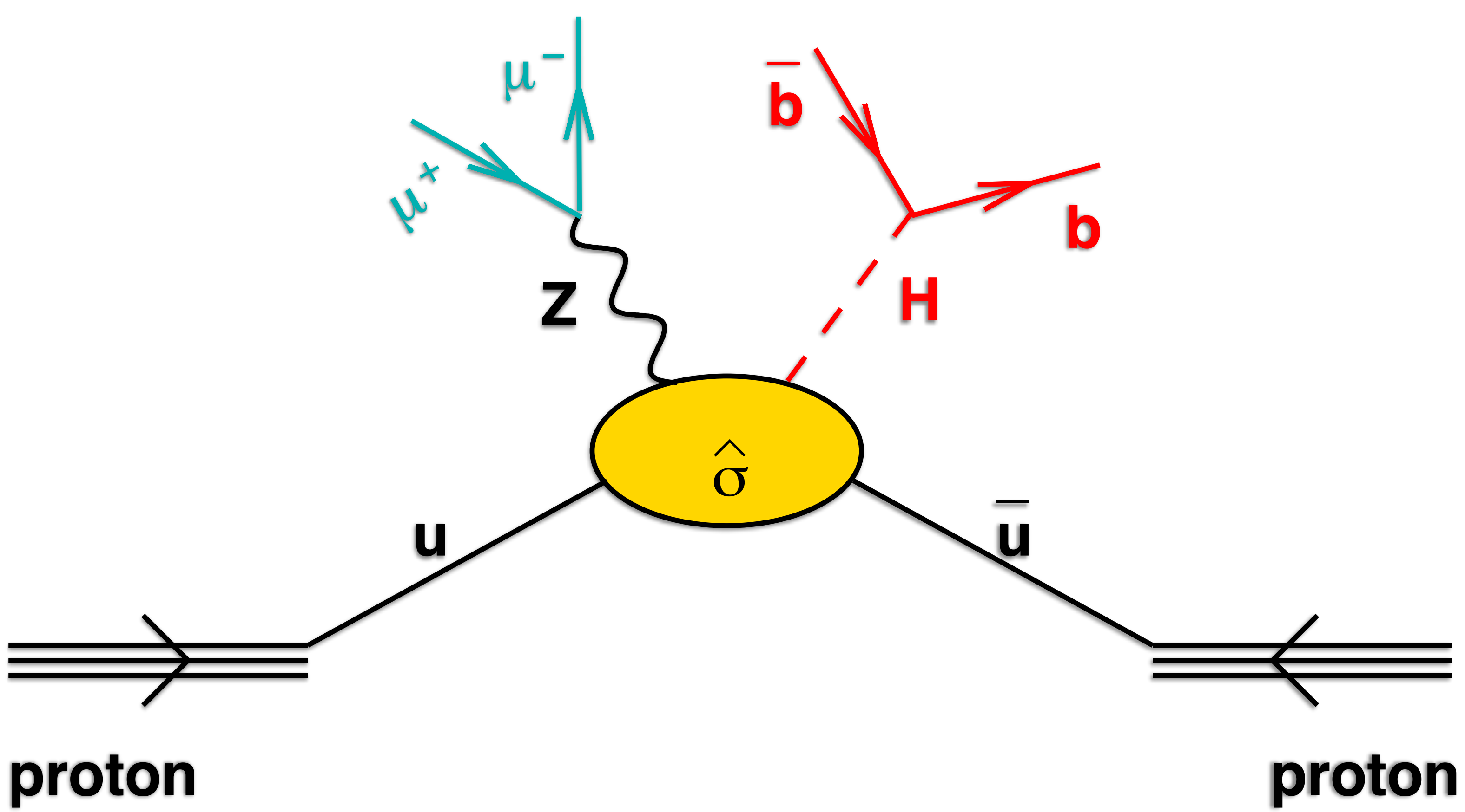
(Next-to-next-to-Leading Order)



THE MASTER EQUATION

$$\sigma\left(h_1 h_2 \rightarrow ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n \left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right)$$

$$\times\, \hat{\sigma}^{(n)}_{ij \rightarrow ZH+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right) ,$$



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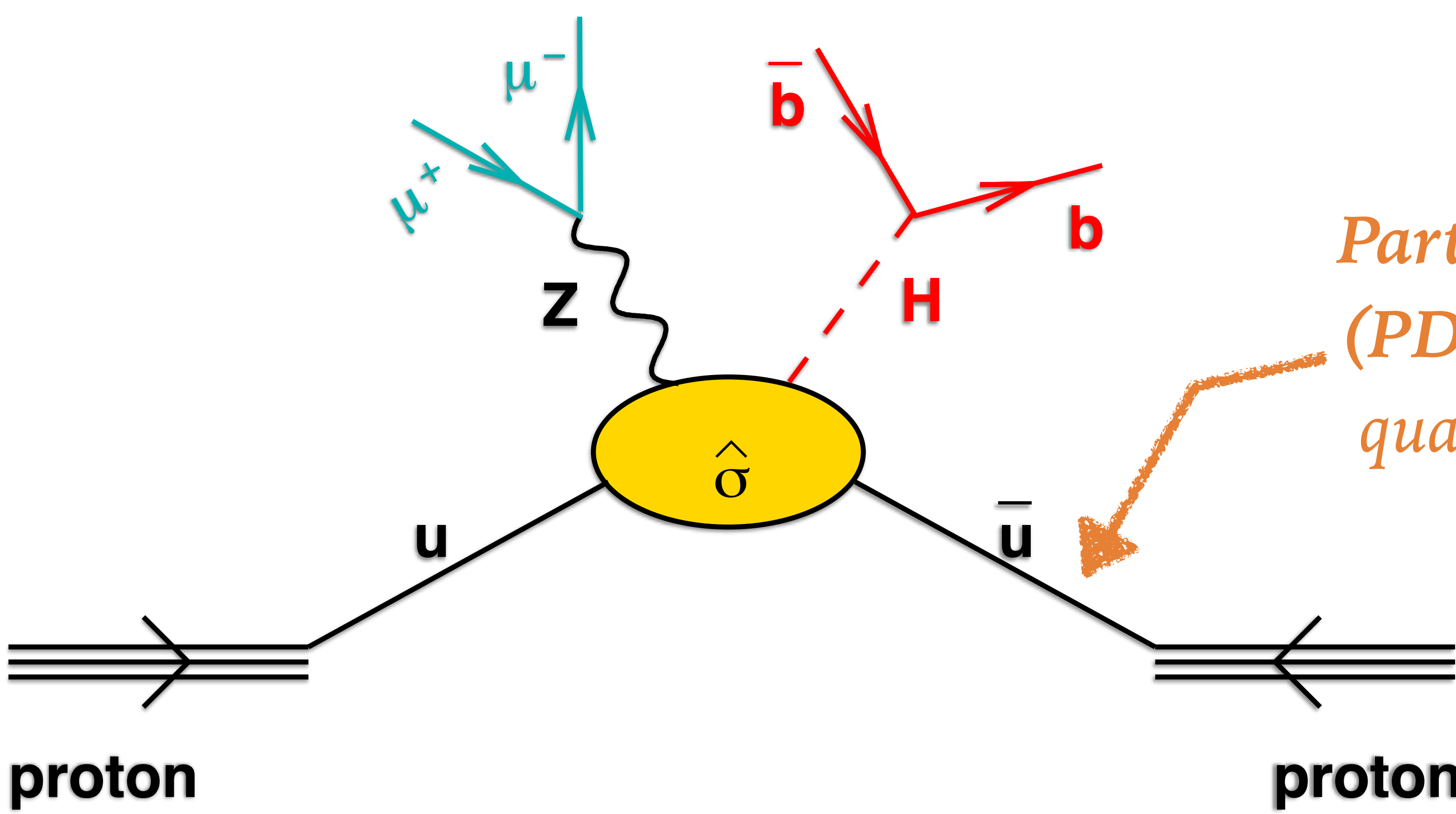

Parton distribution function (PDF): e.g. number of up anti-quarks carrying fraction x_2 of proton's momentum

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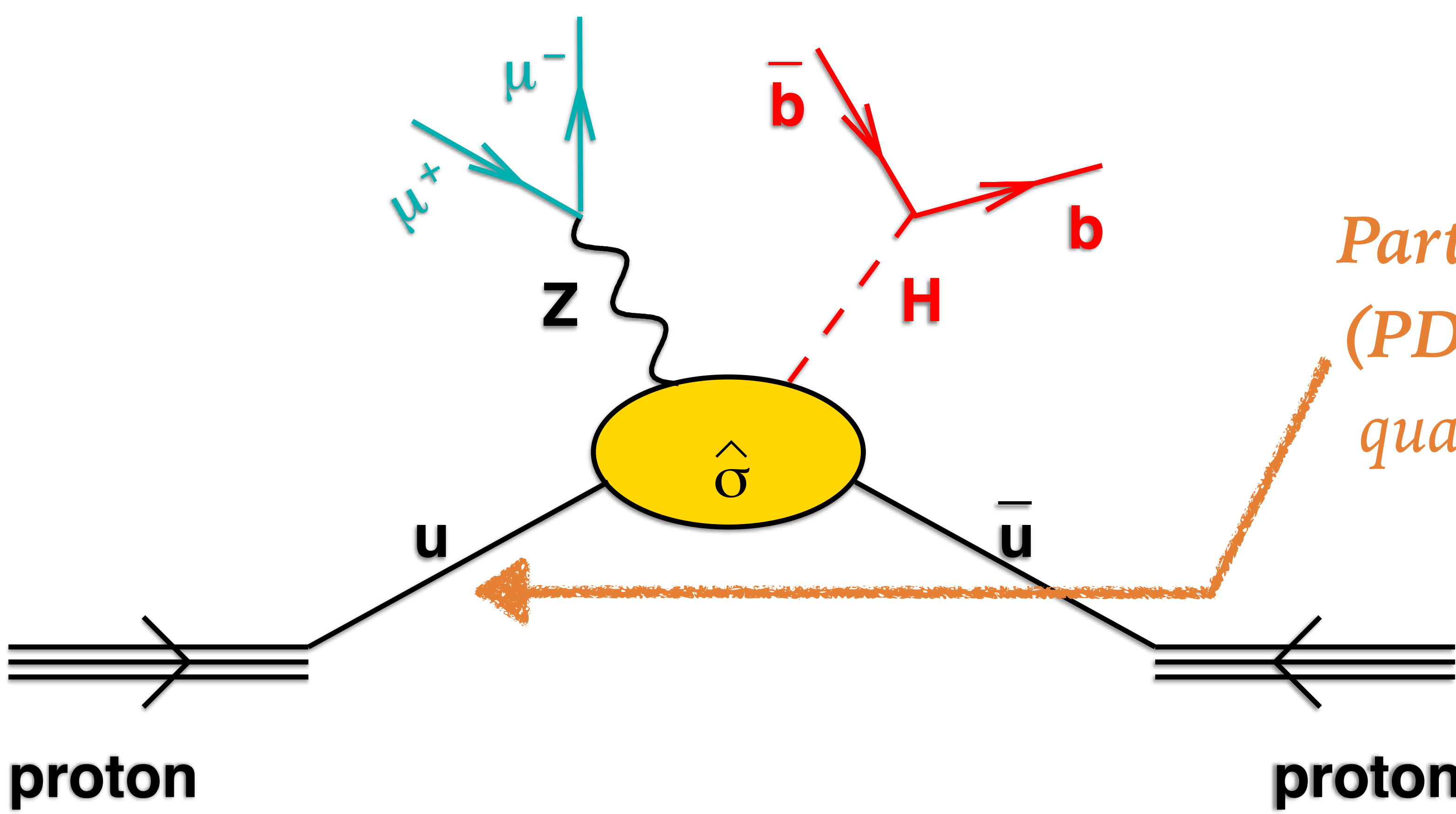

Parton distribution function (PDF): e.g. number of up quarks carrying fraction x_1 of proton's momentum

THE MASTER EQUATION

Perturbative sum over powers of the strong coupling: typically we use first 2-3 orders

$$\sigma\left(h_1 h_2 \to ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n\left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right)$$

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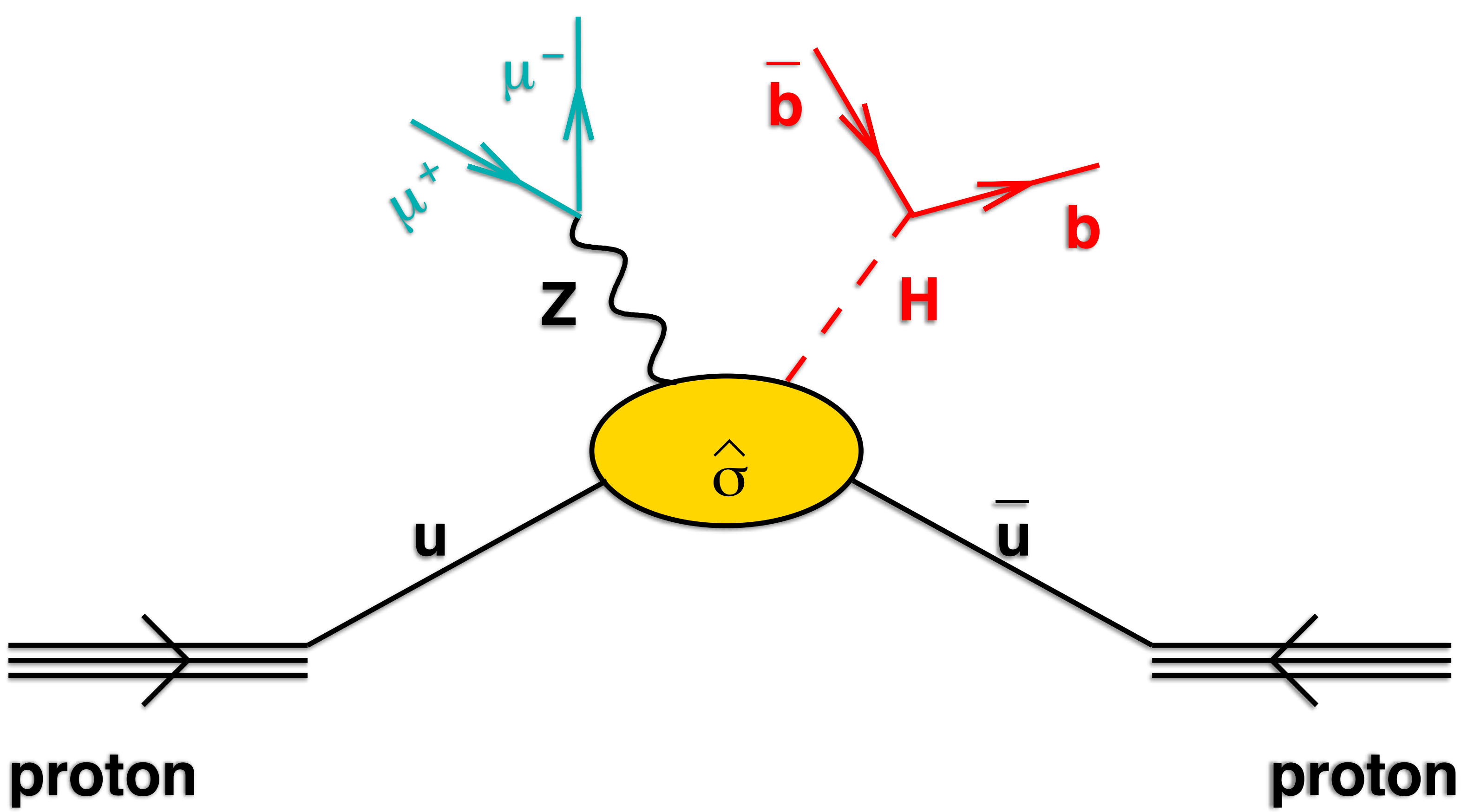

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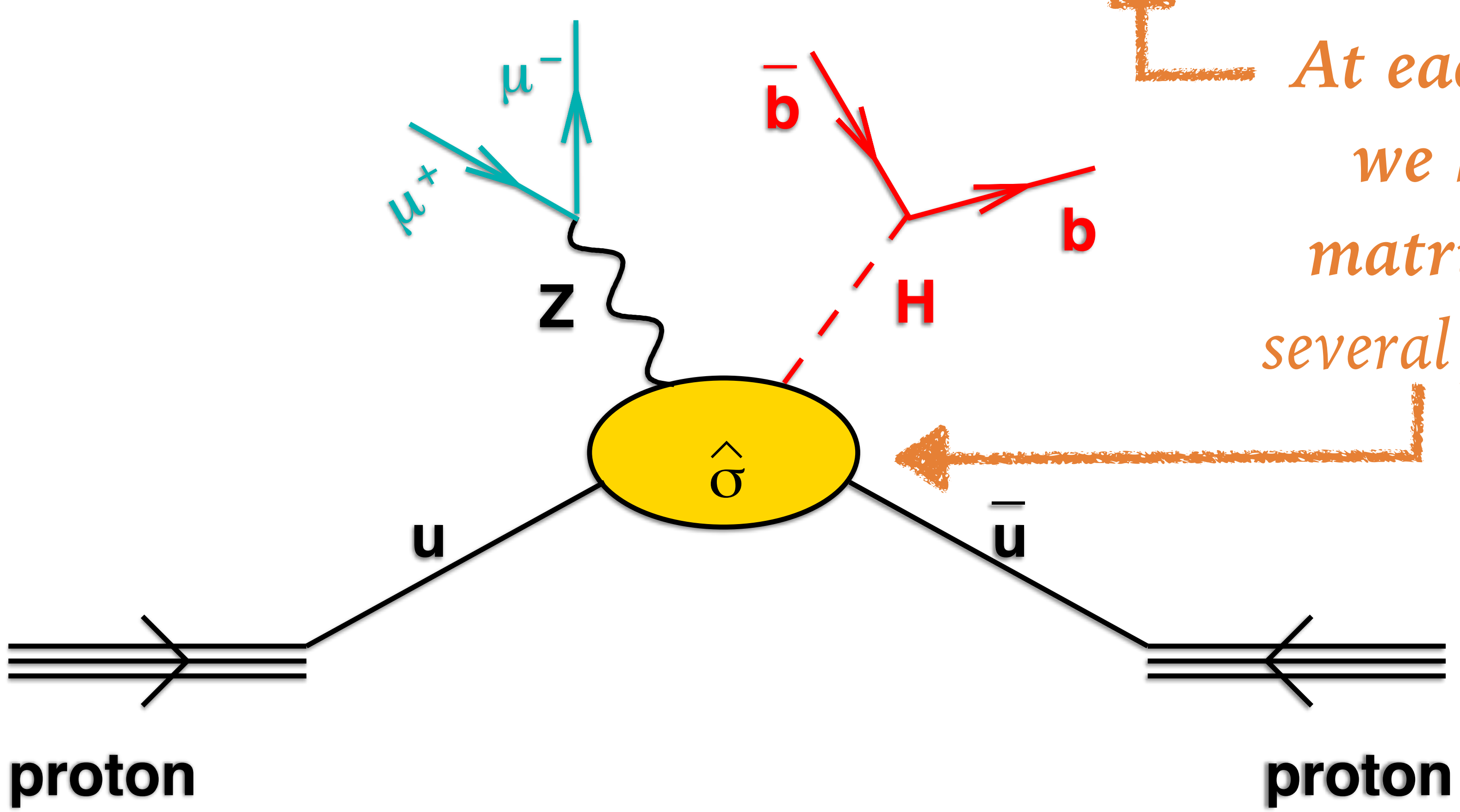
$$\times\, \hat{\sigma}^{(n)}_{ij\to ZH+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$

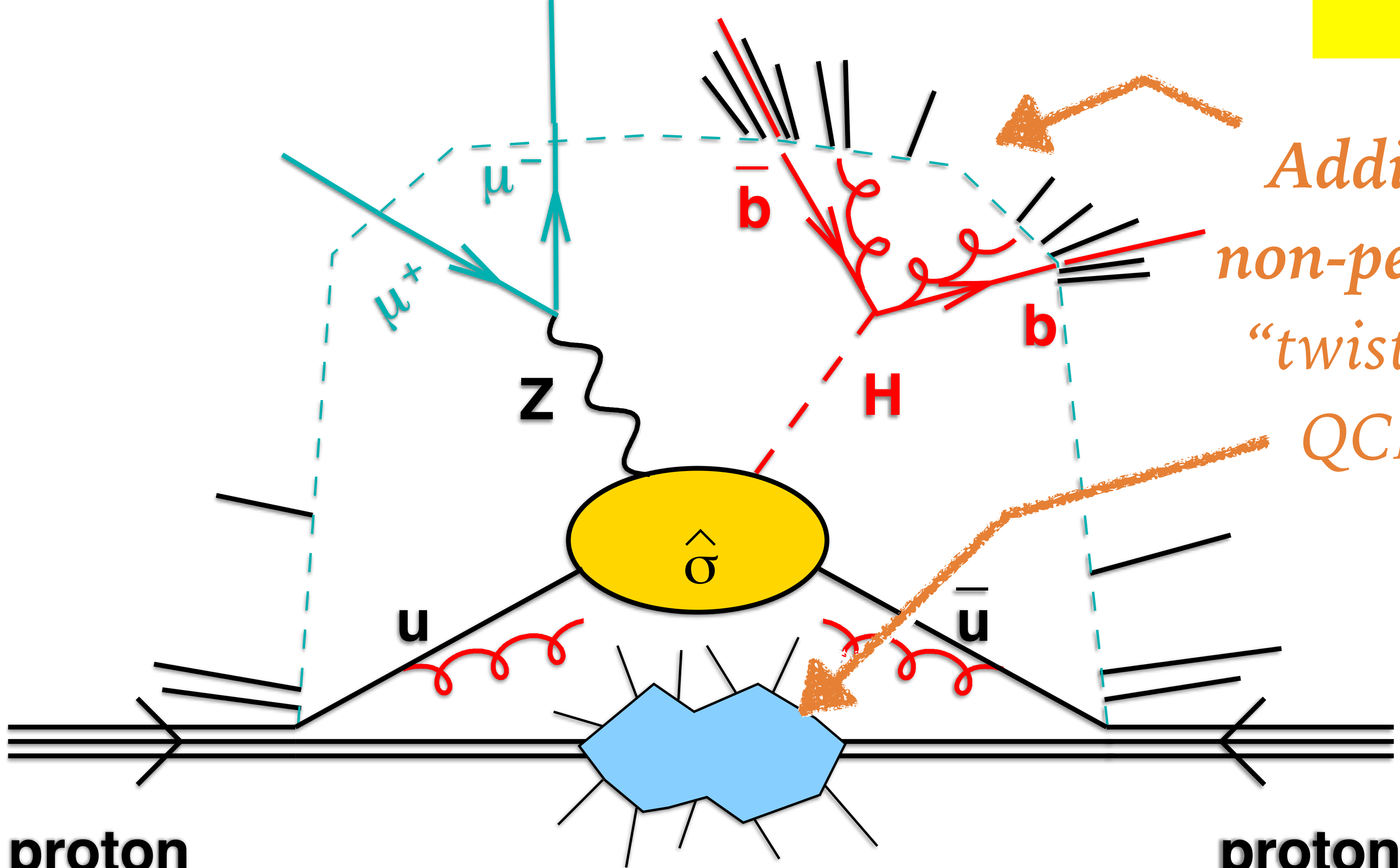
At each perturbative order n we have a specific "hard matrix element" (sometimes several for different subprocesses)



THE MASTER EQUATION

$$\sigma\left(h_1 h_2 \to ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n\left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right)$$

$$\times\, \hat{\sigma}^{(n)}_{ij \to ZH+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$



Additional corrections from non-perturbative effects (higher "twist", suppressed by powers of QCD scale (Λ) / hard scale)

THE STRONG COUPLING

All couplings run (QED, QCD, EW), i.e. they depend on the momentum scale (Q^2) of your process.

The QCD coupling, $\alpha_s(Q^2)$, runs **fast:**

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)\,,$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}\,, \qquad b_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2} = \frac{153 - 19n_f}{24\pi^2}$$

Note sign: Asymptotic Freedom, due to gluon to self-interaction

2004 Novel prize: Gross, Politzer & Wilczek

- At high scales Q, coupling becomes small
  ➥quarks and gluons are almost free, interactions are weak
- At low scales, coupling becomes strong
  ➥quarks and gluons interact strongly — confined into hadrons
  Perturbation theory fails.

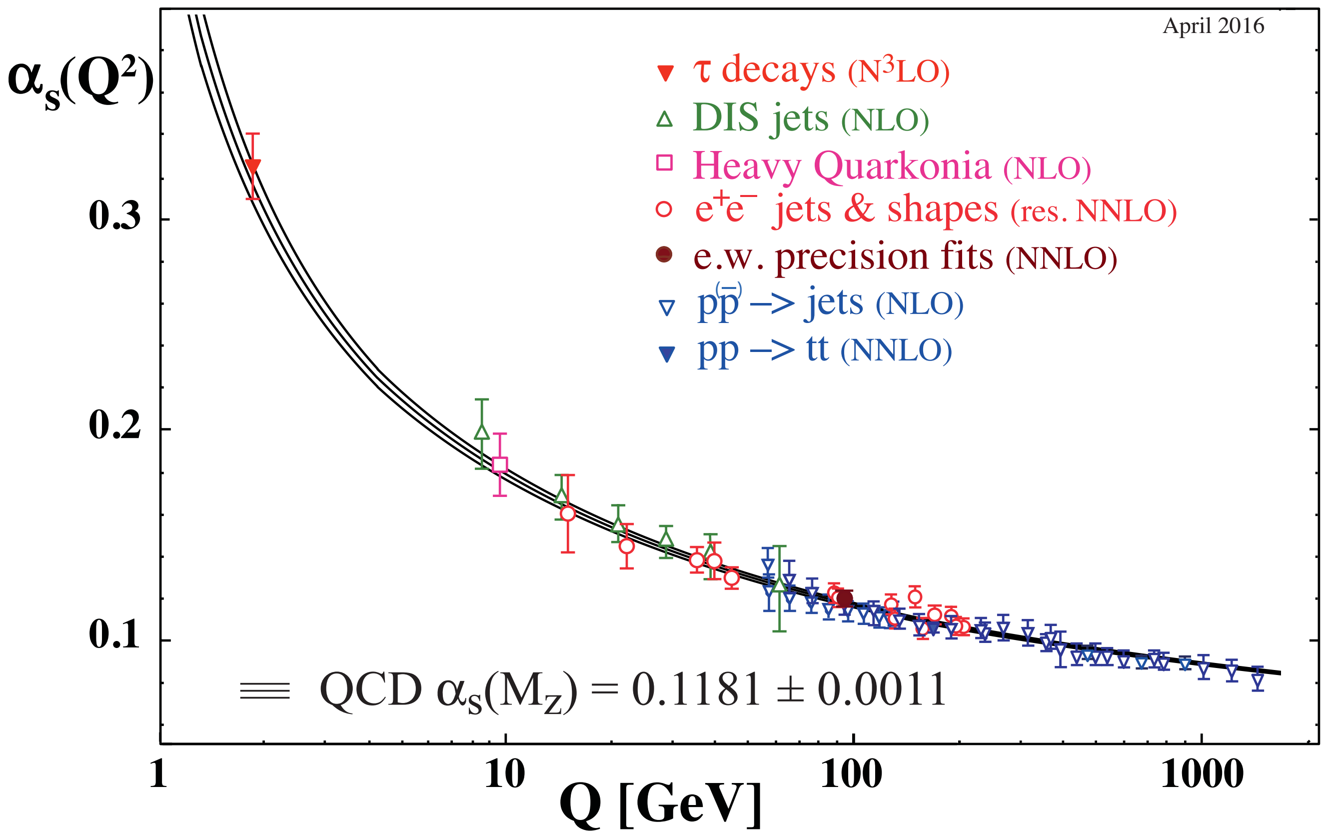
$C_A = 3$, n_f = number of light quark flavours; Q ($\to \mu_R$) is the "renormalisation scale"

THE STRONG COUPLING V. SCALE

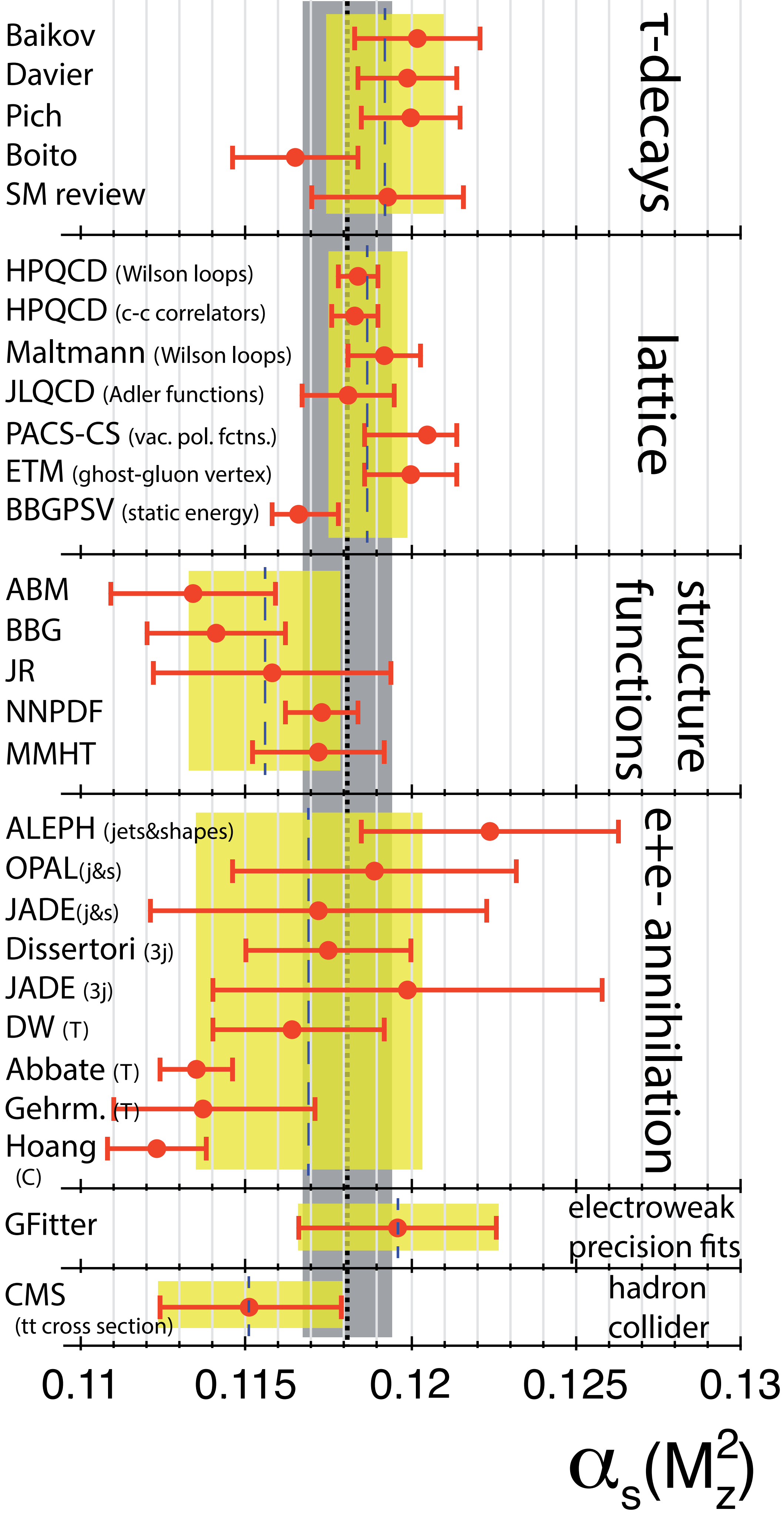
Solve $Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \quad \Rightarrow \quad \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$

$\Lambda \simeq 0.2$ GeV (aka Λ_{QCD}) is the fundamental scale of QCD, at which coupling blows up.

- Λ sets the scale for hadron masses (NB: Λ not unambiguously defined wrt higher orders)
- Perturbative calculations valid for scales $Q \gg \Lambda$.



PDG World Average: $\alpha_s(M_Z) = 0.1181 \pm 0.0011$ (0.9%)



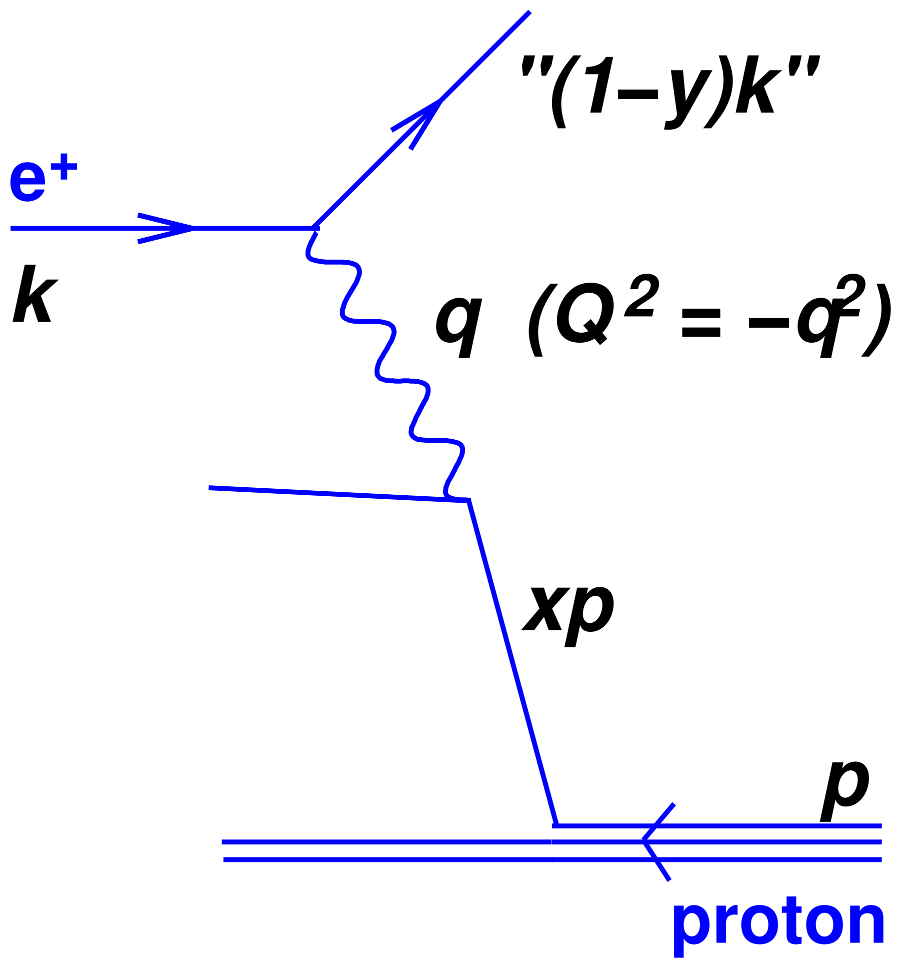
STRONG-COUPLING DETERMINATIONS

Bethke, Dissertori & GPS in PDG '16

- Most consistent set of independent determinations is from lattice
- Two best determinations are from same group (HPQCD, 1004.4285, 1408.4169)
  $\alpha_s(M_Z) = 0.1183 \pm 0.0007$ (0.6%) [heavy-quark correlators]
  $\alpha_s(M_Z) = 0.1183 \pm 0.0007$ (0.6%) [Wilson loops]
- Many determinations quote small uncertainties (≲1%). All are disputed!
- Some determinations quote anomalously small central values (~0.113 v. world avg. of 0.1181±0.0011). Also disputed

PARTON DISTRIBUTION FUNCTIONS (PDFs)

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



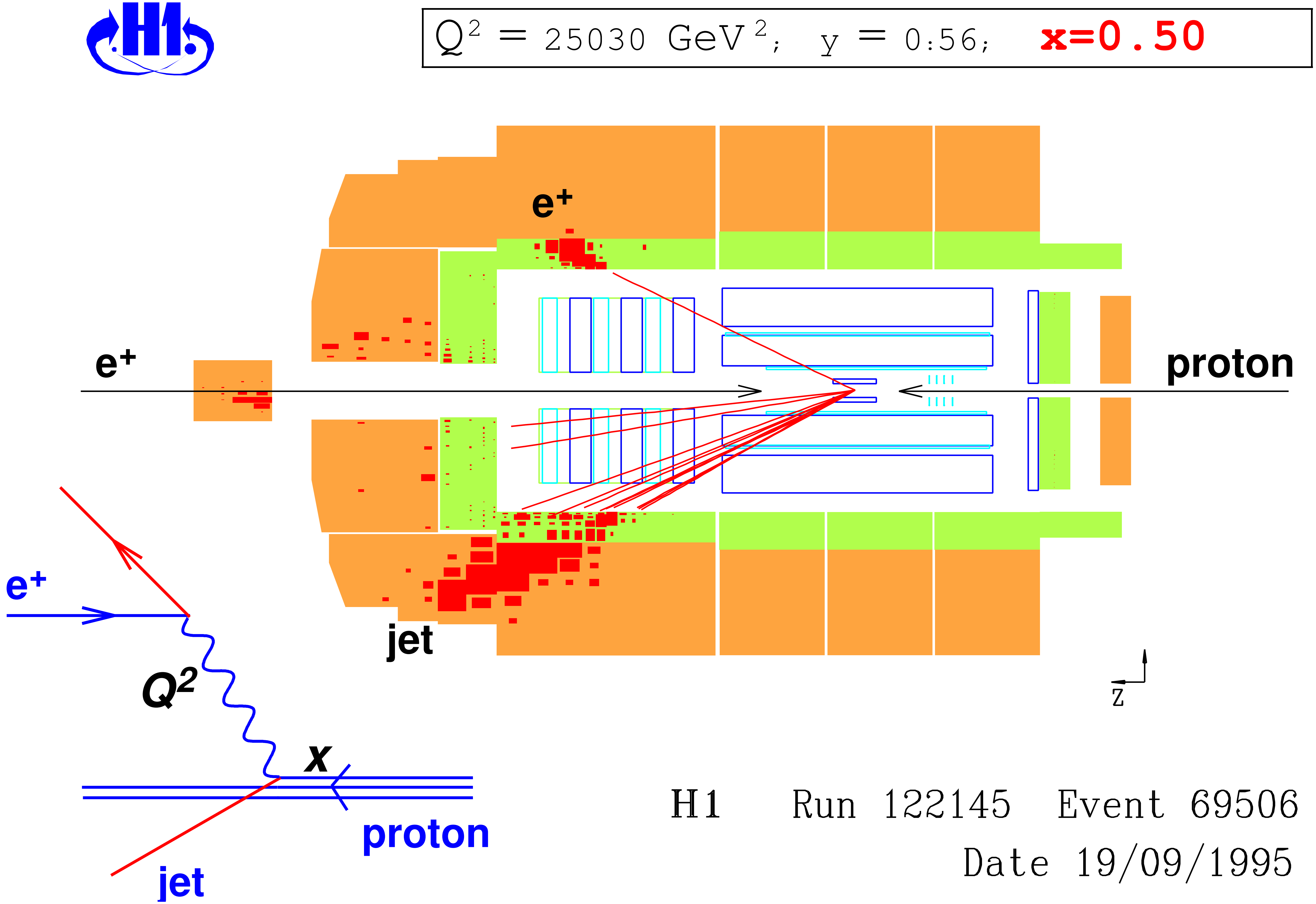
Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality ↔ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

DEEP INELASTIC SCATTERING



Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

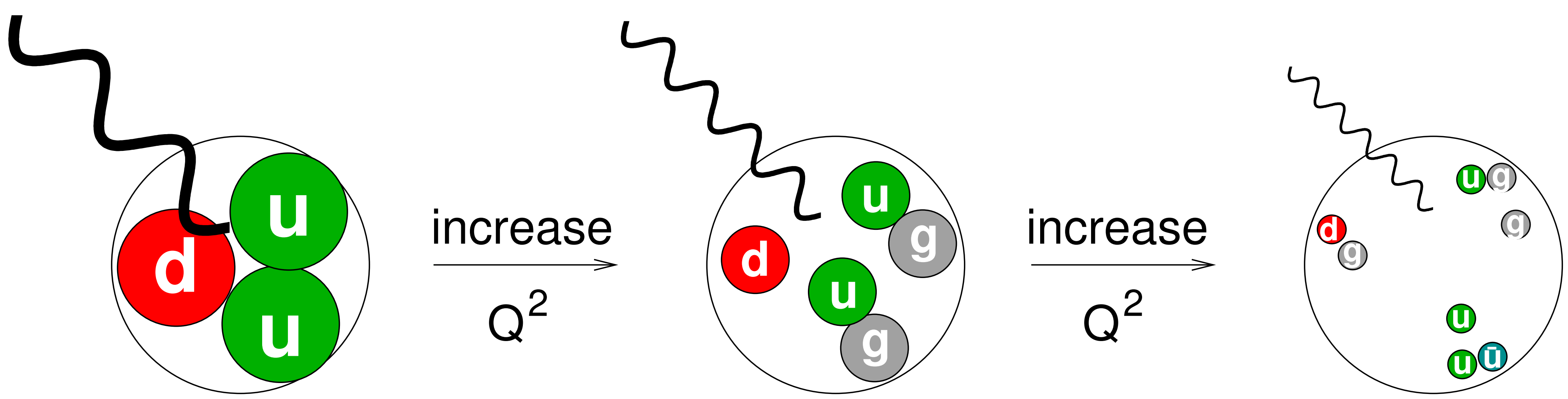
- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

PARTON DISTRIBUTION AND DGLAP

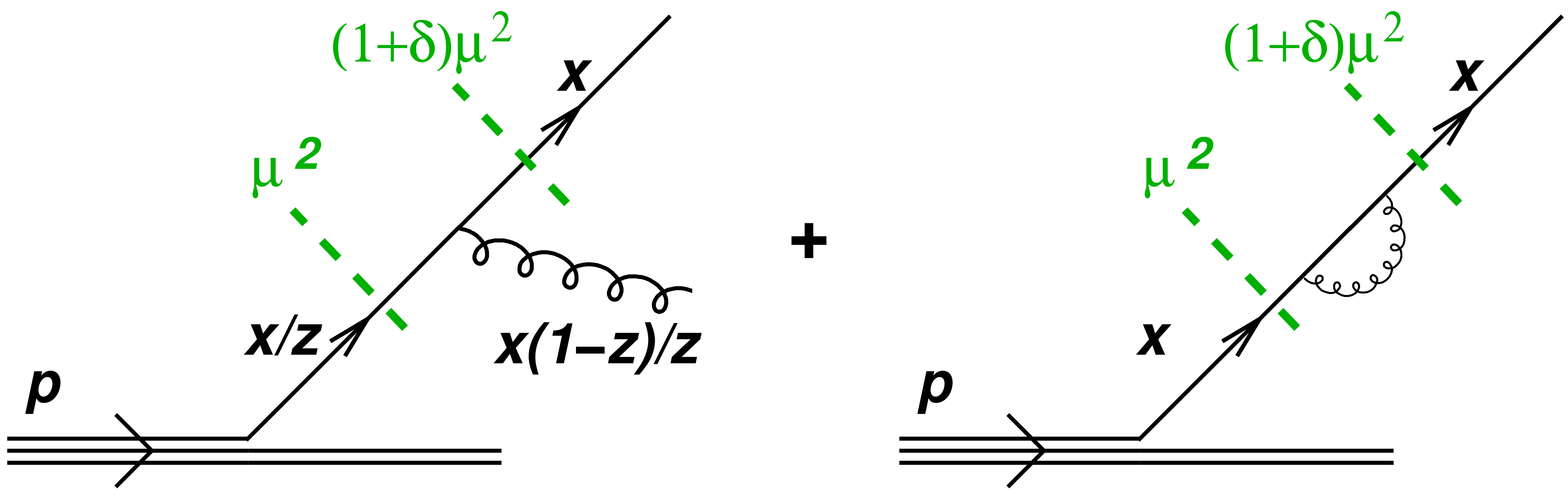
- Write up-quark distribution in proton as

$$u(x, \mu_F^2)$$

- μ_F is the **factorisation scale** — a bit like the renormalisation scale (μ_R) for the running coupling.
- As you vary the factorisation scale, the parton distributions evolve with a renormalisation-group type equation



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations *take derivative* wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F\frac{1+z^2}{1-z}$

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\,[g(z)]_+\,f(z) = \int_0^1 dz\,g(z)\,f(z) - \int_0^1 dz\,g(z)\,f(1)$$

$z=1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z=1$

DGLAP EQUATION

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

2015 EPS HEP prize to Bjorken, Altarelli, Dokshitzer, Lipatov & Parisi

NLO DGLAP

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

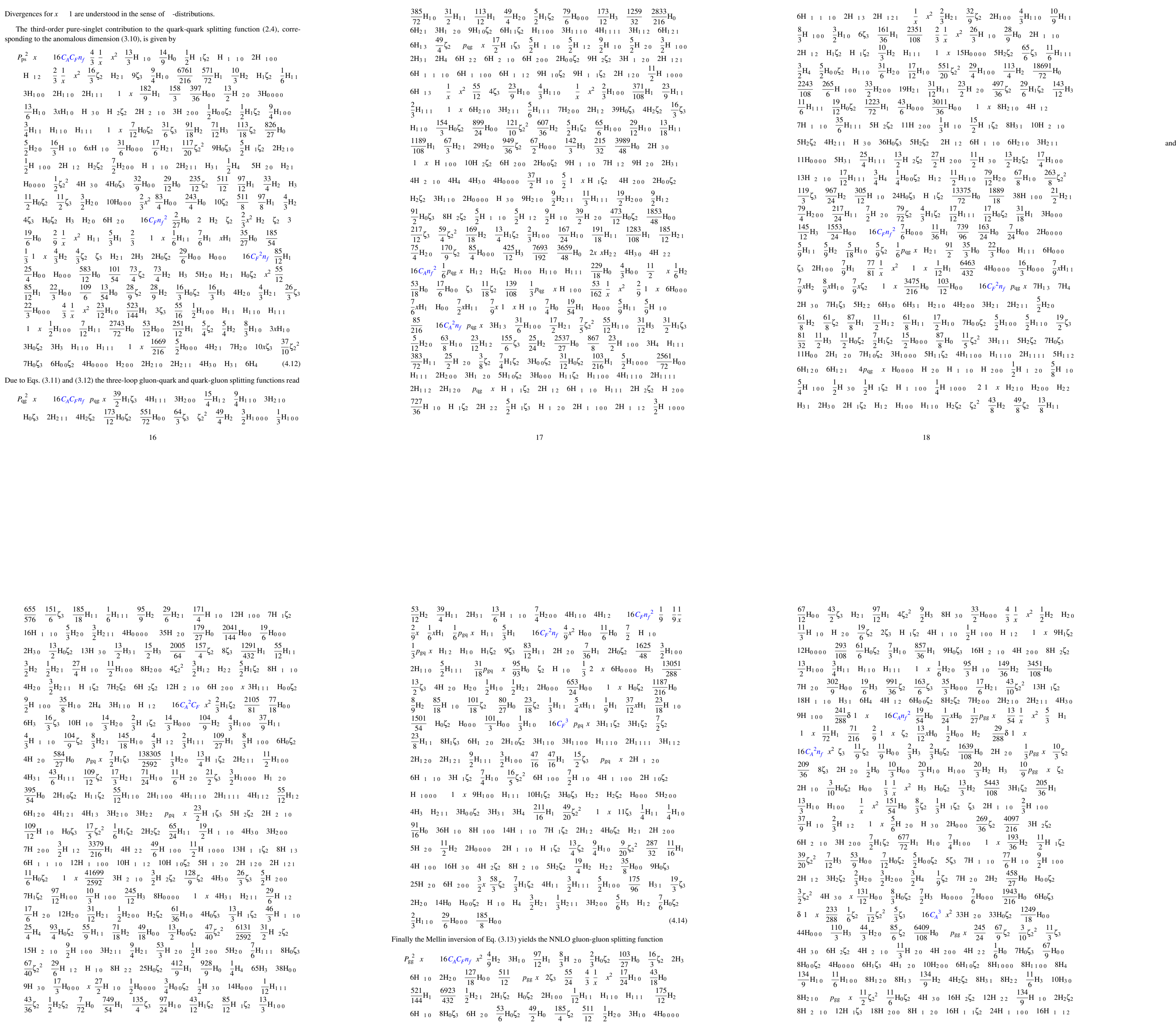
$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2 \left[\frac{8}{3}{\rm H}_0 - \frac{56}{9} \right] + (1+x)\left[5{\rm H}_0 - 2{\rm H}_{0,0} \right] \right)$$

$$P^{(1)}_{\rm qg}(x) = 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x){\rm H}_{-1,0} - 2p_{\rm qg}(x){\rm H}_{1,1} + x^2 \left[\frac{44}{3}{\rm H}_0 - \frac{218}{9} \right] \right.$$
$$+4(1-x)\left[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1 \right] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0 \Big) + 4\, C_F n_f \Big(2p_{\rm qg}(x)\Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2$$
$$-\zeta_2 \Big] + 4x^2 \left[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2} \right] + 2(1-x)\left[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4} \right] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0 \Big)$$

$$P^{(1)}_{\rm gq}(x) = 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1 \right] - x^2 \left[\frac{8}{3}{\rm H}_0 - \frac{44}{9} \right] + 4\zeta_2 - 2 \right.$$
$$-7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\left[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9} \right] - 2p_{\rm gq}(-x){\rm H}_{-1,0} \Big) - 4\, C_F n_f \Big(\frac{2}{3}x$$
$$-p_{\rm gq}(x)\left[\frac{2}{3}{\rm H}_1 - \frac{10}{9} \right] \Big) + 4\, C_F{}^2 \Big(p_{\rm gq}(x)\left[3{\rm H}_1 - 2{\rm H}_{1,1} \right] + (1+x)\left[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0 \right] - 3{\rm H}_{0,0}$$
$$+1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x \Big)$$

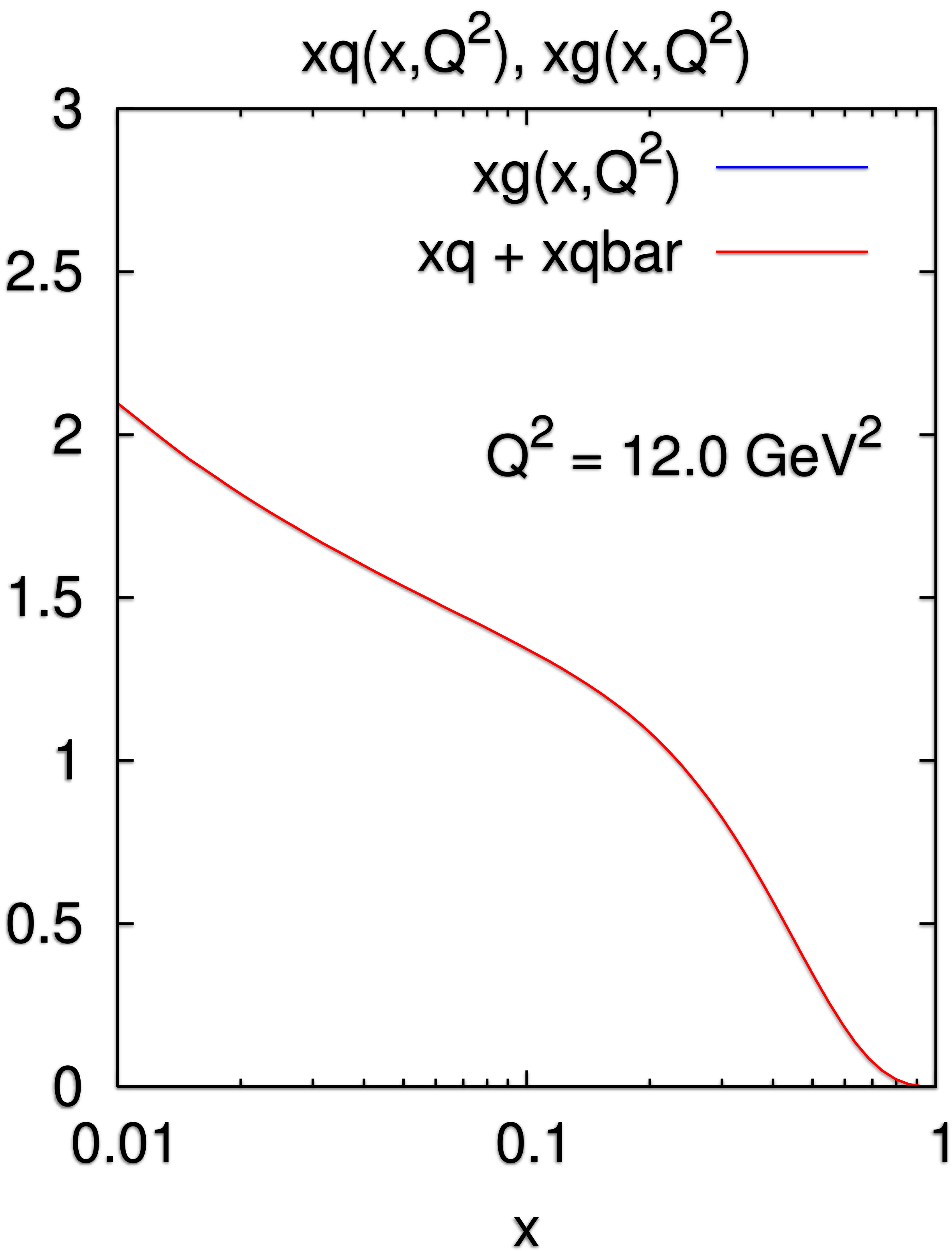
$$P^{(1)}_{\rm gg}(x) = 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2 \right) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x) \right) + 4\, C_A{}^2 \Big(27$$
$$+(1+x)\left[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2} \right] + 2p_{\rm gg}(-x)\left[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2 \right] - \frac{67}{9}\left(\frac{1}{x} - x^2 \right) - 12{\rm H}_0$$
$$-\frac{44}{3}x^2{\rm H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2 \right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3 \right] \Big) + 4\, C_F n_f \Big(2{\rm H}_0$$
$$+\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5{\rm H}_0 - 2{\rm H}_{0,0} \right] - \frac{1}{2}\delta(1-x) \Big) \ .$$

NNLO DGLAP



NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

DGLAP evolution (initial quarks only)



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g \leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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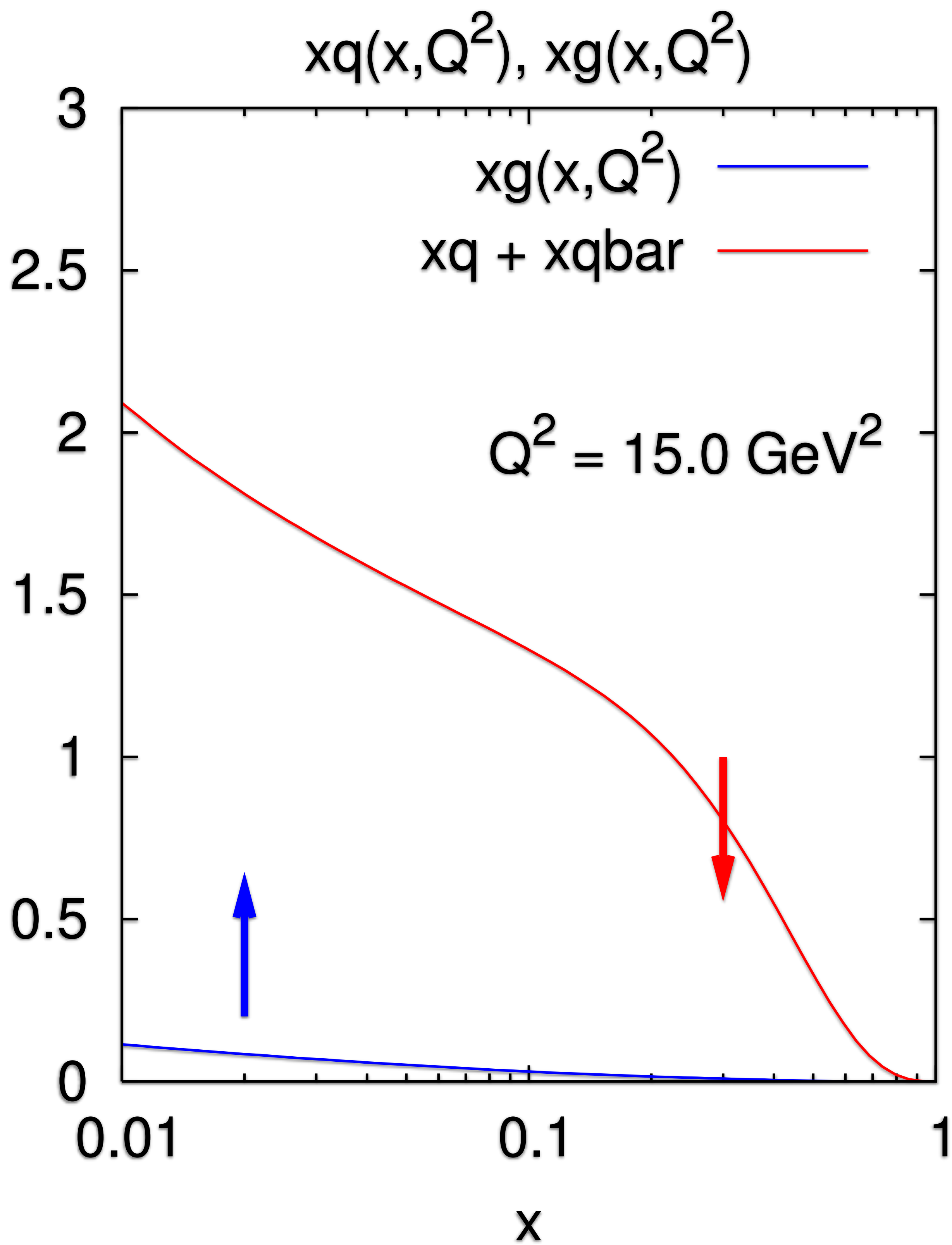

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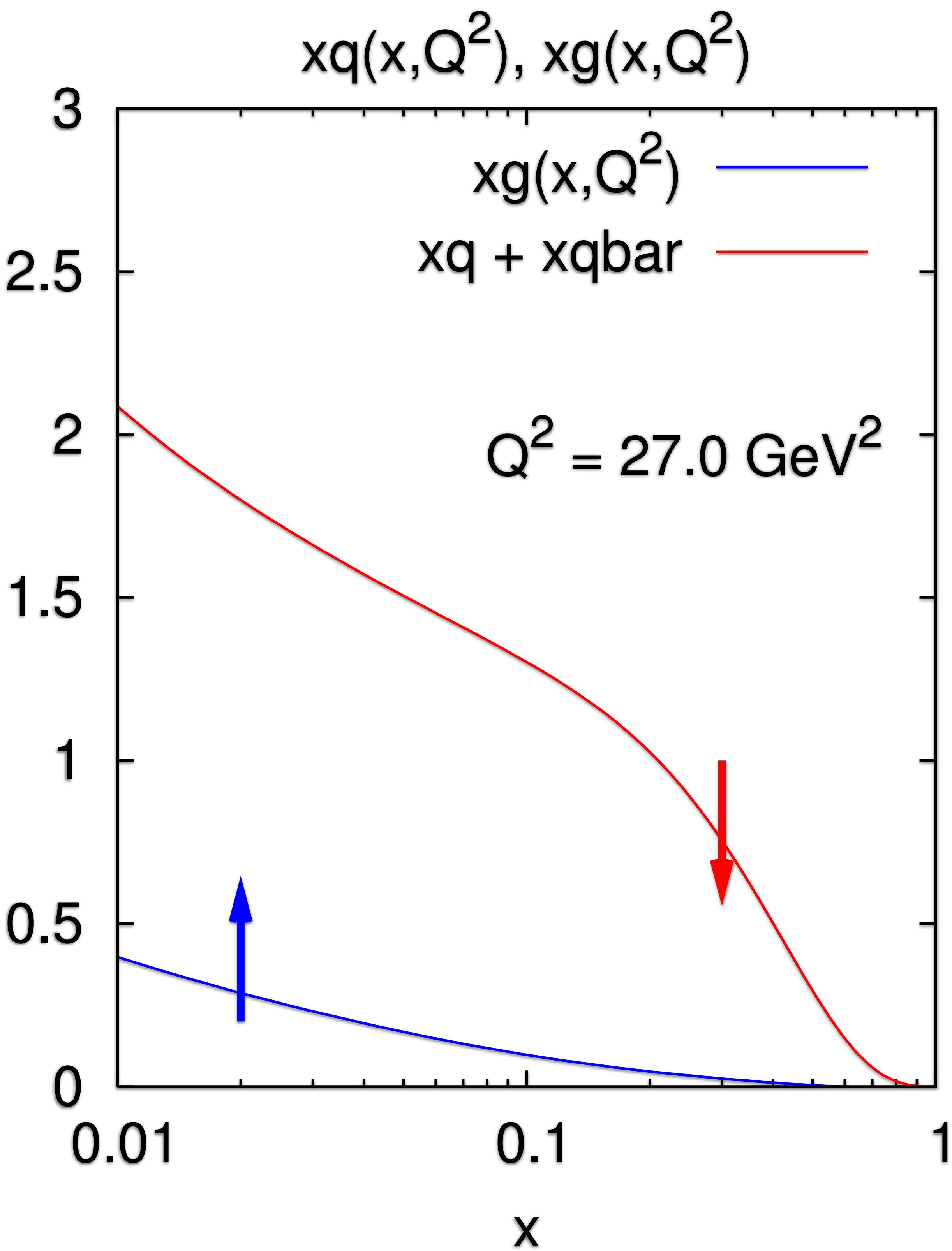

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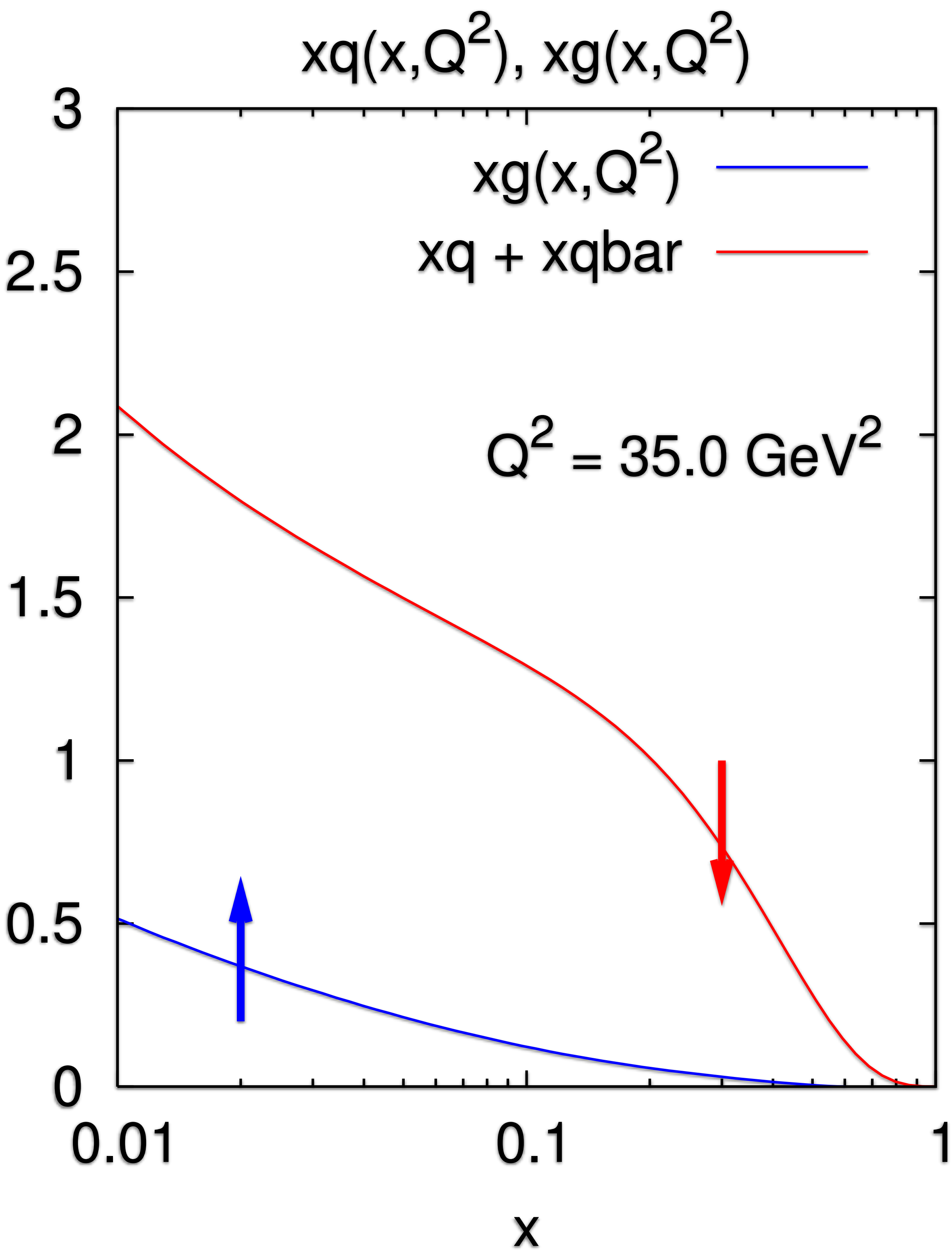

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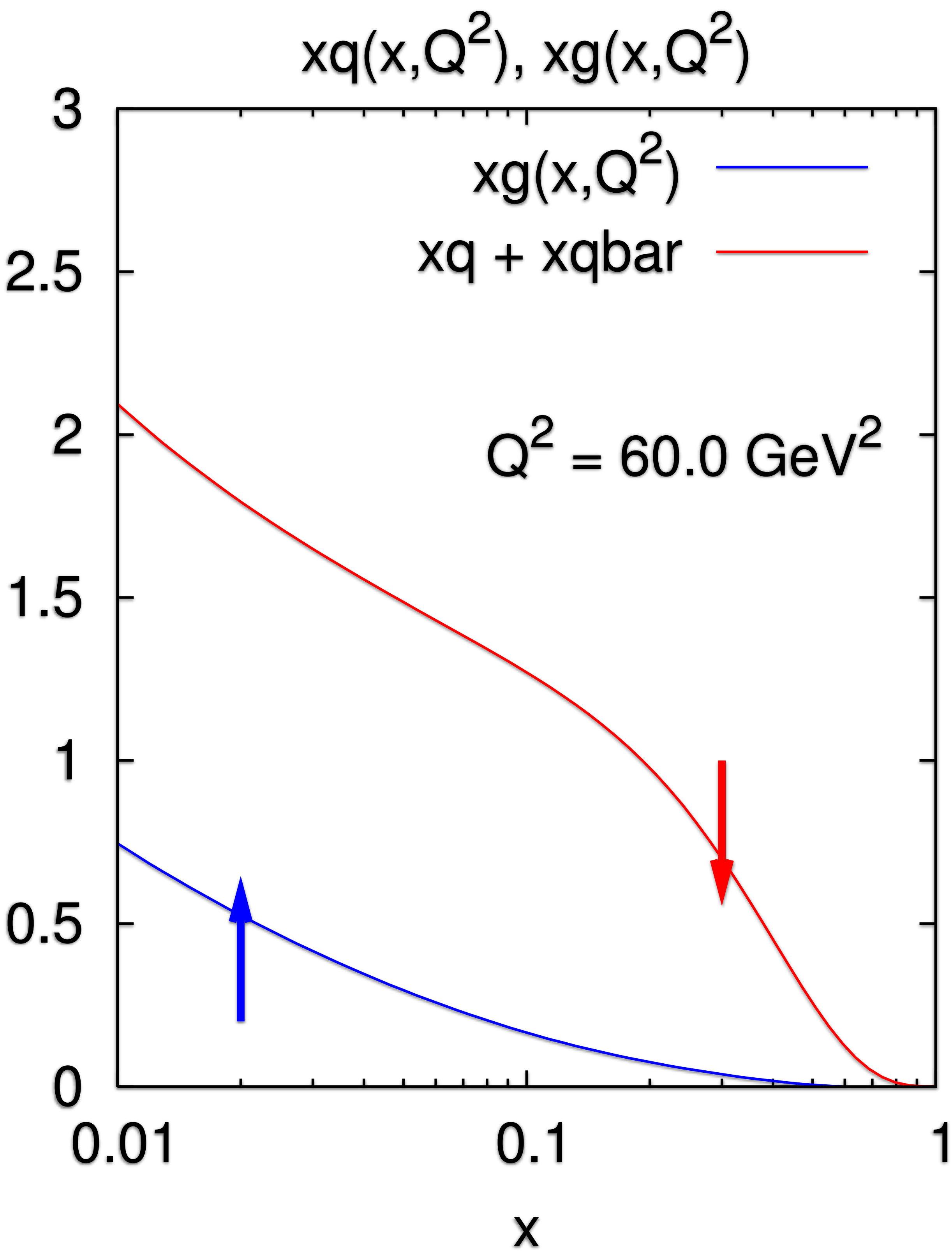

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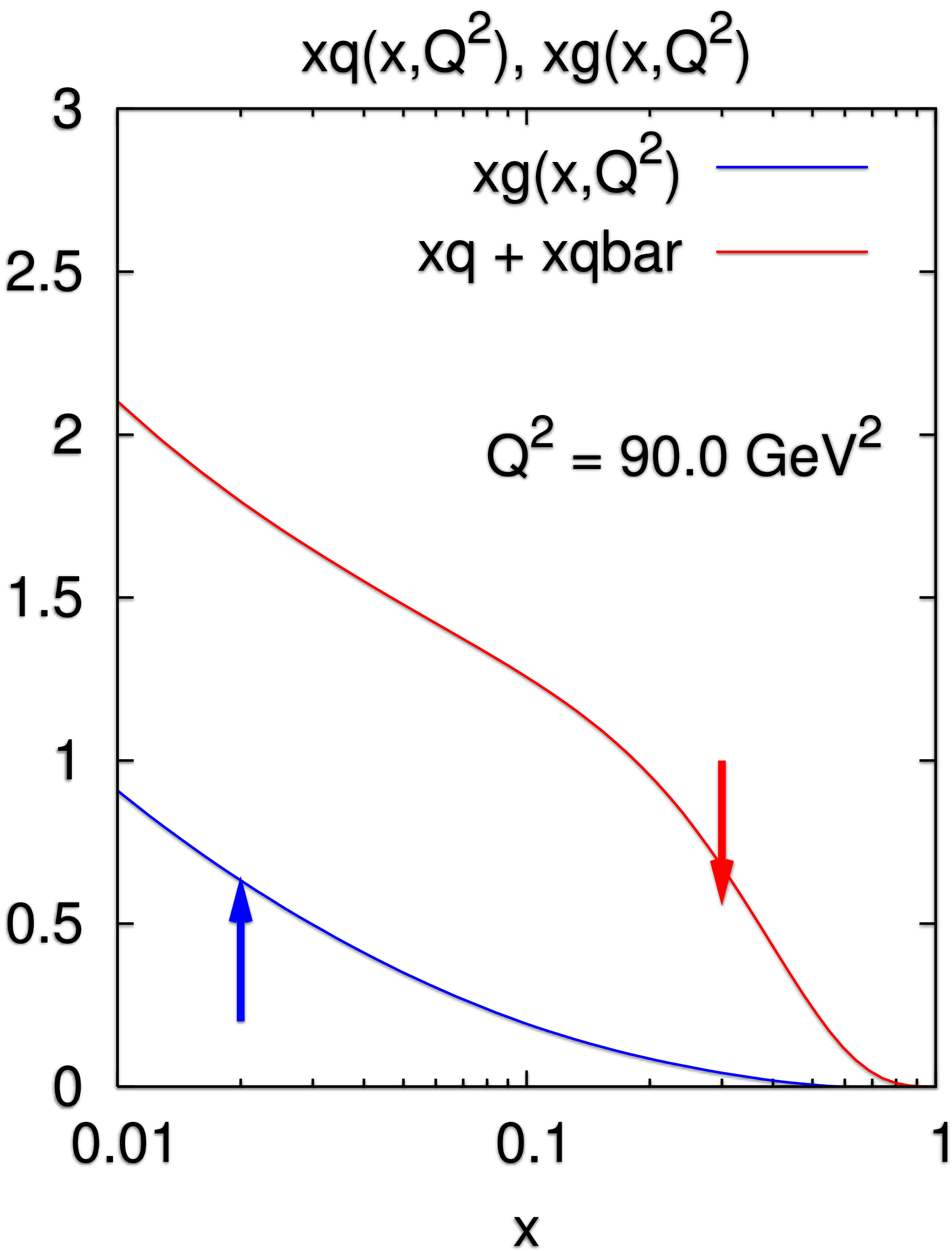

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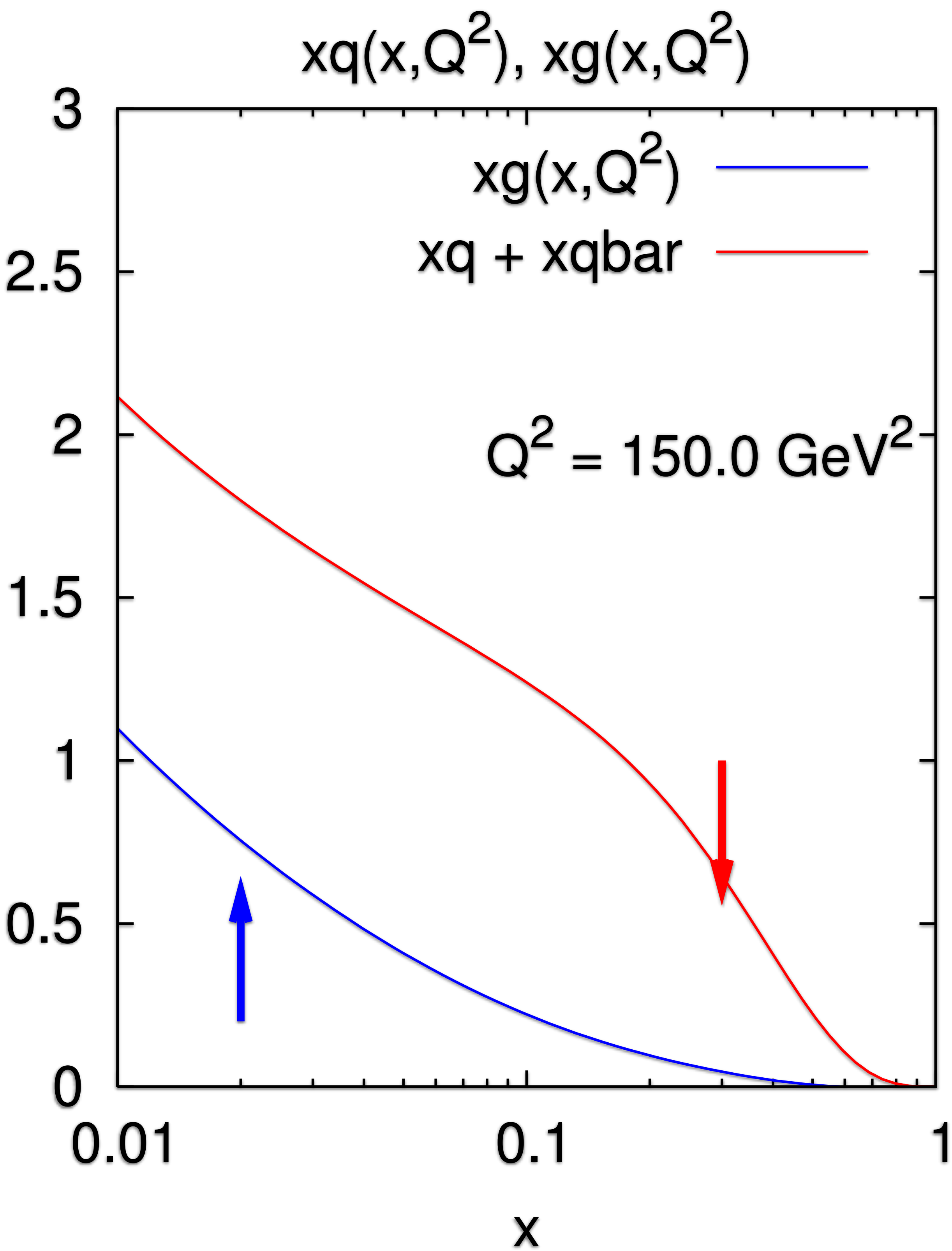

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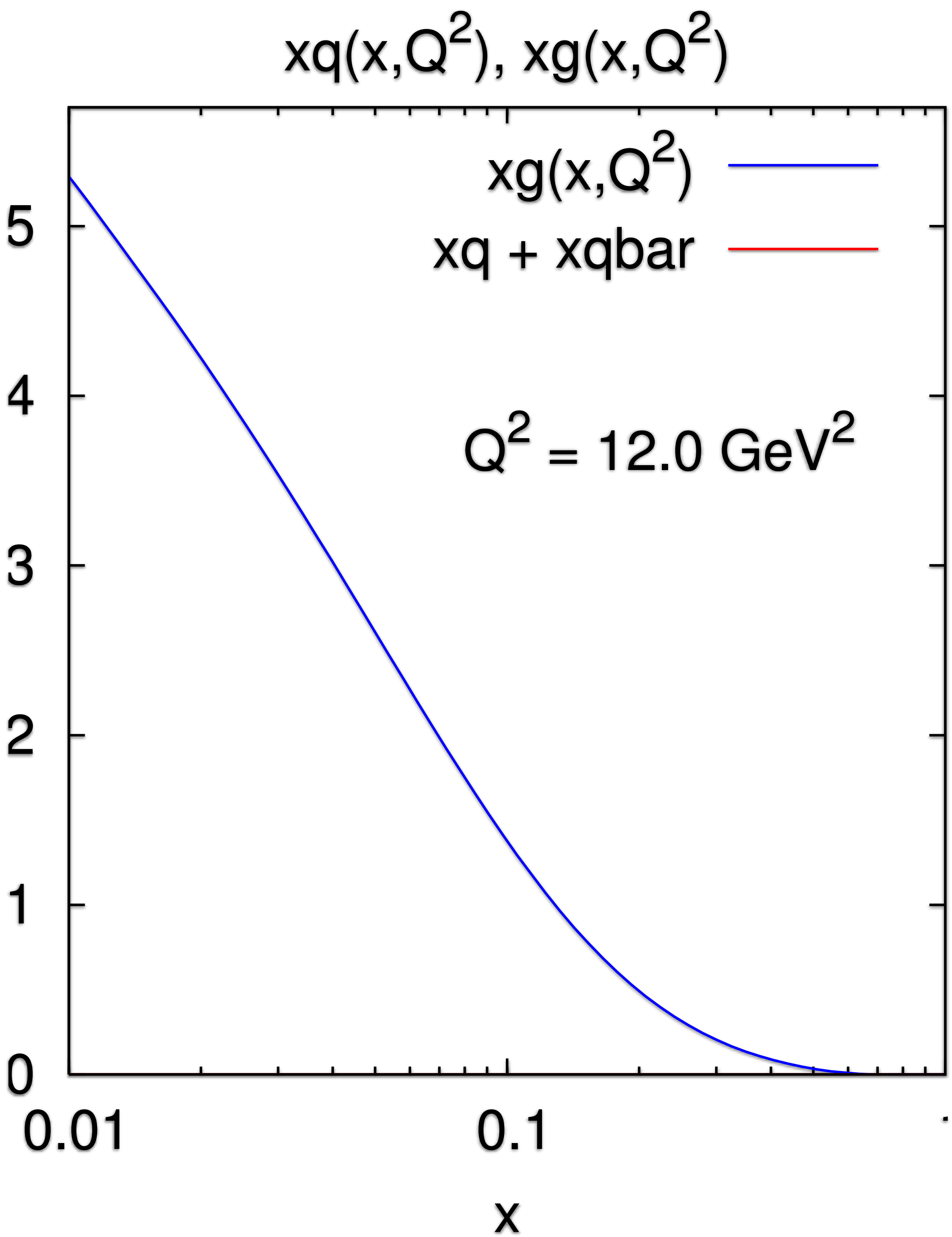

2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
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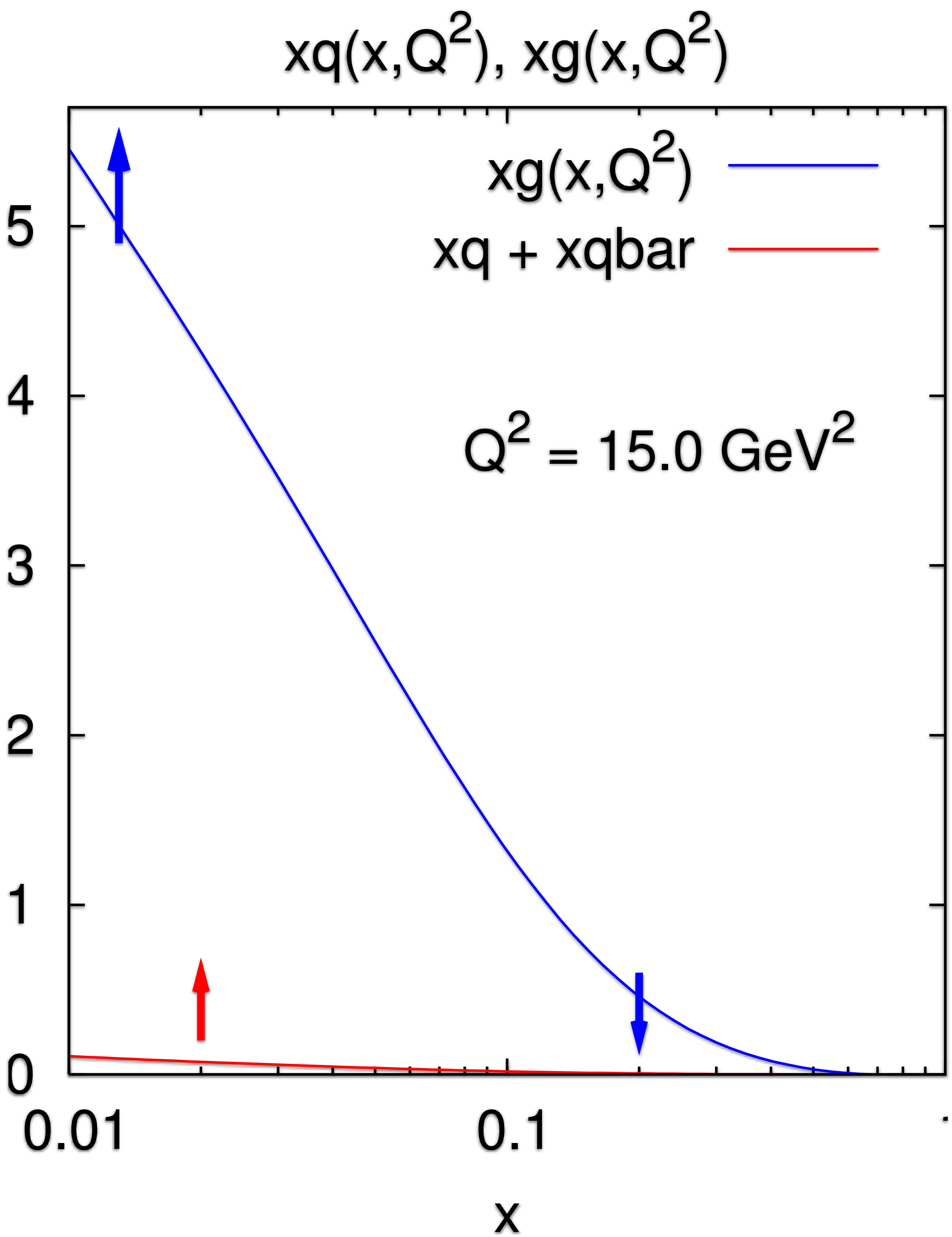
- gluon is depleted at large x.
- high-x gluon feeds growth of small x gluon & quark.

DGLAP evolution (initial gluons only)



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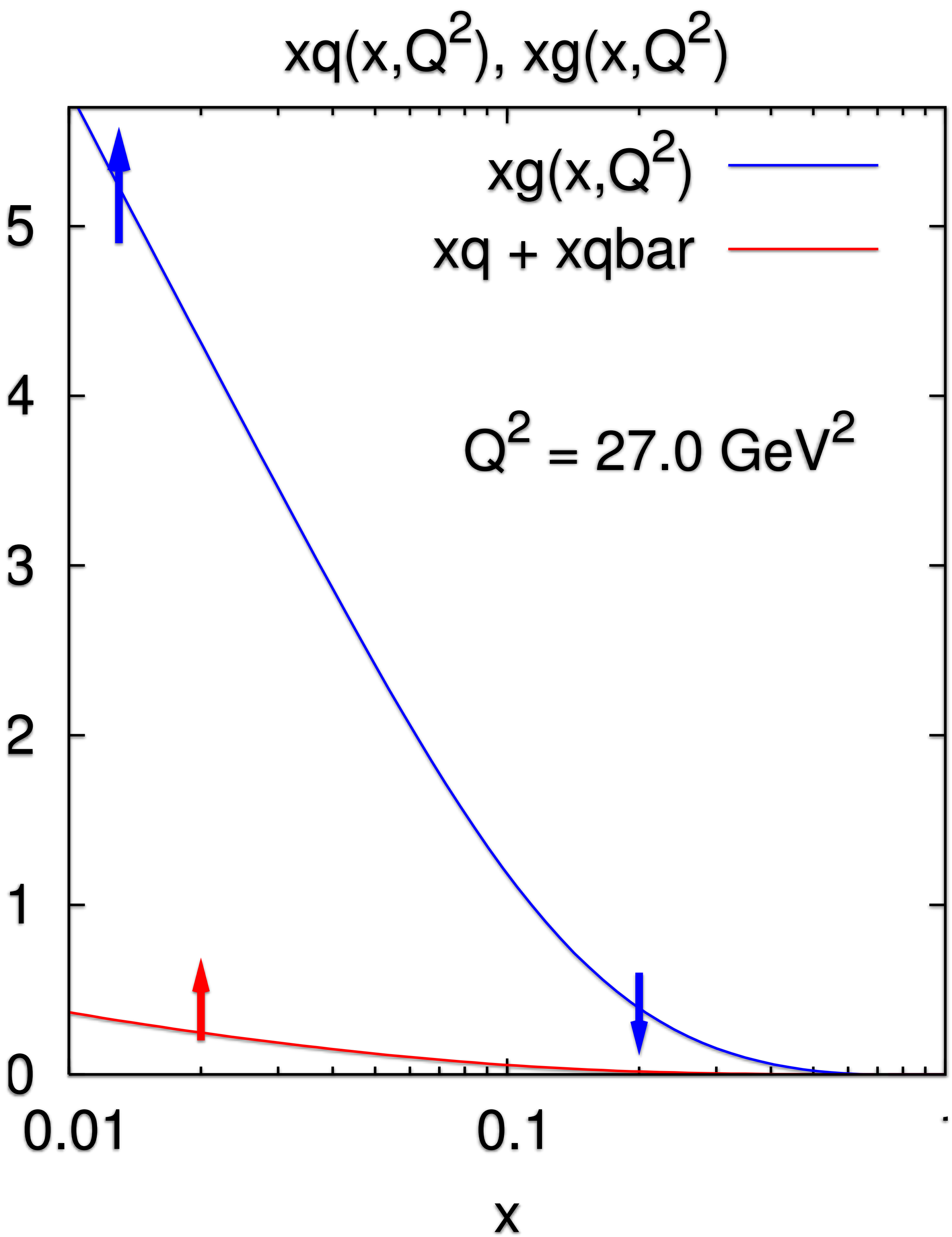

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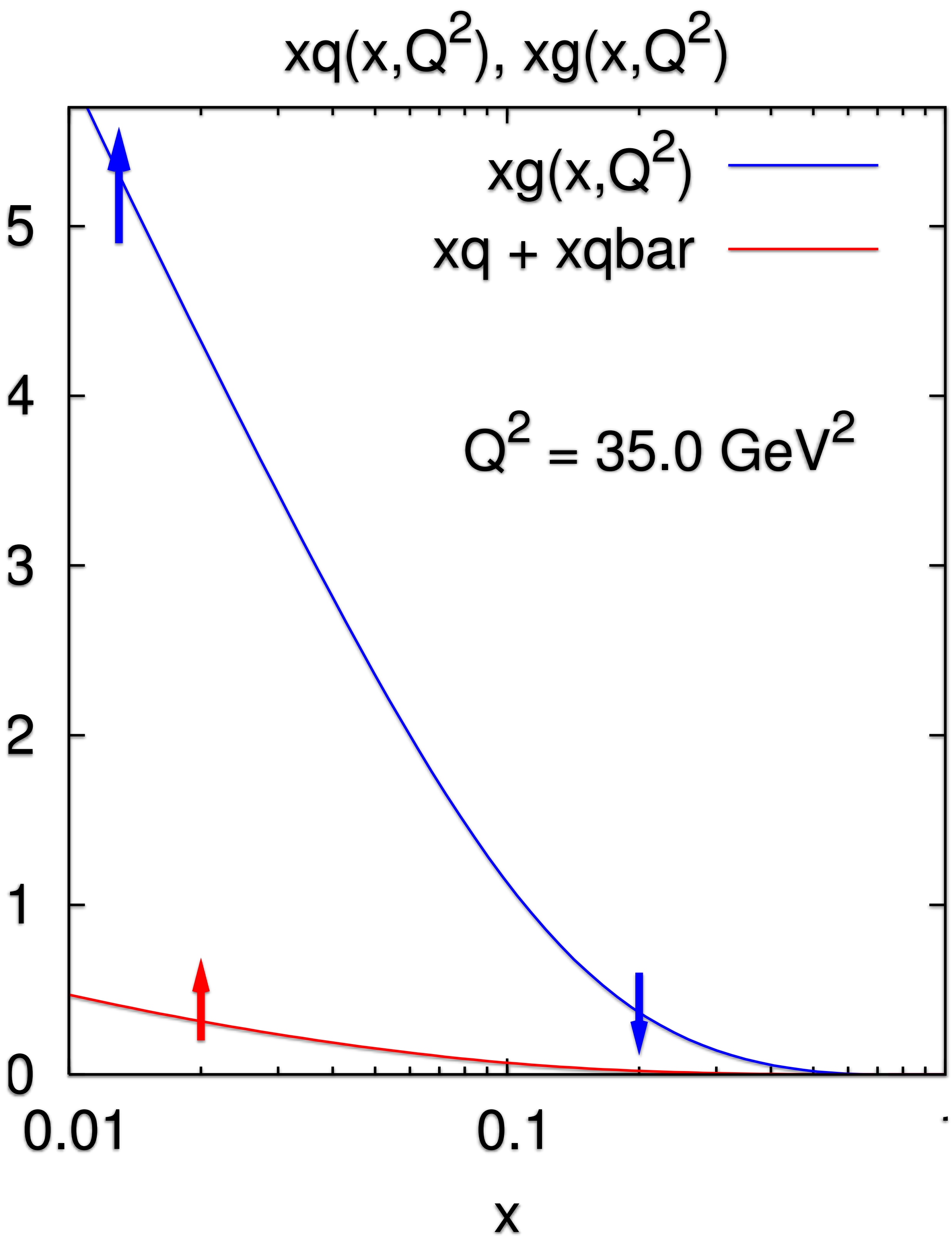

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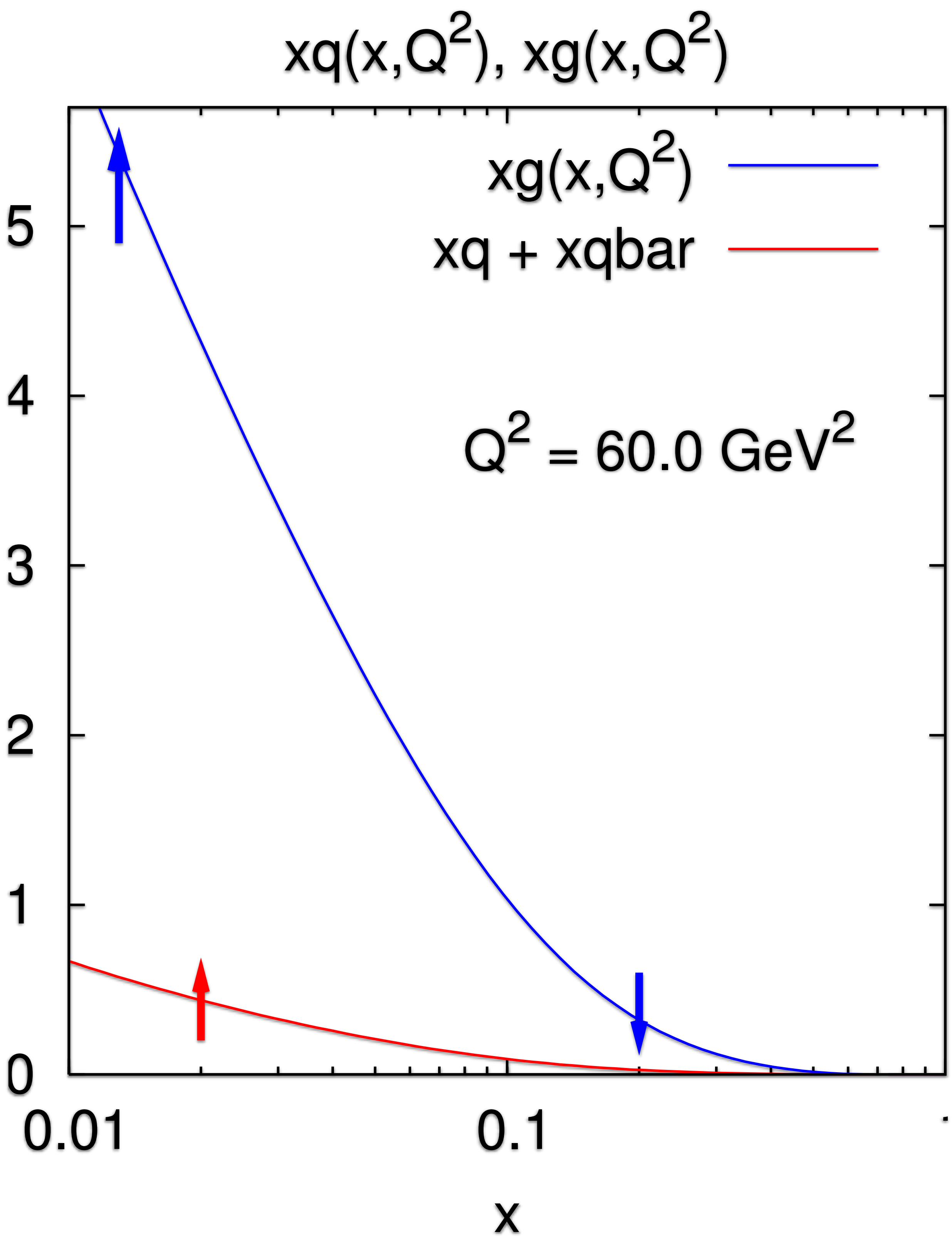

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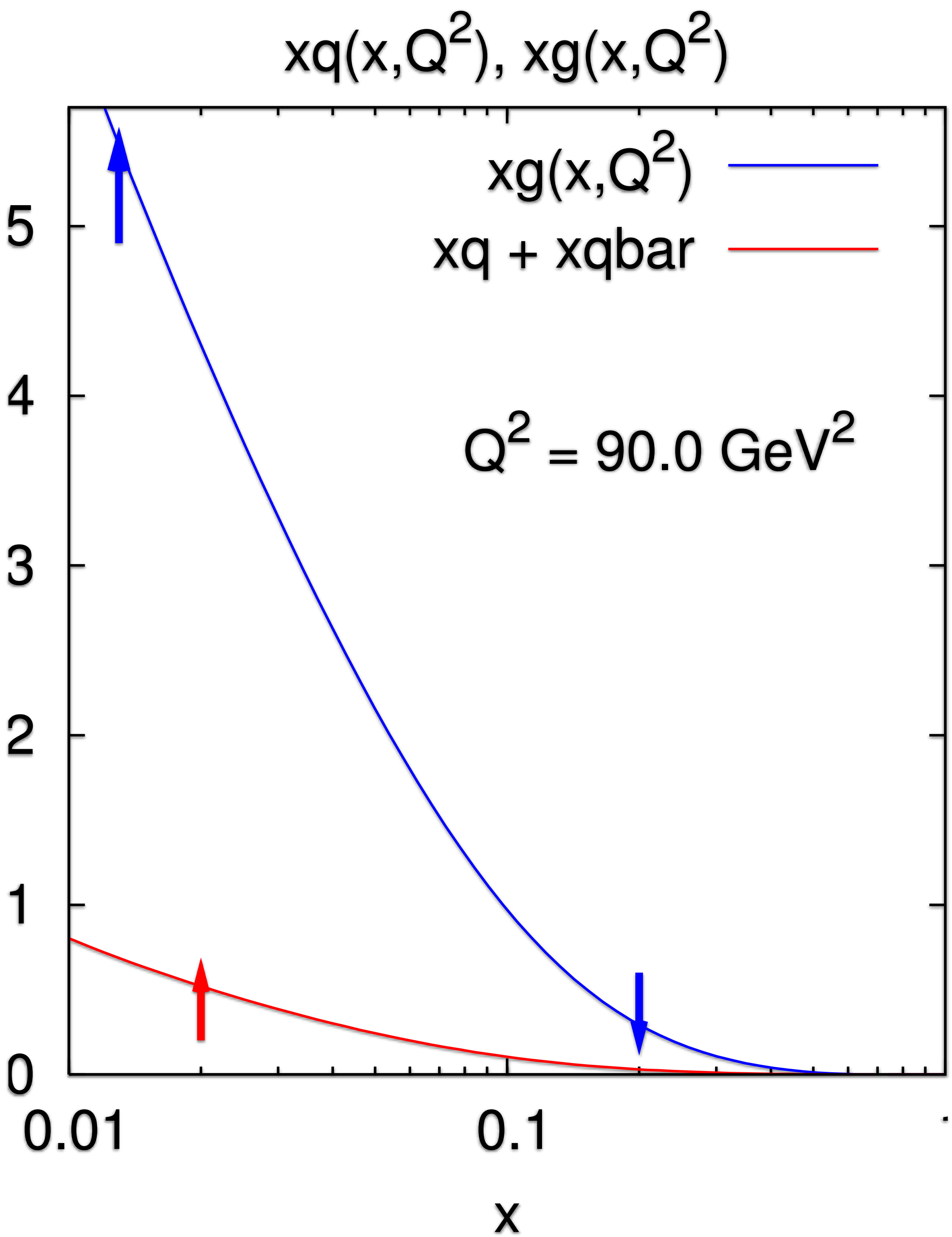

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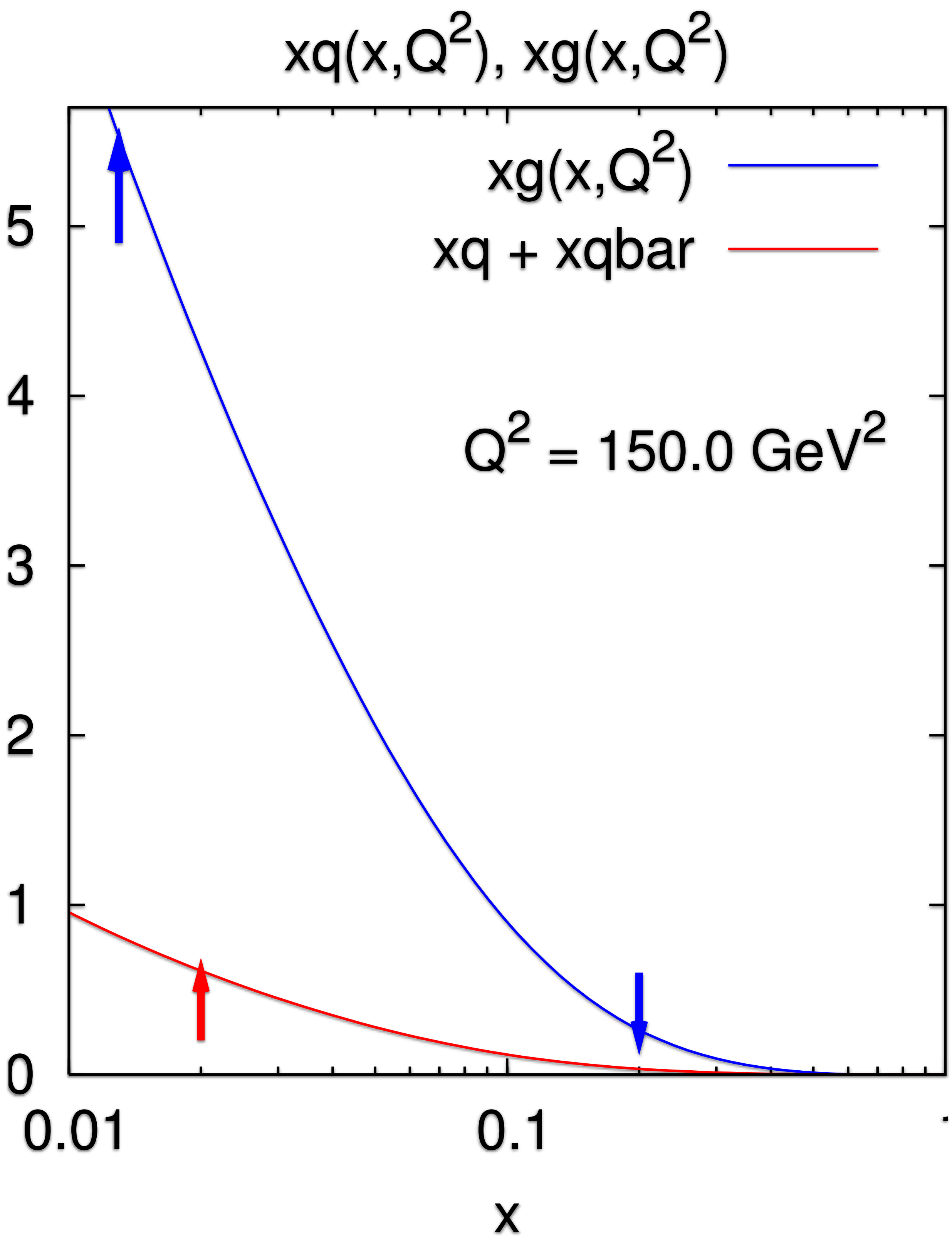

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DGLAP evolution:

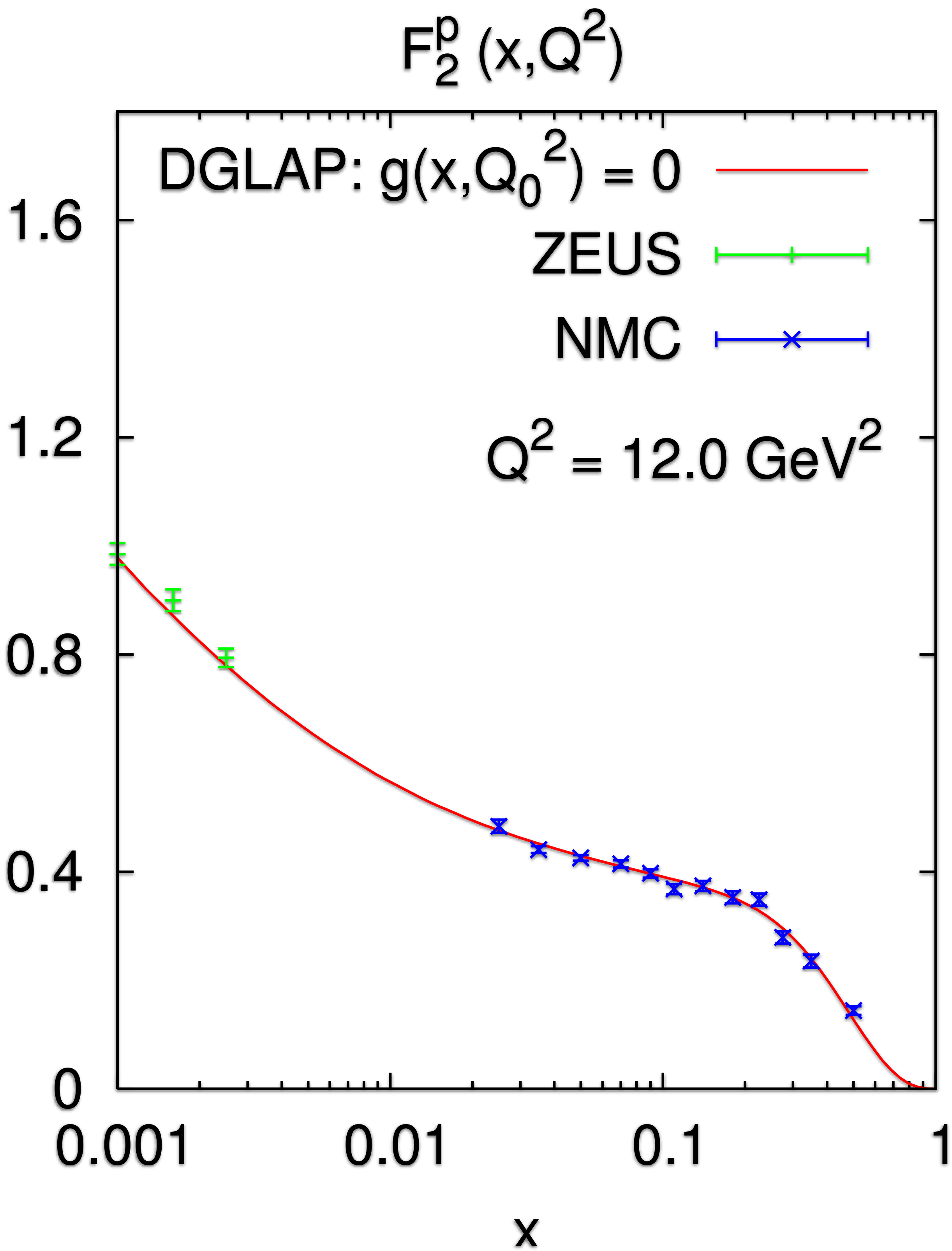
➤ partons lose momentum and shift towards smaller x

➤ high-x partons drive growth of low-x gluon

determining the gluon

which is critical at hadron colliders (e.g. Higgs dominantly produced by gluon-gluon fusion), but not directly probed in Deep-Inelastic-Scattering

Consider DIS data — $F_2(x,Q^2)$ — in a world where the proton just had quarks



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\ \mathrm{GeV}^2$.

NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

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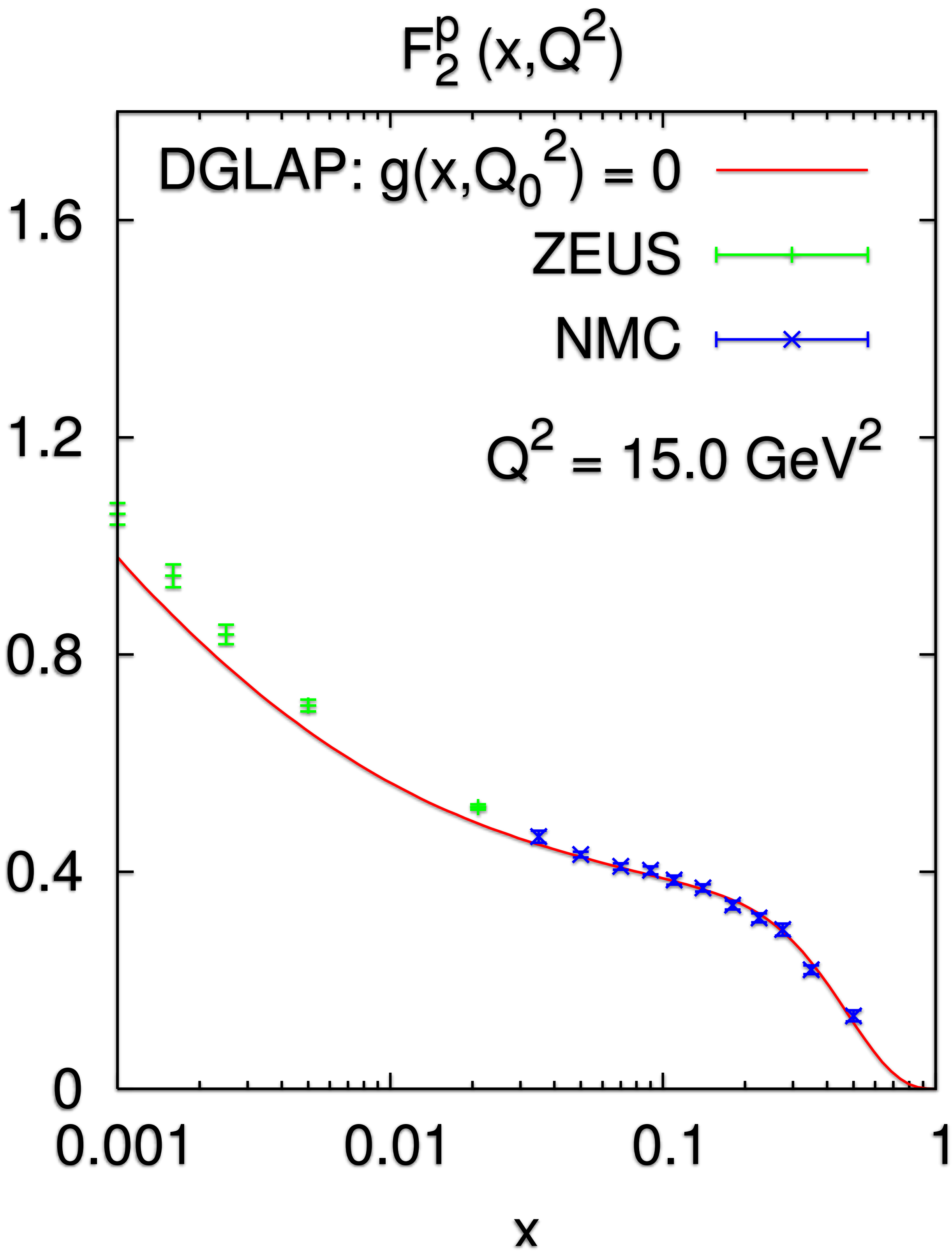

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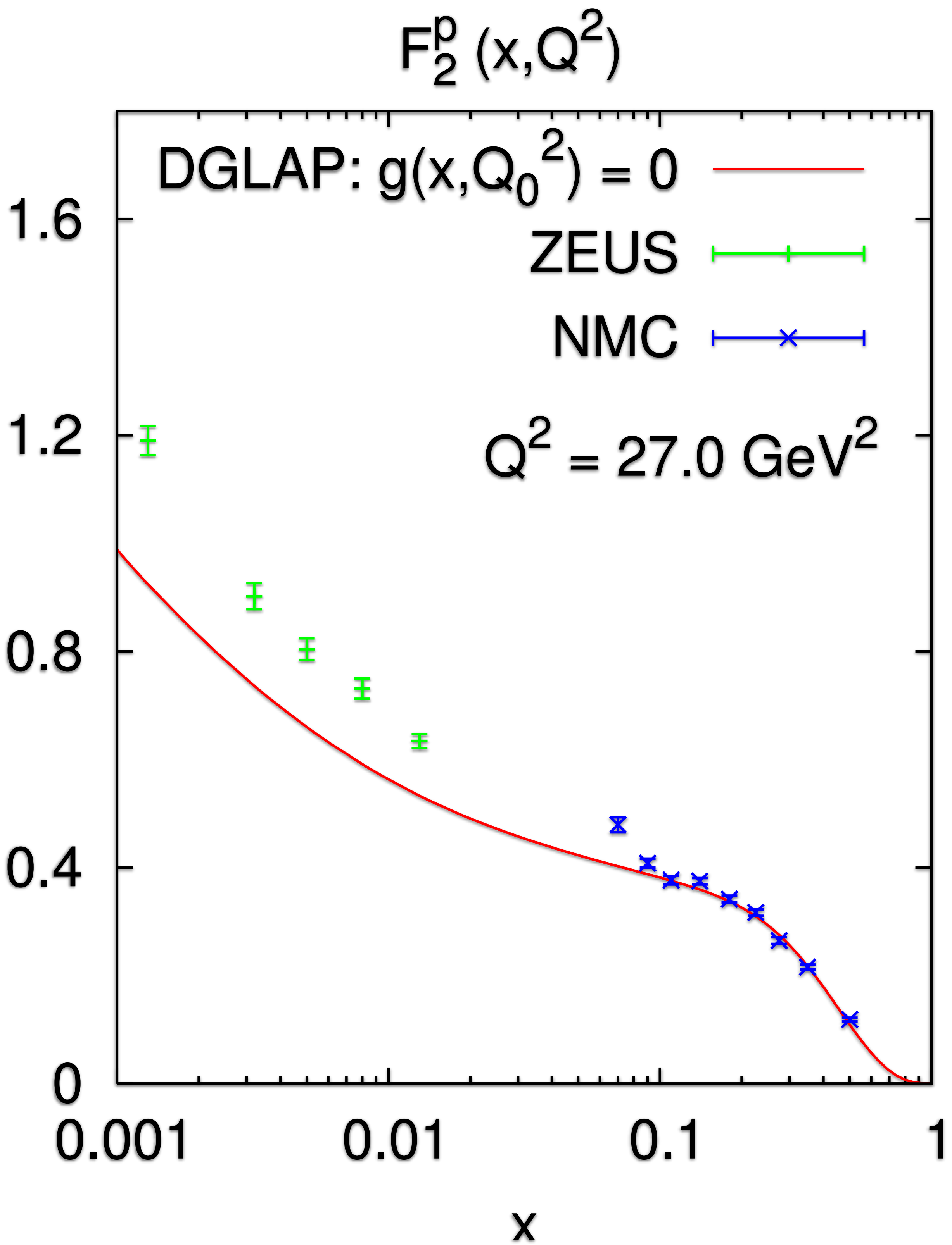

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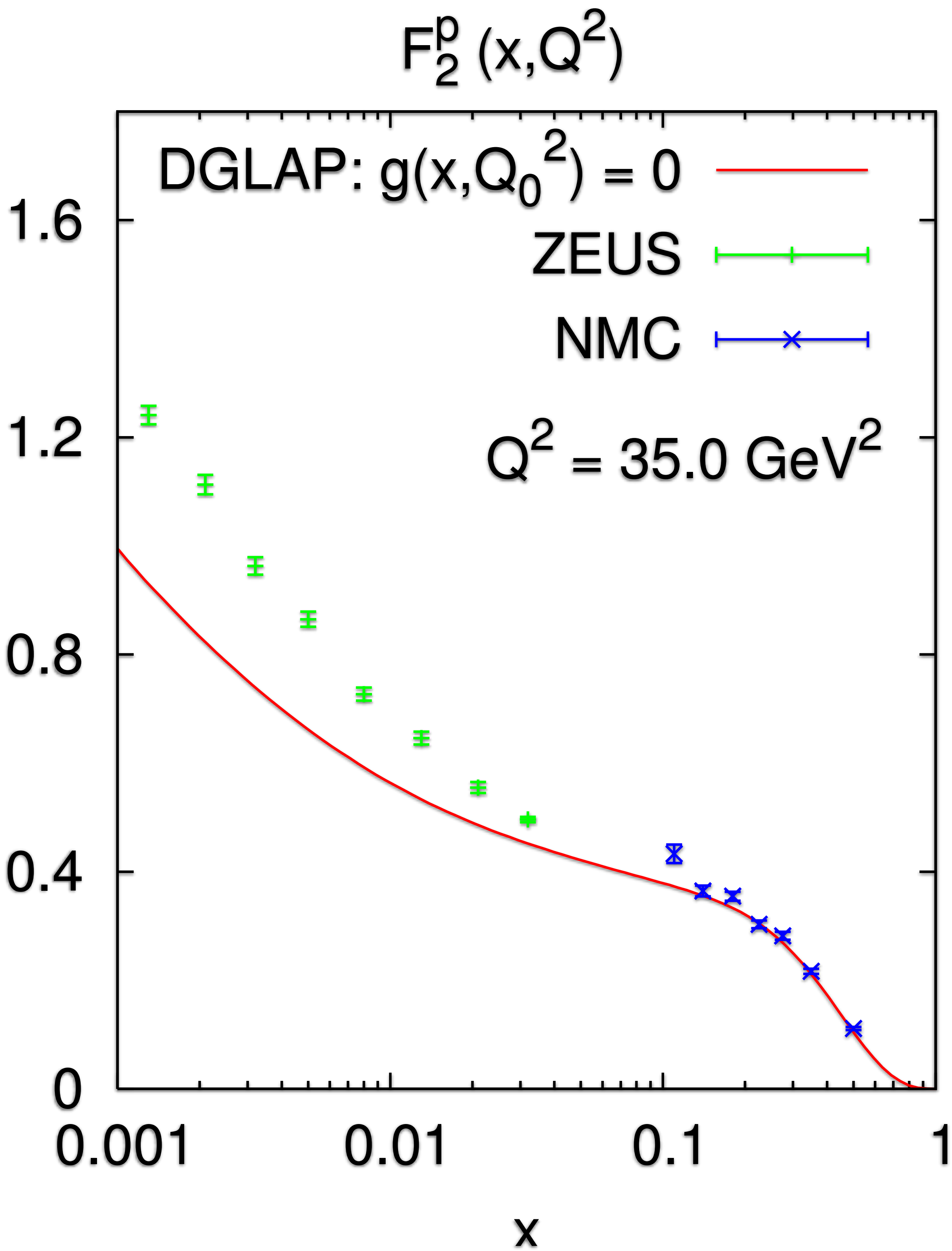
Consider DIS data — $F_2(x,Q^2)$ — in a world where the proton just had quarks



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

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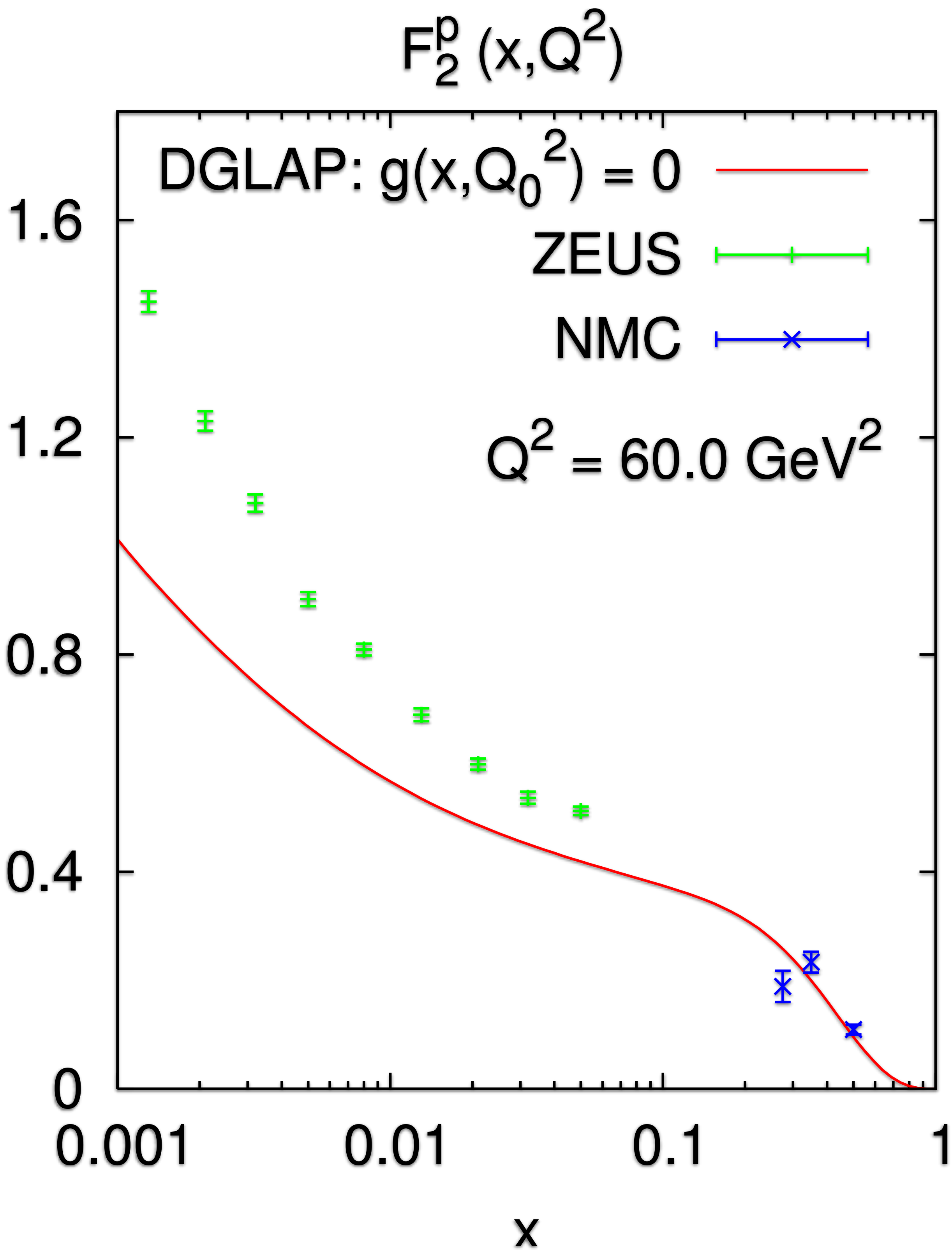

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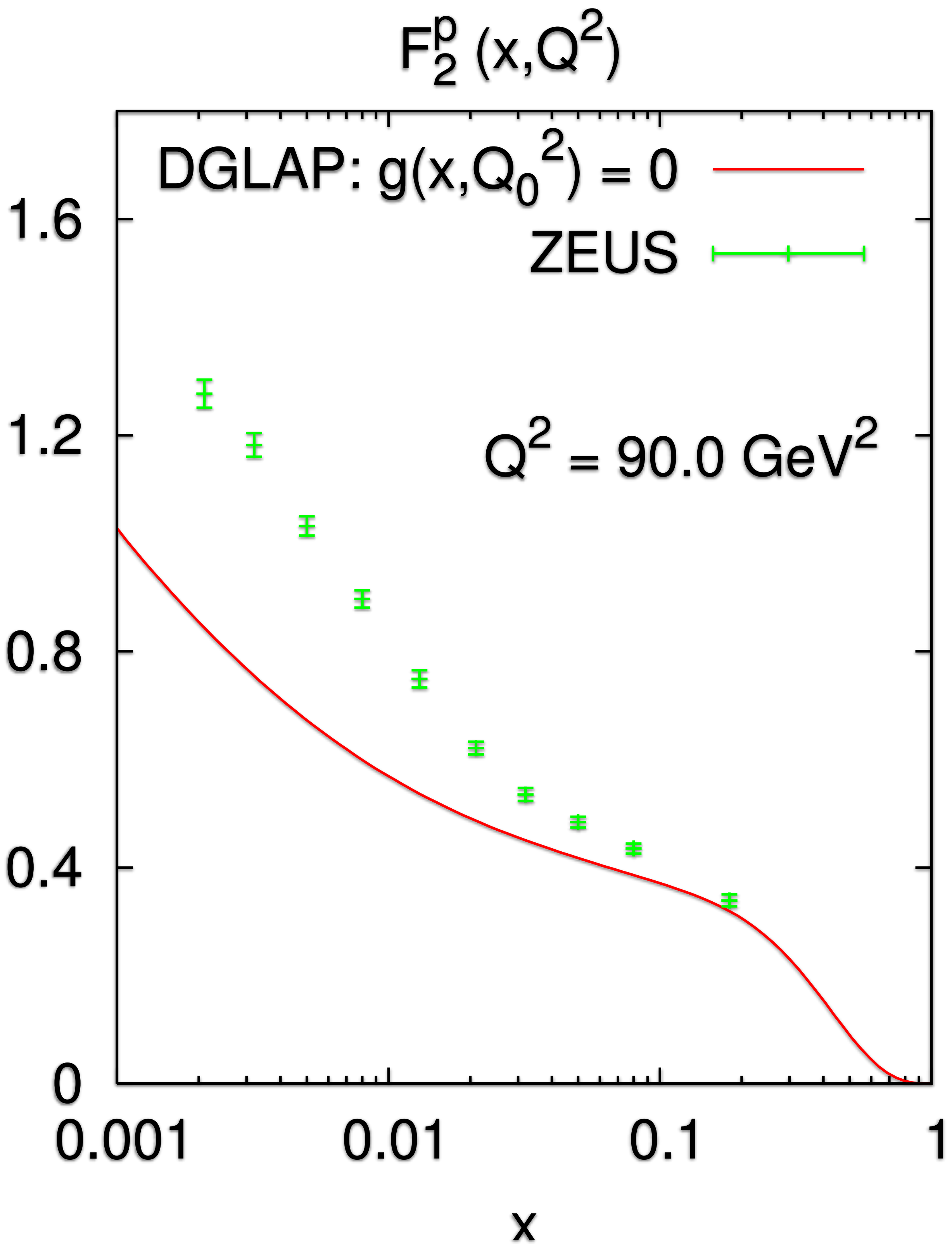

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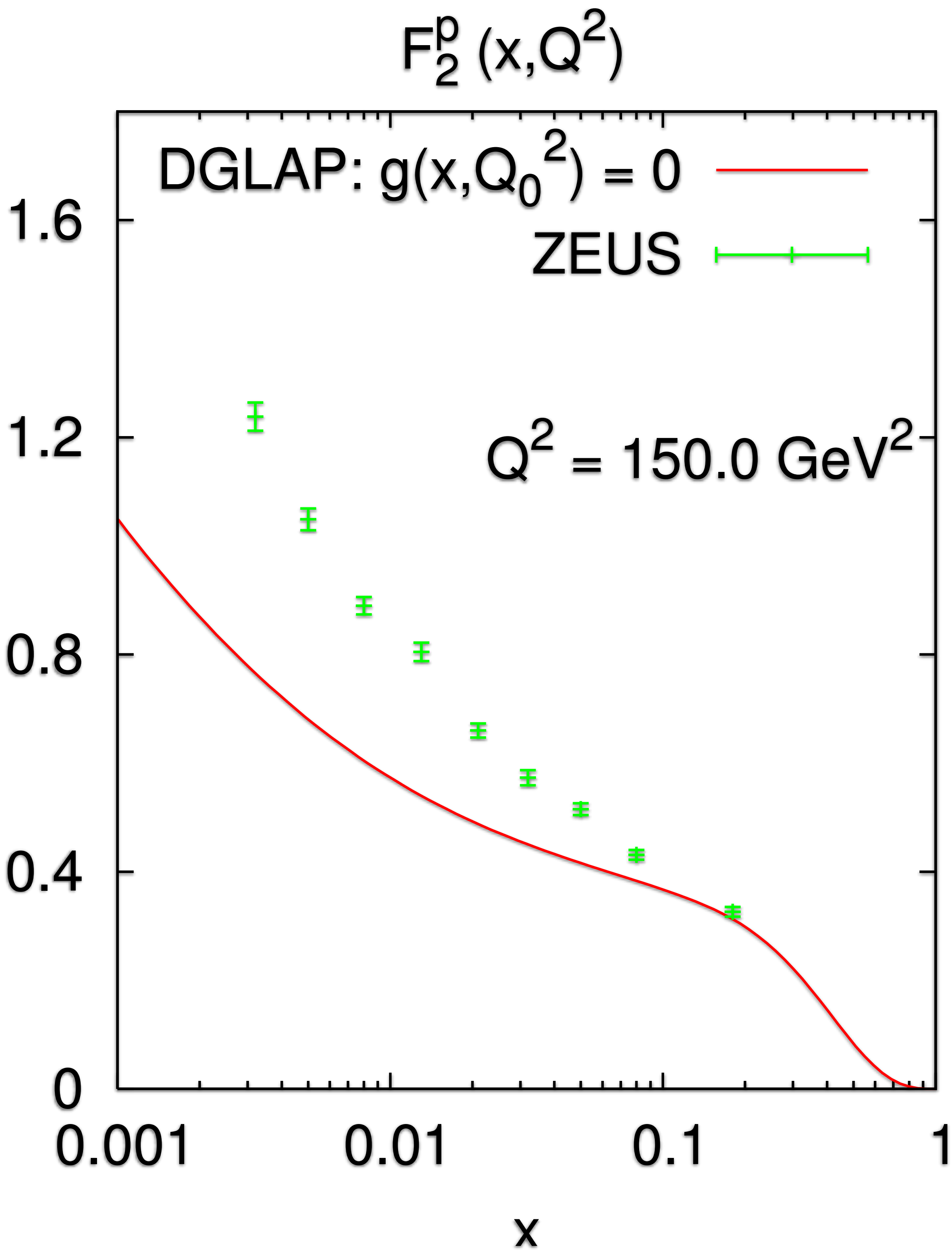

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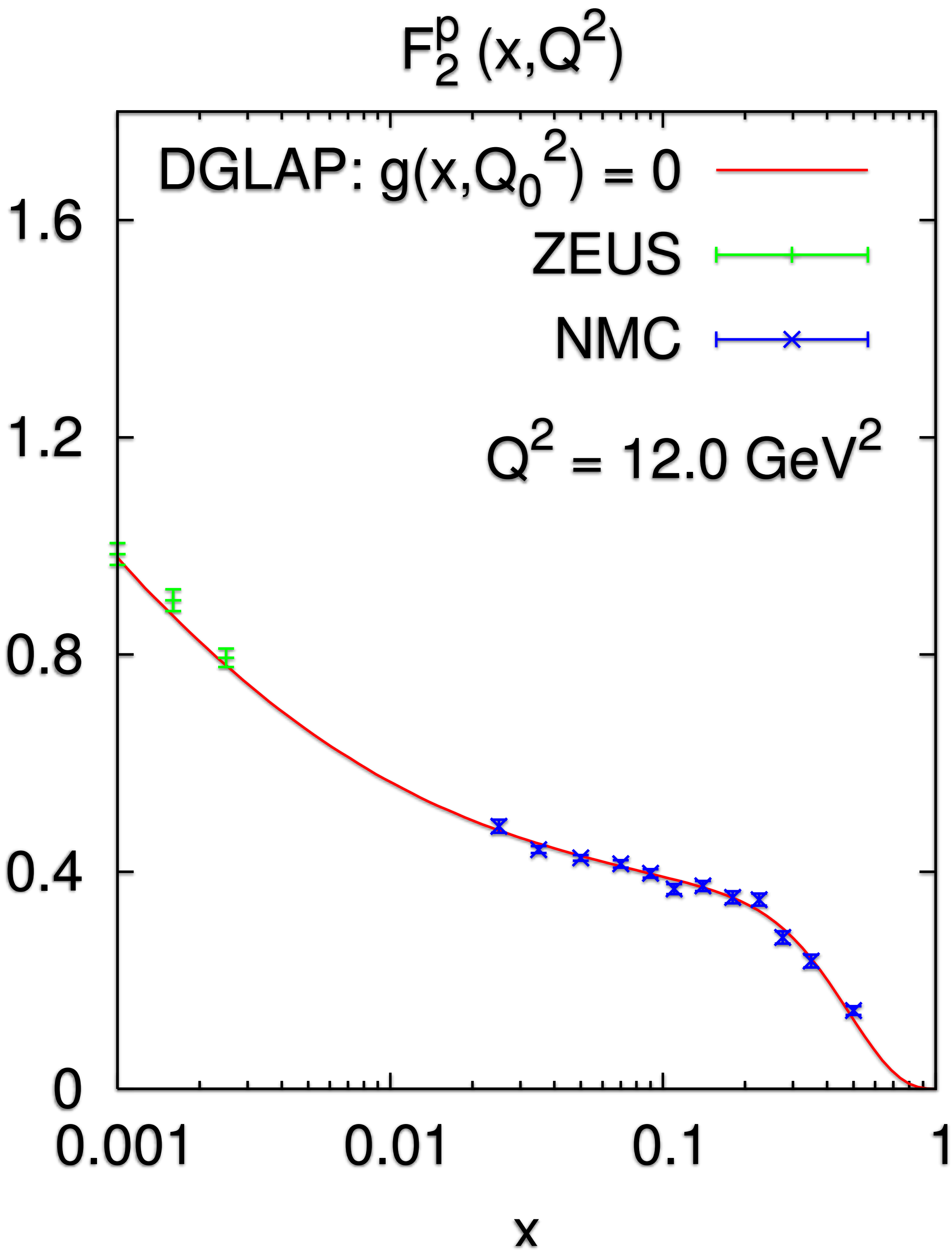
COMPLETE FAILURE

to reproduce data evolution

Consider DIS data — $F_2(x,Q^2)$ — with specially tuned gluon



If gluon $\neq 0$, splitting

$$g \to q\bar{q}$$

generates extra quarks at large Q2 ➠ faster rise of F2

Global PDF fits (CT, MMHT, NNPDF, etc.) choose gluon distribution that leads to the correct Q2 evolution.

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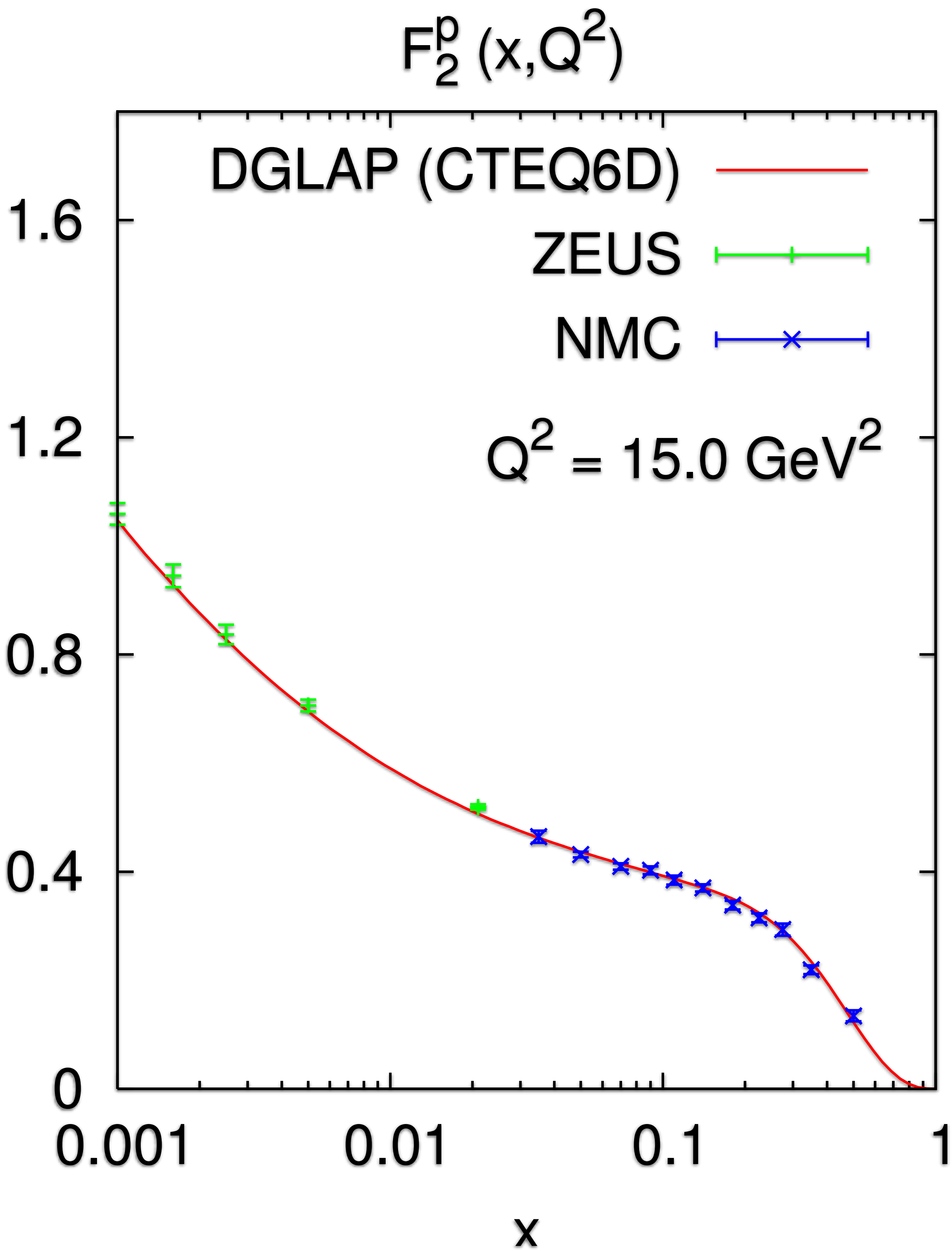

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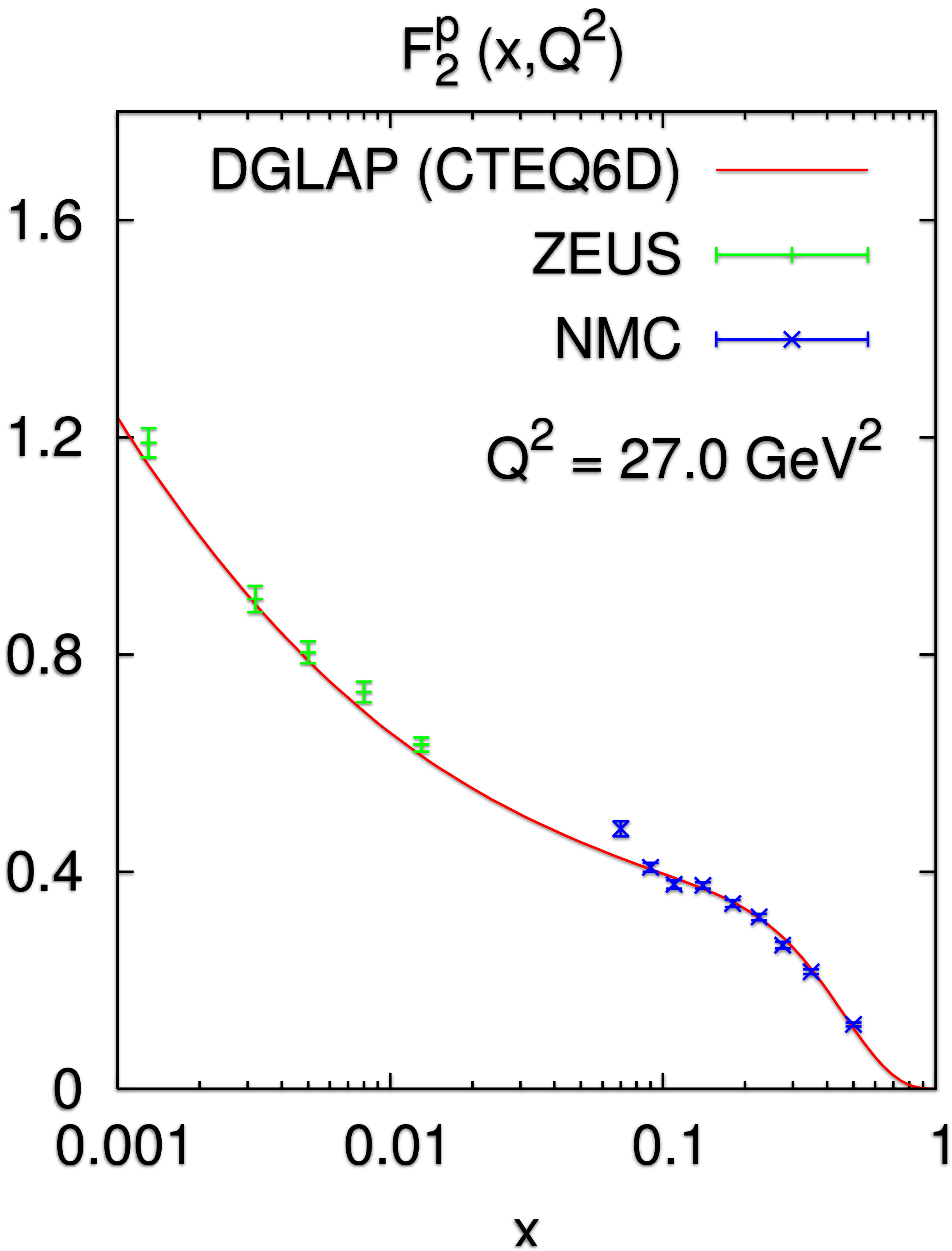

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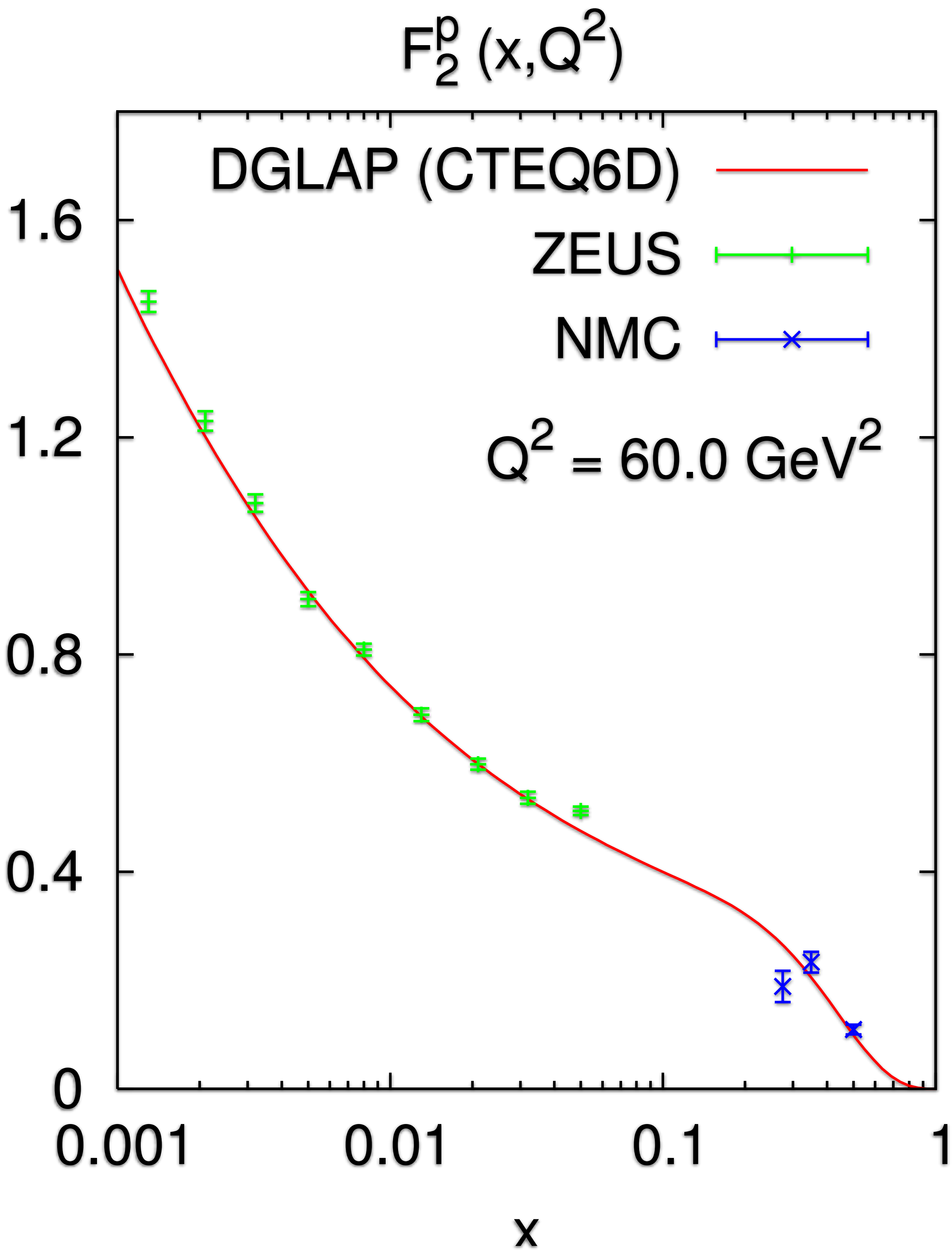

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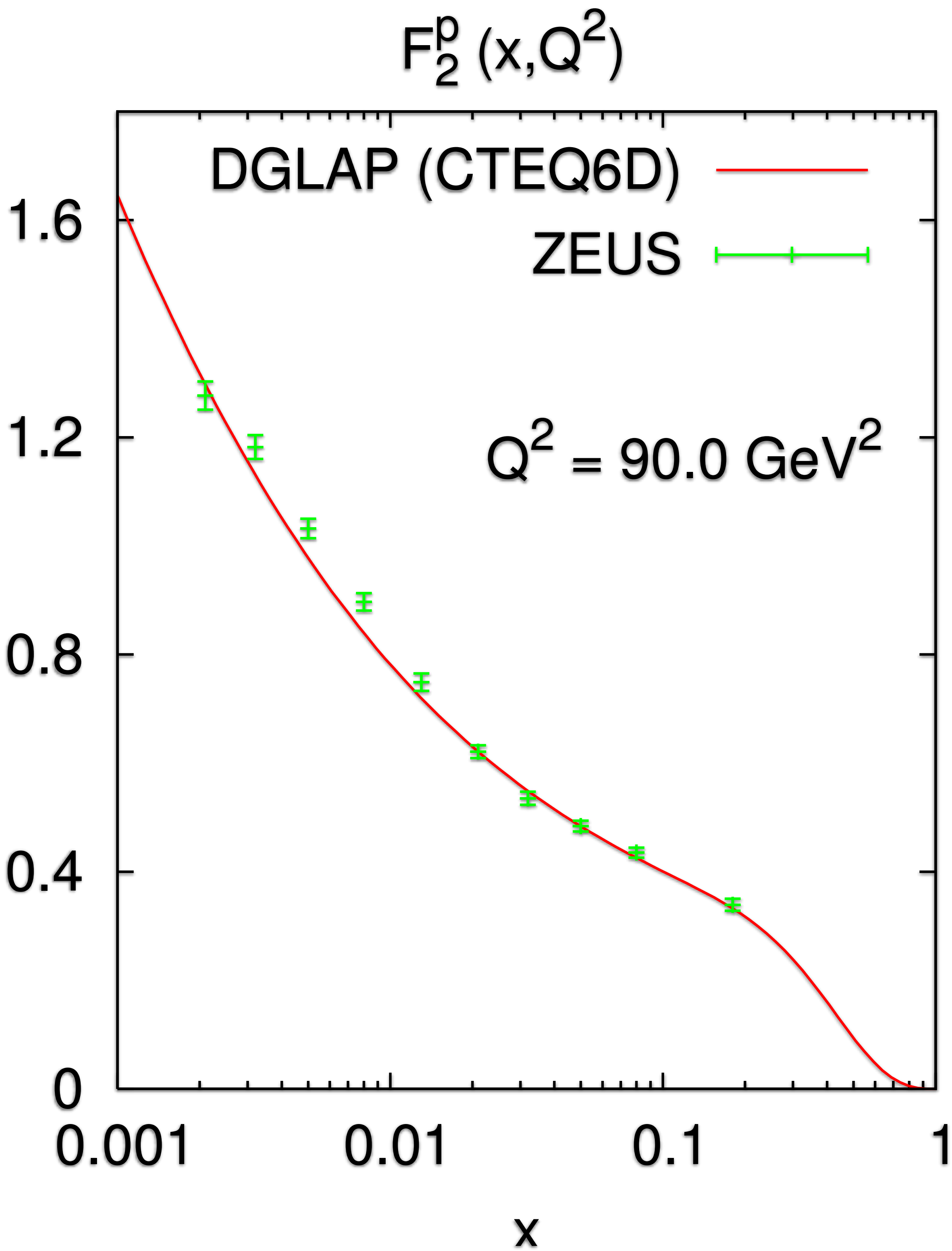

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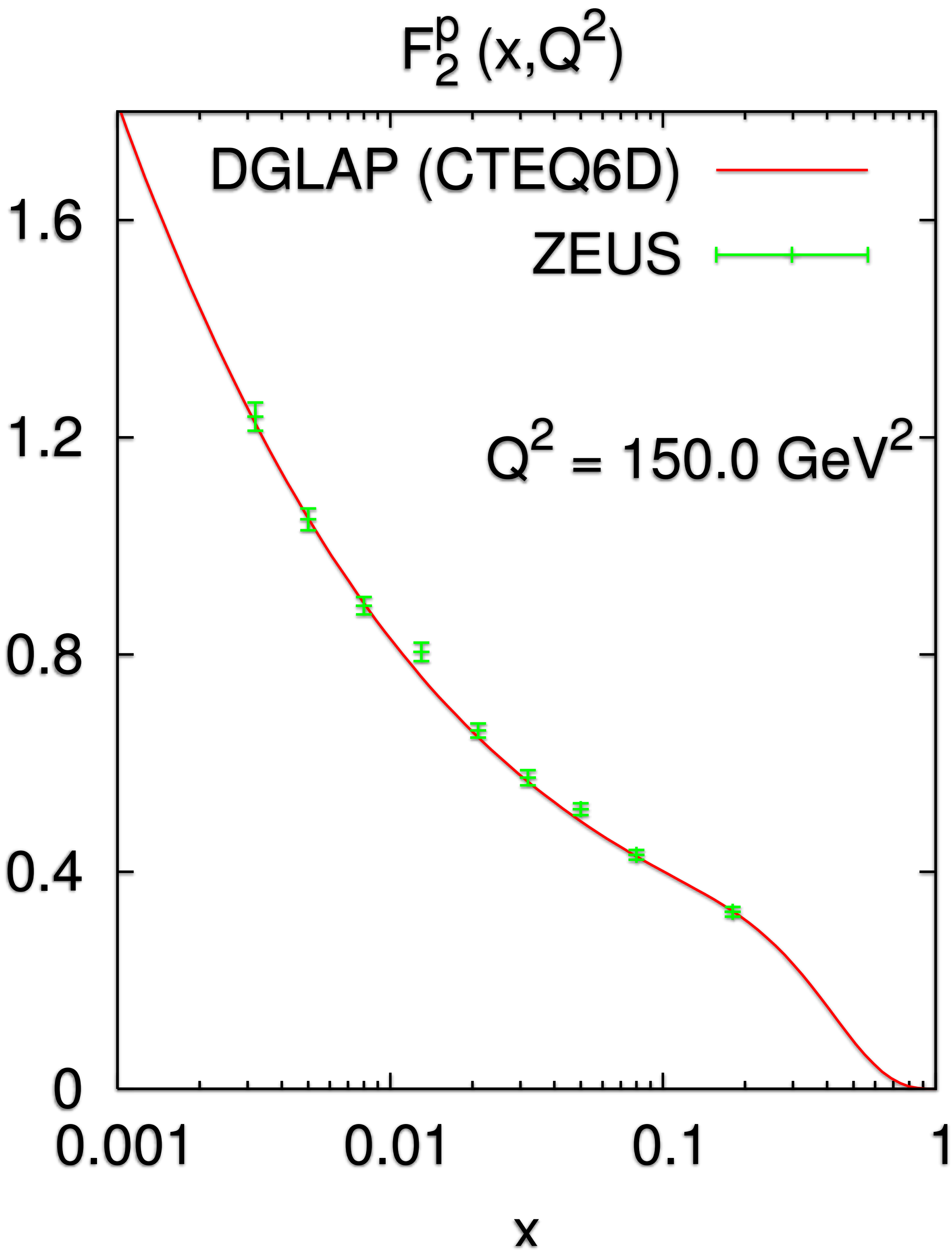

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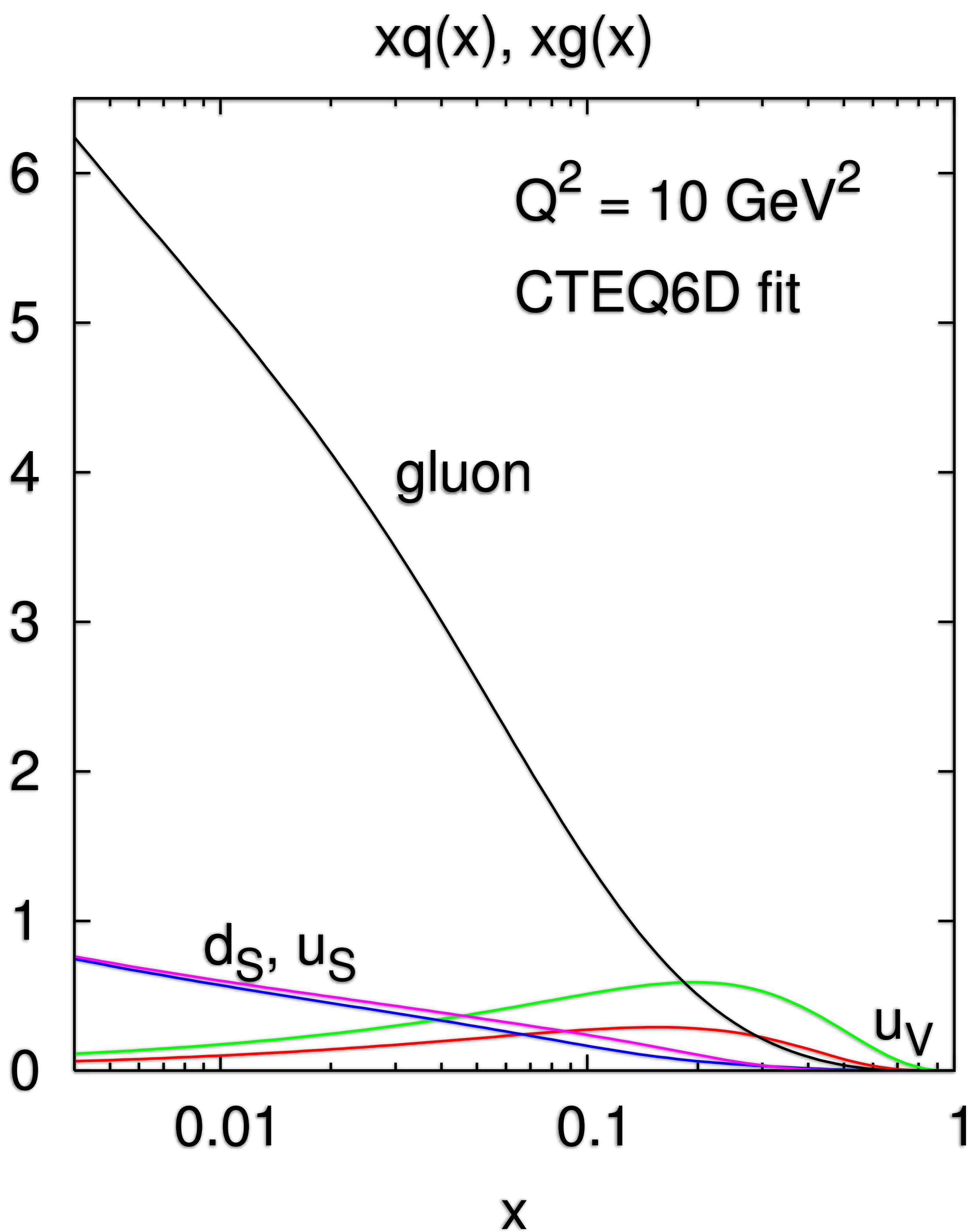
SUCCESS

Resulting gluon distribution, compared to quarks



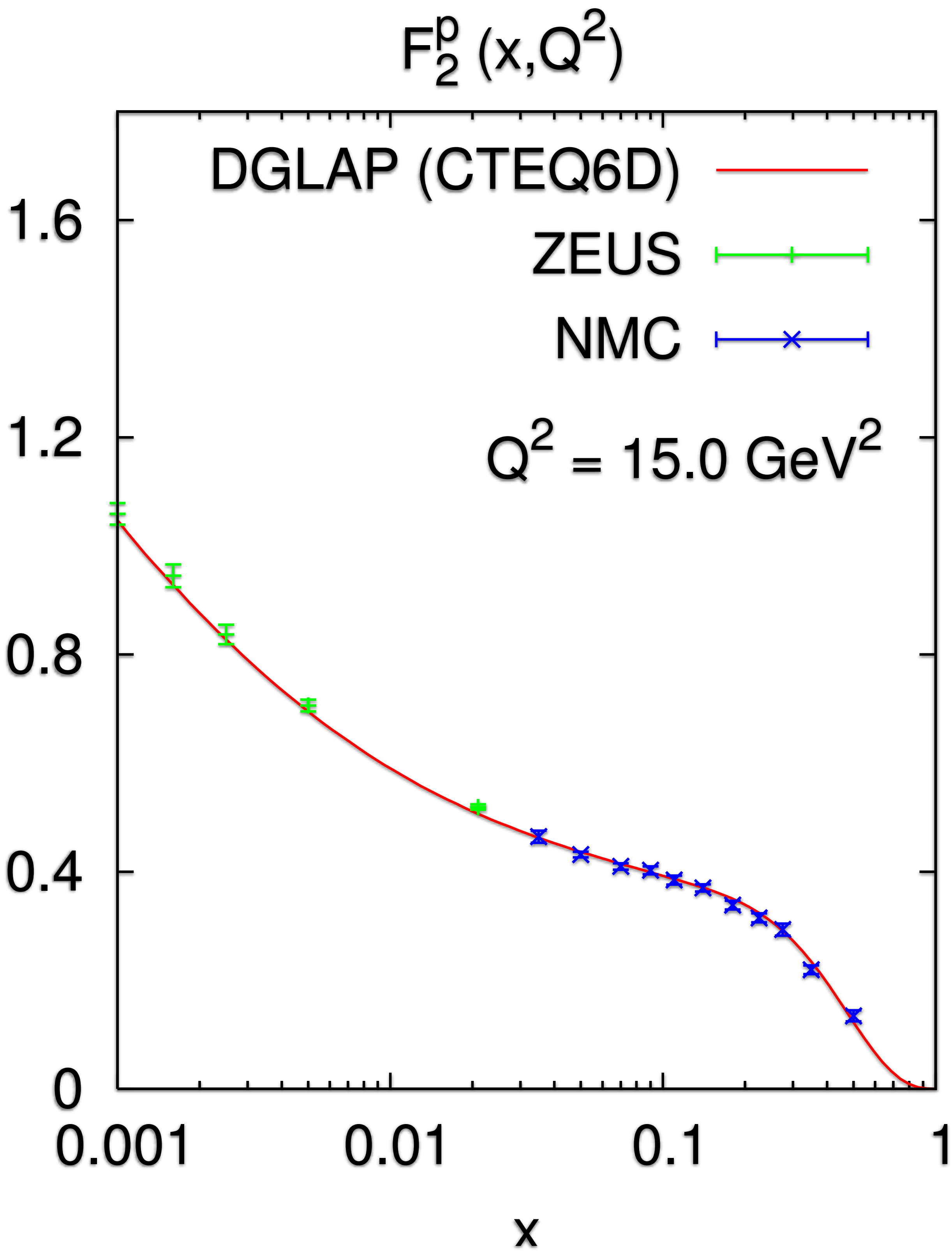
Resulting gluon distribution is **HUGE!**

Carries 47% of proton's momentum
(at scale of 100 GeV)

Crucial in order to satisfy momentum sum rule.

Large value of gluon has big impact on phenomenology

Consider DIS data — $F_2(x,Q^2)$ — with specially tuned gluon



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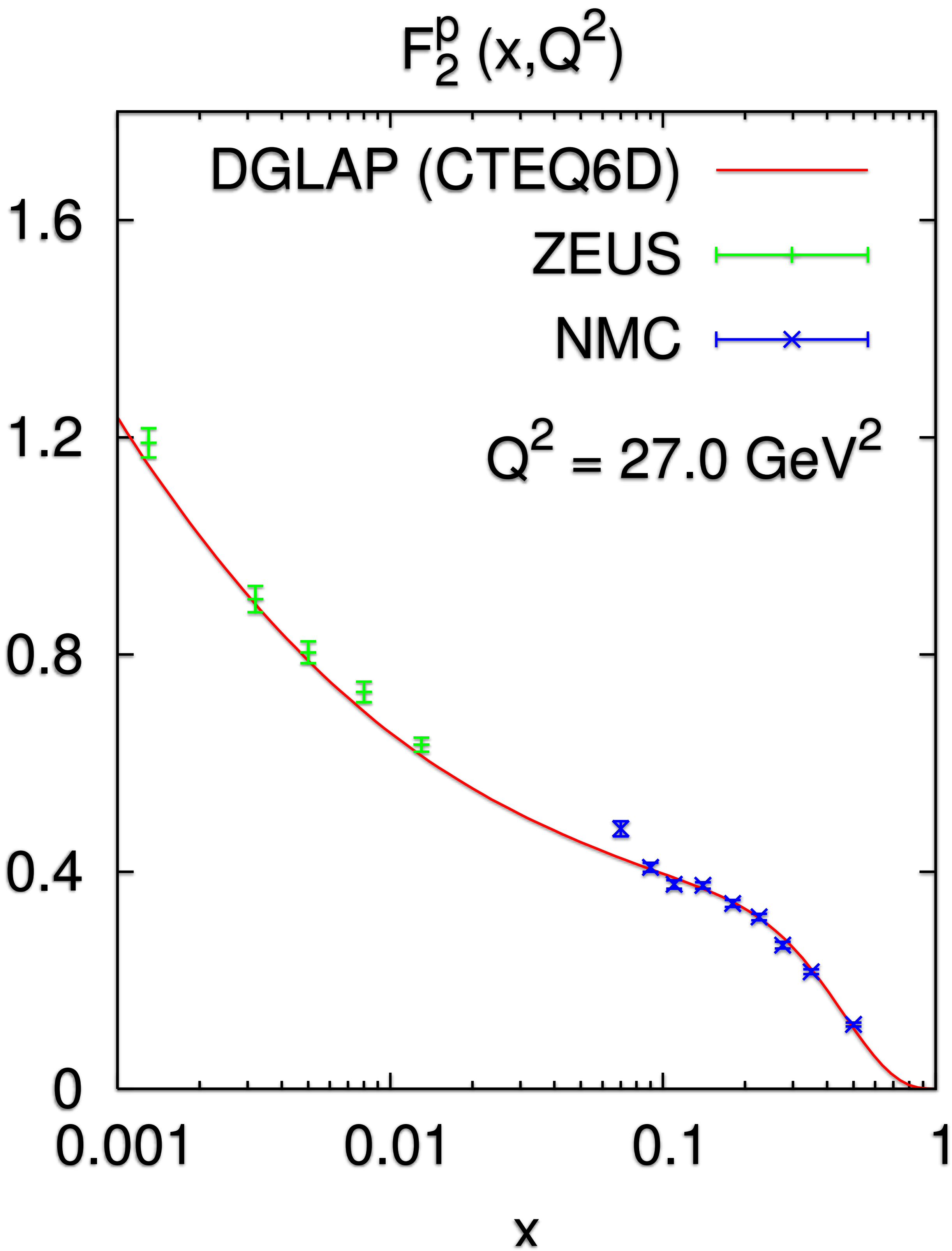

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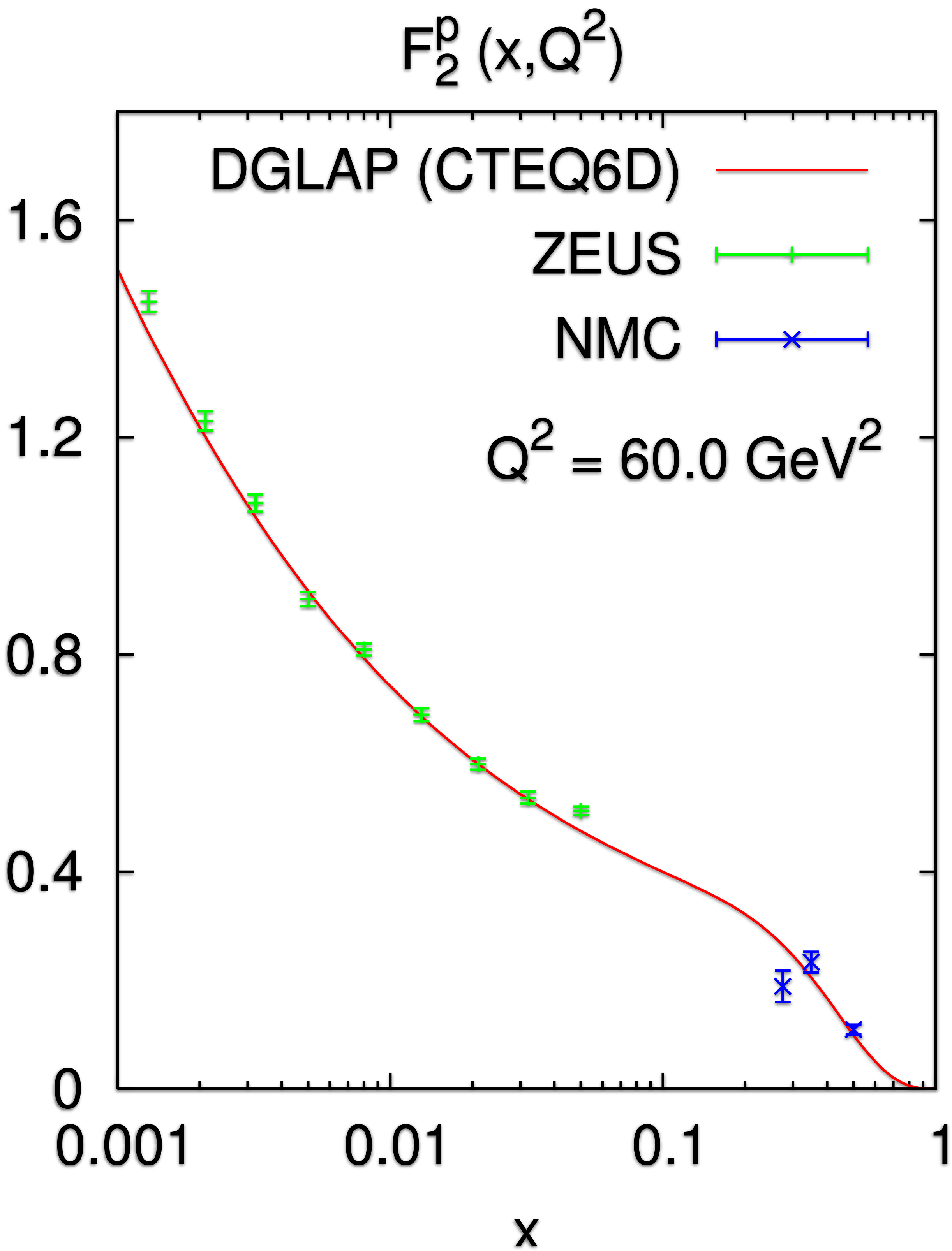

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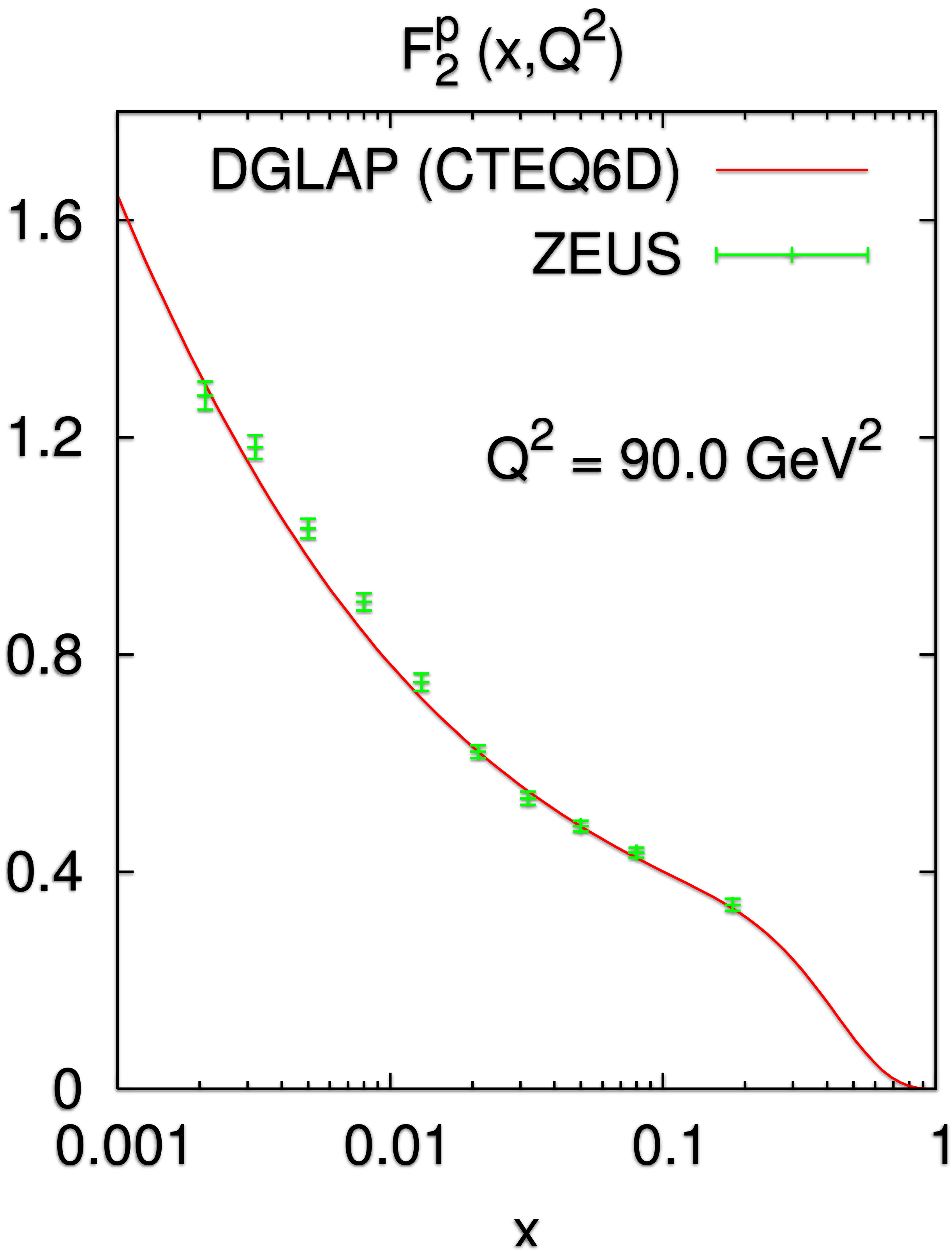

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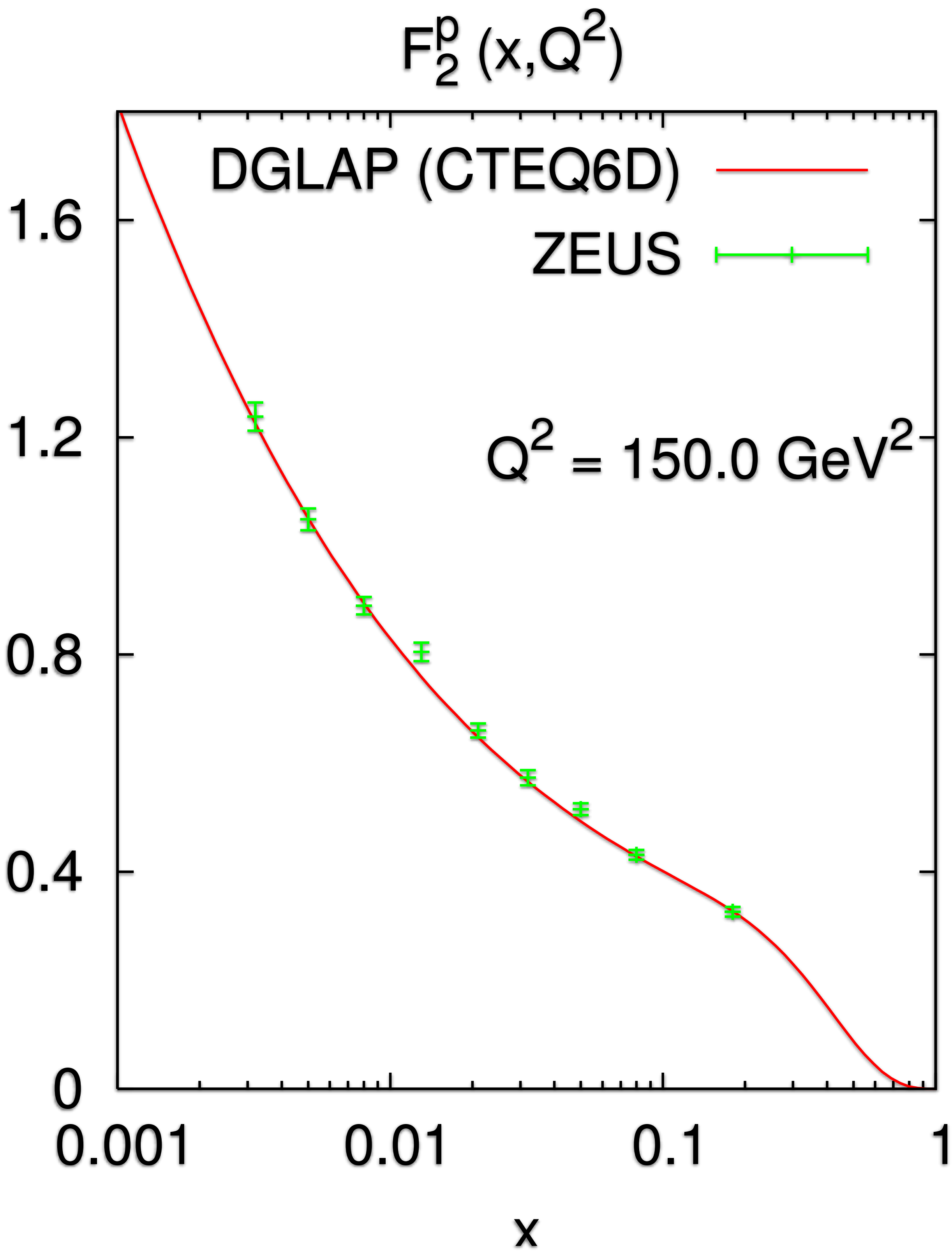

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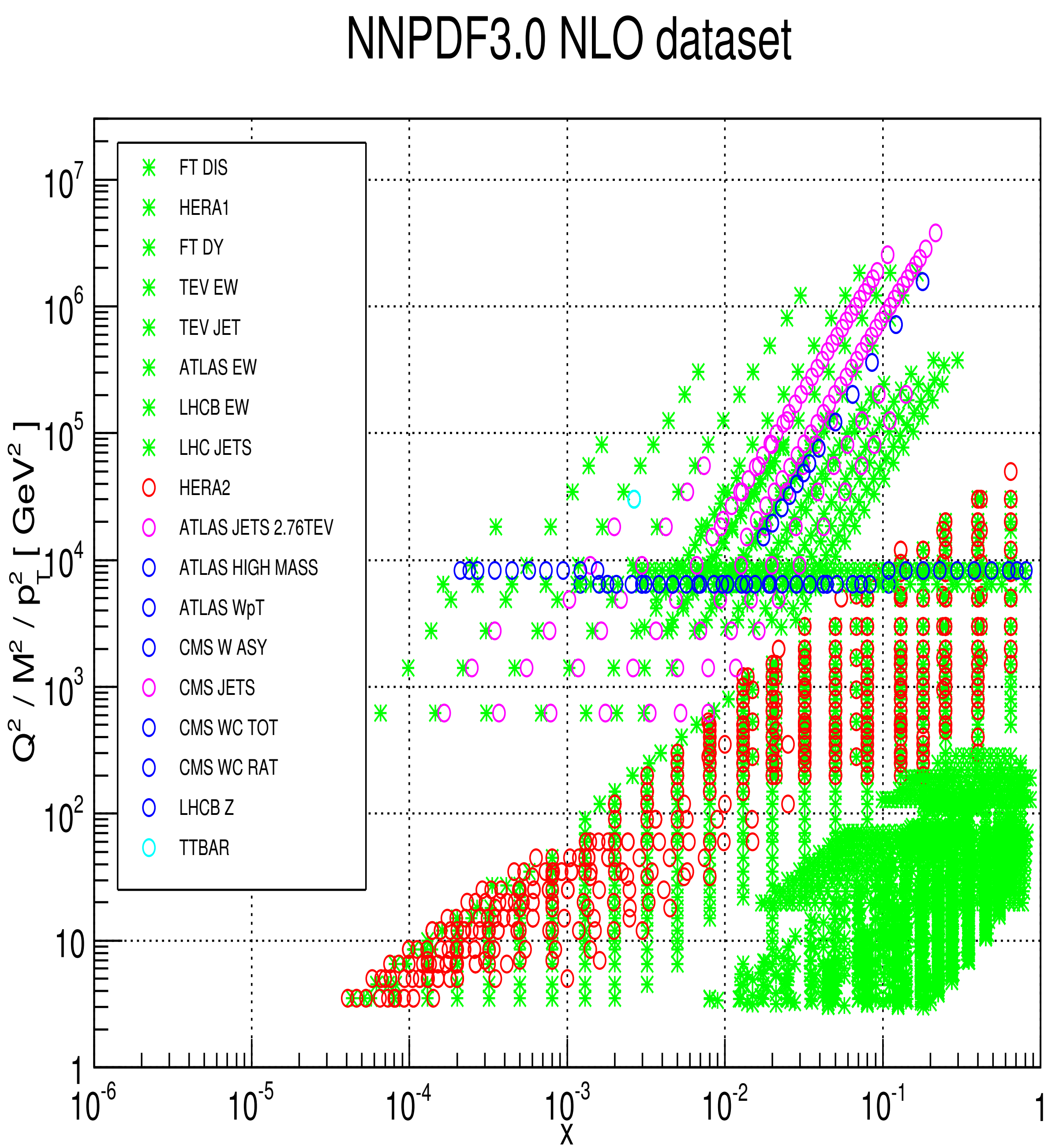
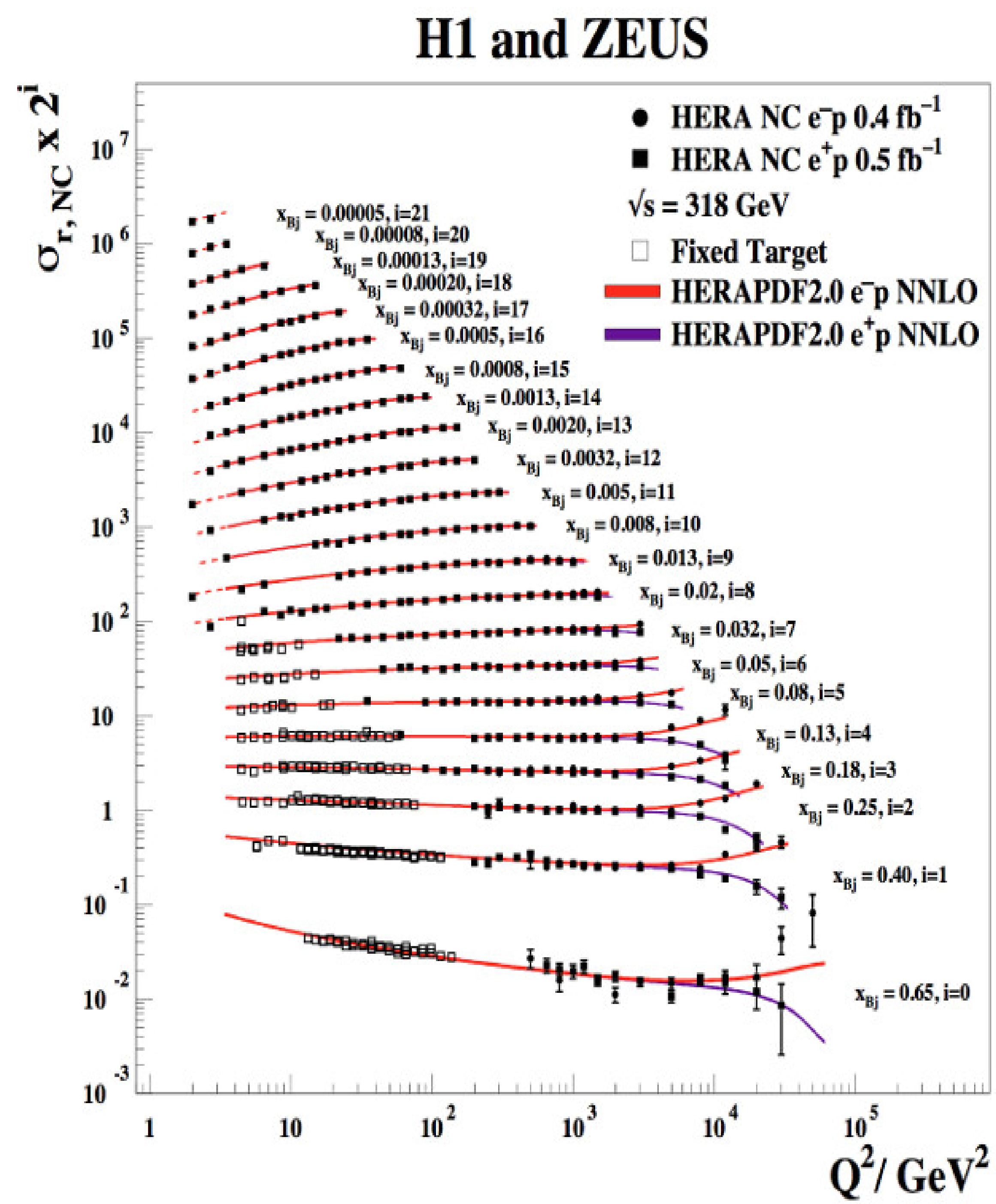
SUCCESS

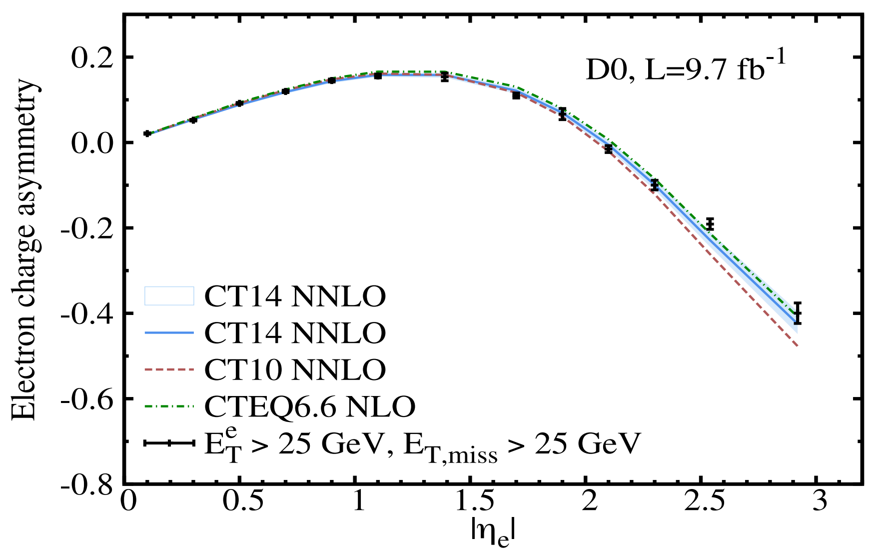
TODAY'S PDF FITS

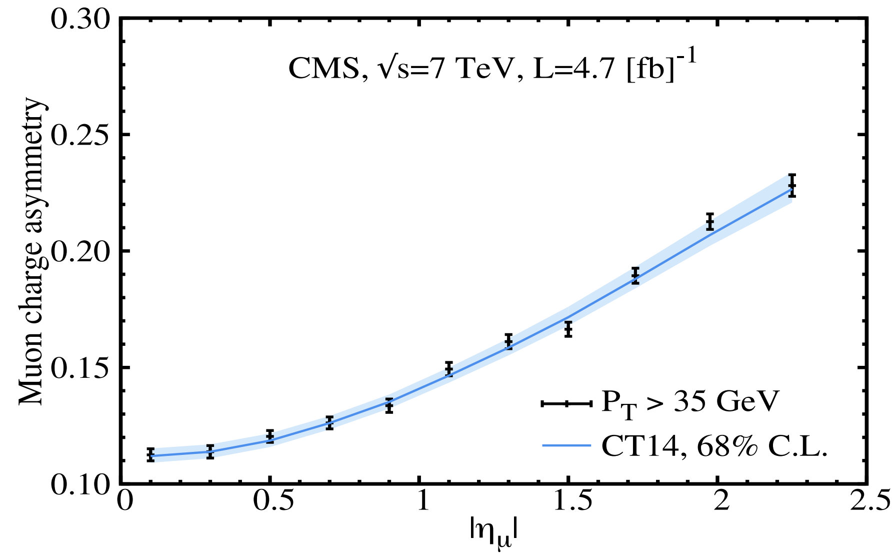


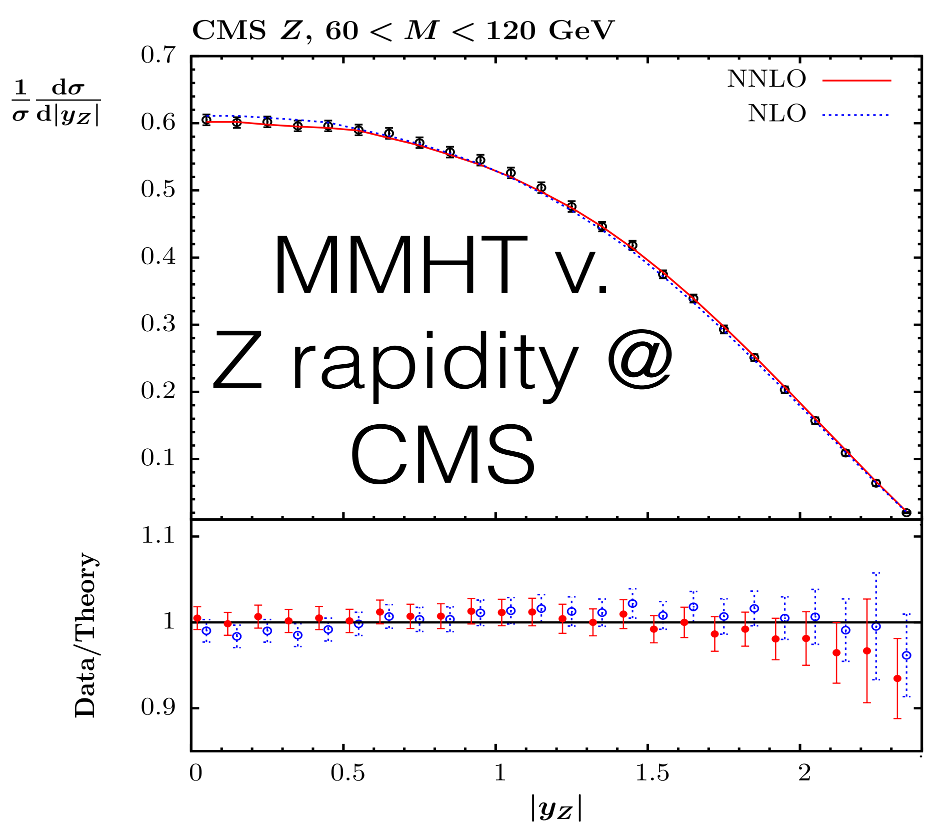


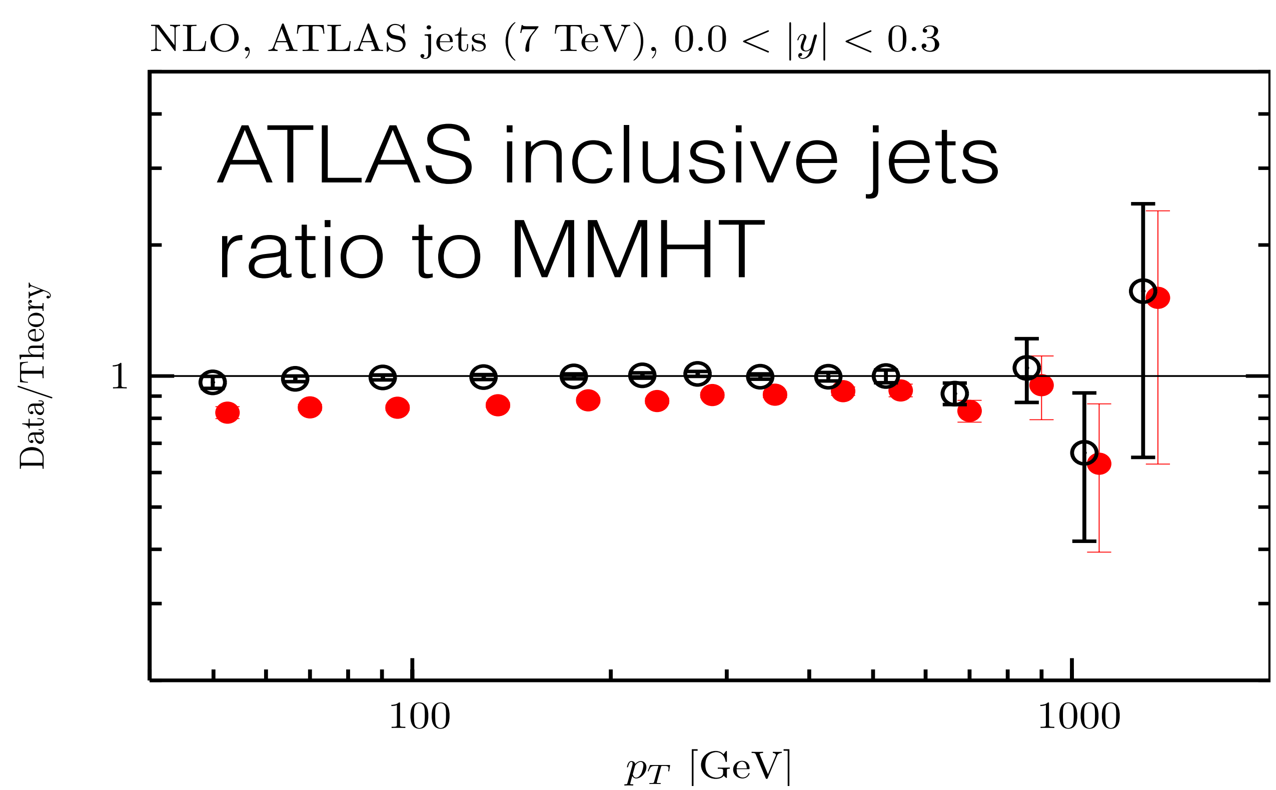
TODAY'S PDF FITS

Lepton charge asym. v. CT14 @ D0 & CMS

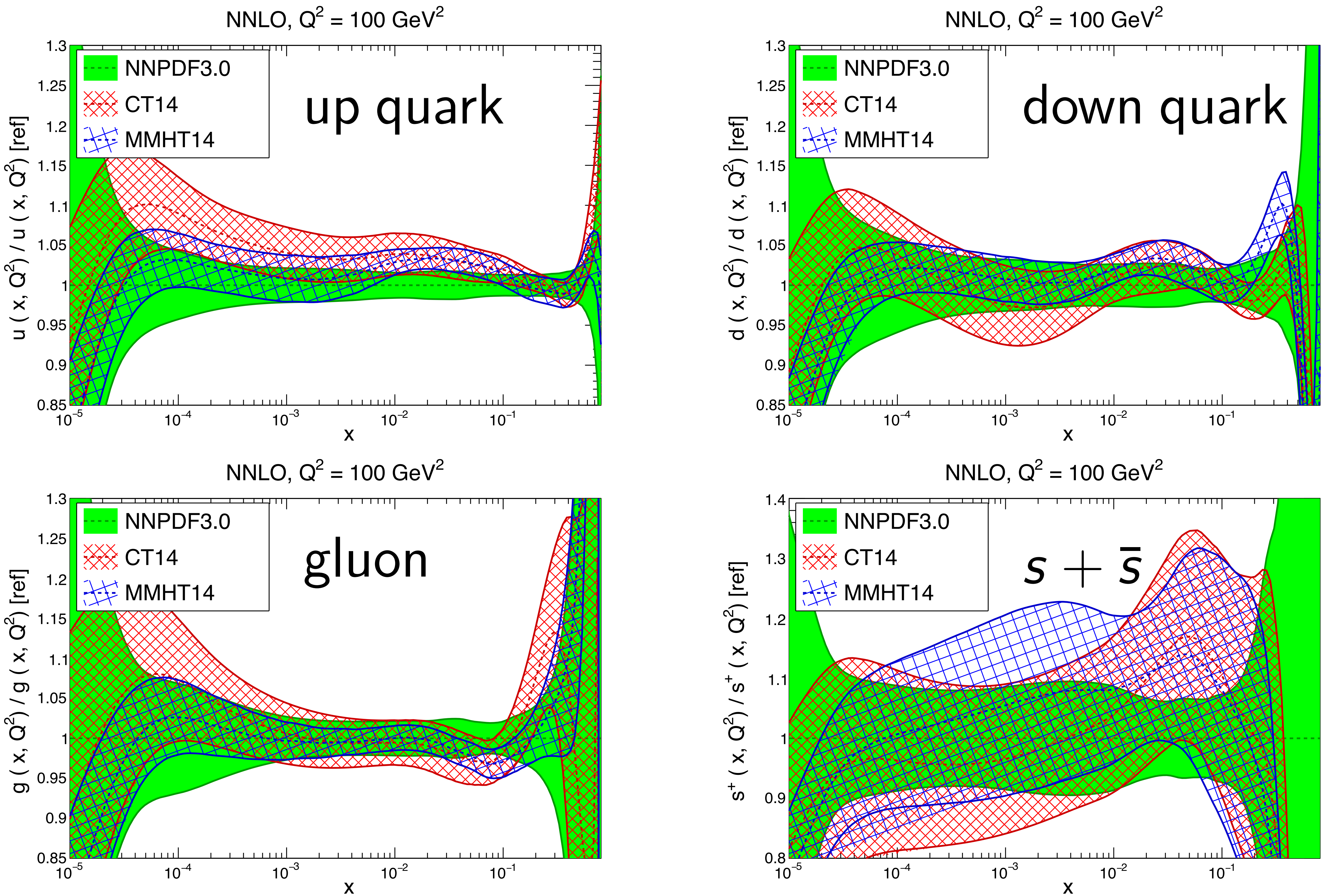




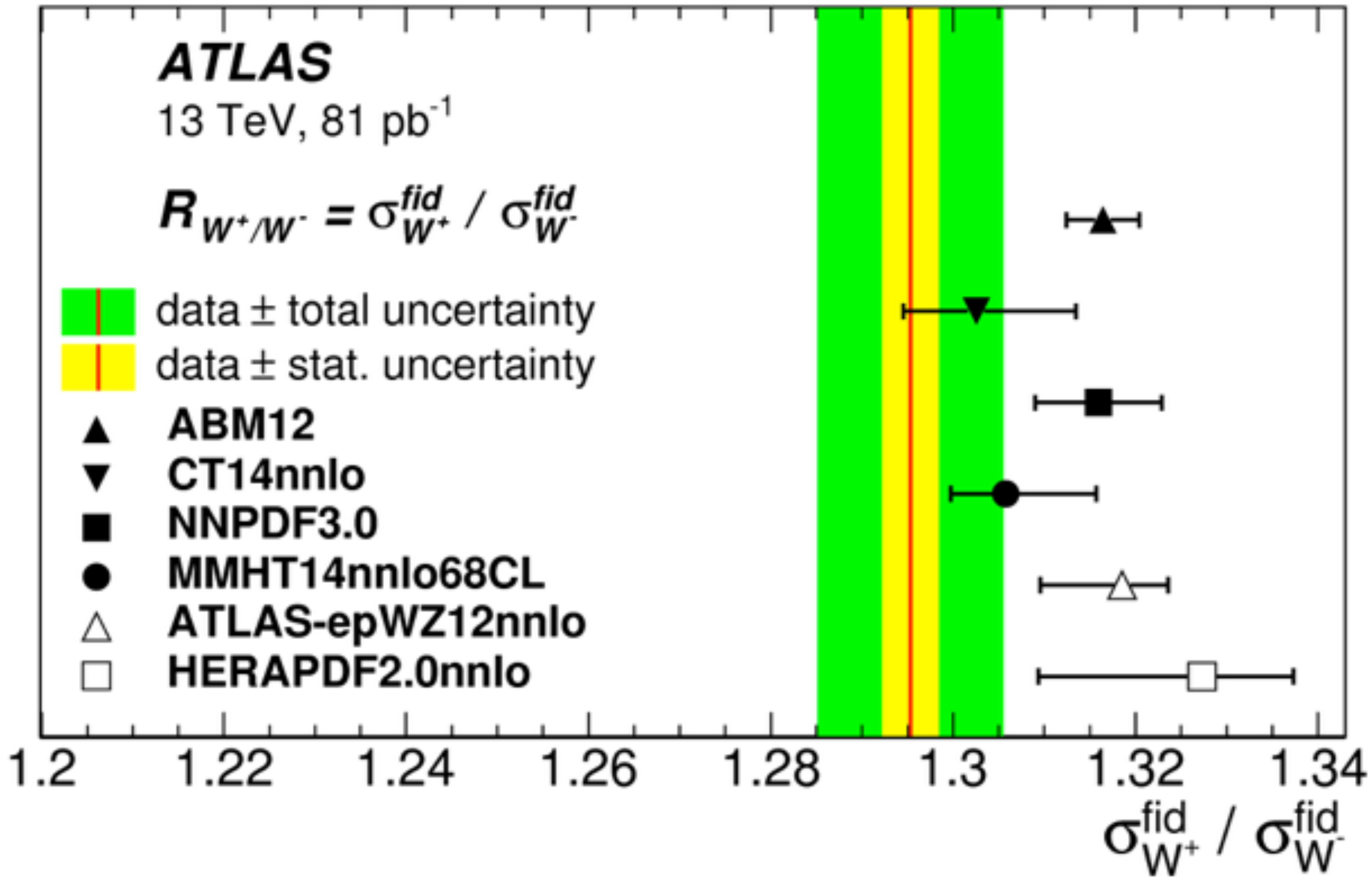




THREE GLOBAL PDF FITS: CT14, MMHT2014, NNPDF30



cross-section ratios (W^+/W^-, ttbar/Z) show tensions with some PDFs



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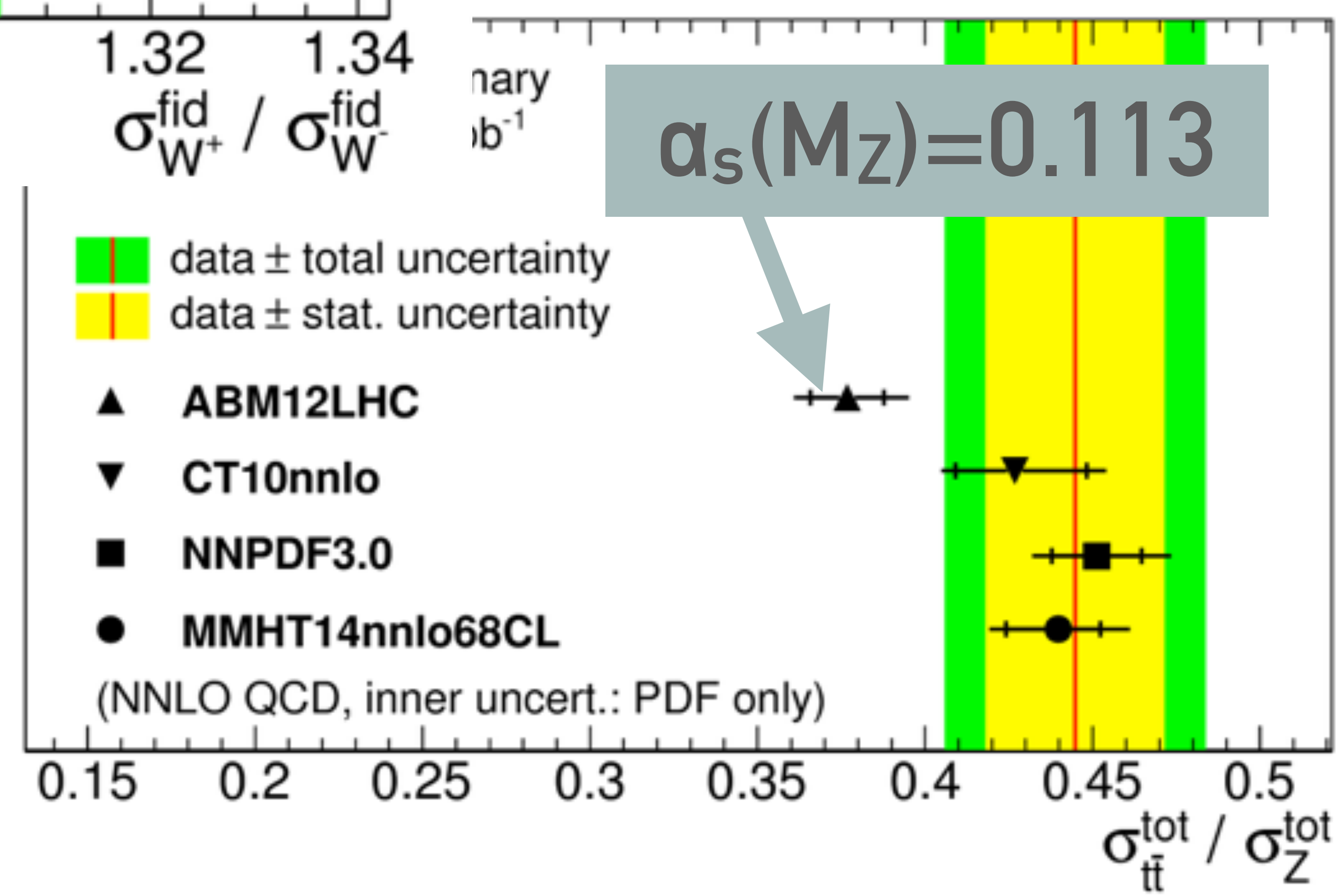


NB: top-quark mass choice affects this plot

FINAL REMARKS ON PDFS

- In range $10^{-3} < x < 0.1$, core PDFs (up, down, gluon) known to ~ 1-2% accuracy
- For many LHC applications, you can use PDF4LHC15 set, which merges CT14, MMHT2014, NNPDF30
- Situation is not full consensus: ABM group claims substantially different gluon distribution

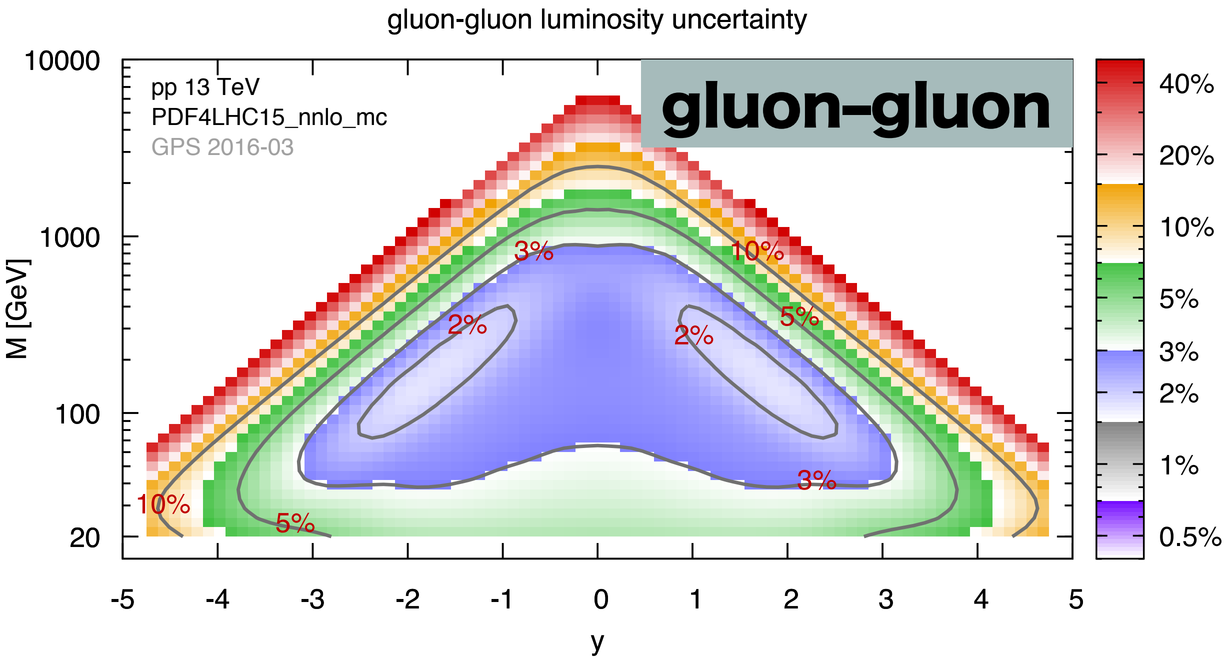
For visualisations of PDFs and related quantities, a good place to start is

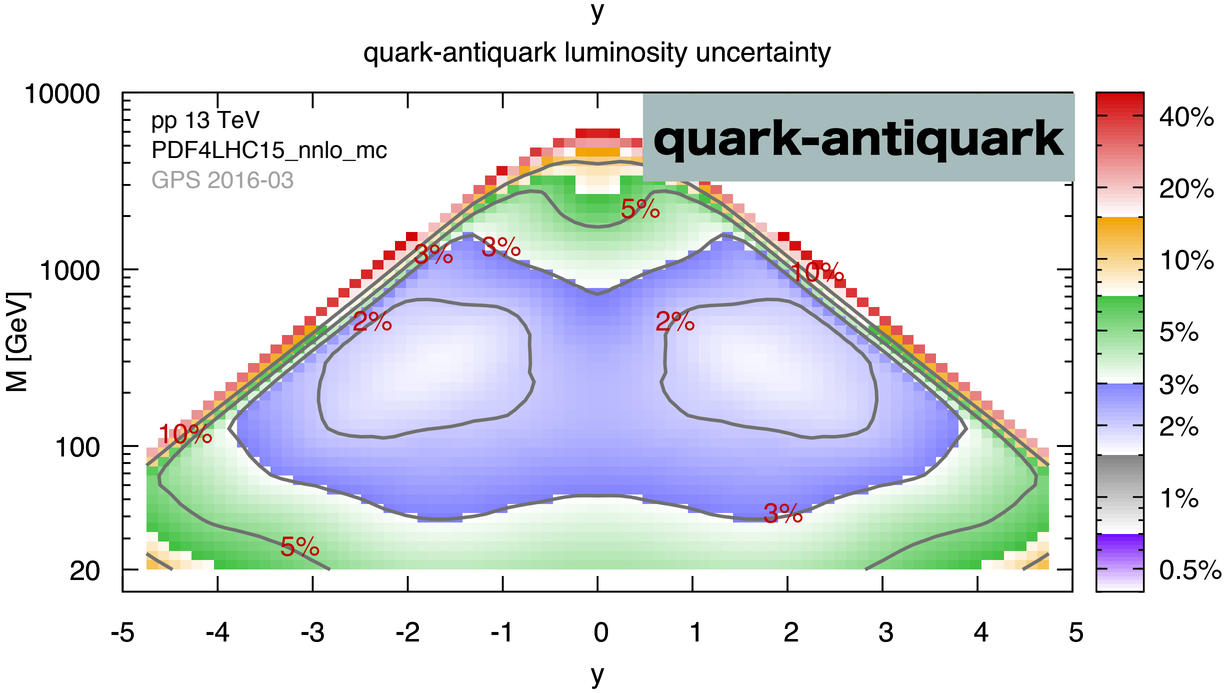
http://apfel.mi.infn.it/ (ApfelWeb)

EXTRA SLIDES

PDFs: WHAT ROUTE FOR PROGRESS?

- Current status is 2–3% for core "precision" region
- Path to 1% is not clear — e.g. Z p_T's strongest constraint is on qg lumi, which is already best known (why?)
- It'll be interesting to revisit the question once ttbar, incl. jets, Z p_T, etc. have all been incorporated at NNLO
- **Can expts. get better lumi determination? 0.5%?**





PDF THEORY UNCERTAINTIES