INGREDIENTS FOR ACCURATE COLLIDER PHYSICS (2/2)

Gavin Salam, CERN

PSI Summer School Exothiggs,
Zuoz, August 2016

TUESDAY'S LECTURE

- We discussed the "Master" formula

$$\sigma\left(h_1h_2 \to W+X\right) = \sum_{n=0}^{\infty} \alpha_s^n \left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right) \times \hat{\sigma}^{(n)}_{ij\to W+X}\left(x_1x_2s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$

- and its main inputs
  - the strong coupling α_s
  - Parton Distribution Functions (PDFs)
- **Today:** we discuss the actual scattering cross section

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$$\times\, \hat{\sigma}^{(n)}_{ij\to W+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$

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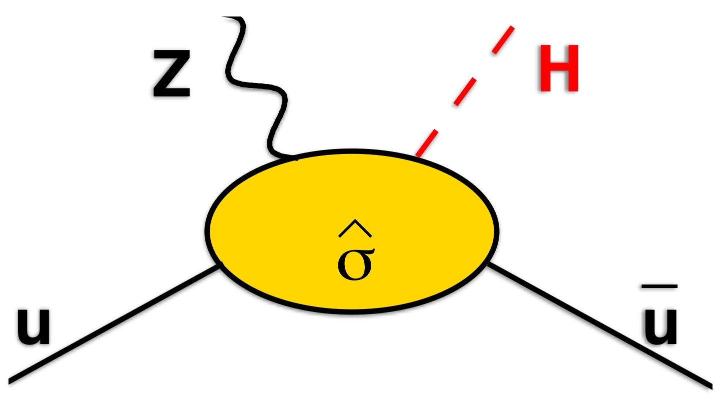


- **Today:** we discuss the actual scattering cross section

the hard cross section

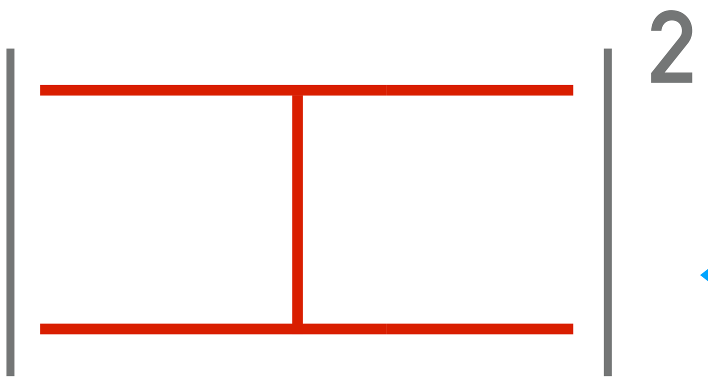
$$\sigma \sim \sigma_2 \alpha_s^2 + \sigma_3 \alpha_s^3 + \sigma_4 \alpha_s^4 + \sigma_5 \alpha_s^5 + \cdots$$

LO **NLO** **NNLO** **N3LO**

INGREDIENTS FOR A CALCULATION (generic 2→2 process)

LO

Tree
2→2



to illustrate the concepts, we don't care what the particles are — just draw lines

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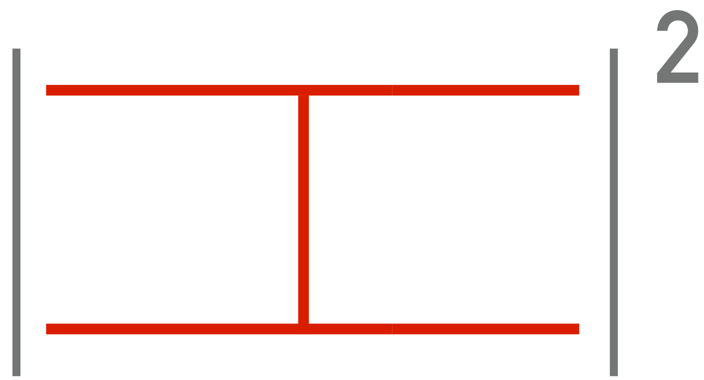

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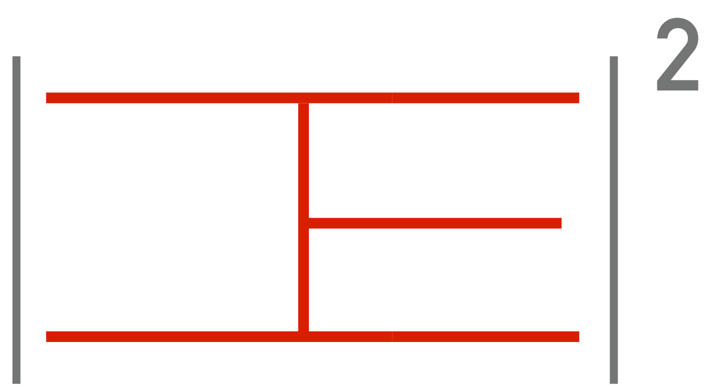
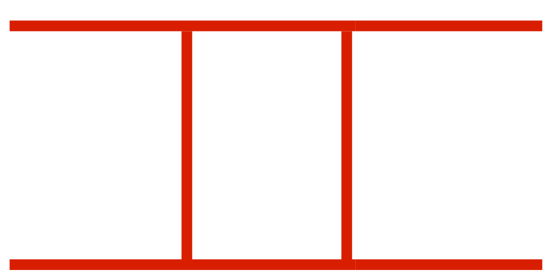
NLO

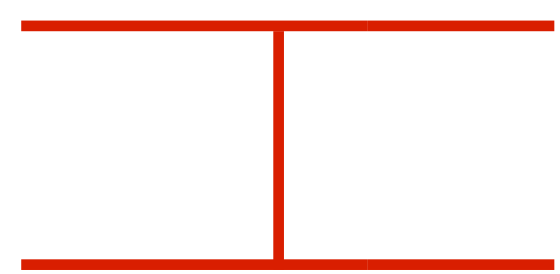
Tree
2→3



1-loop
2→2



×

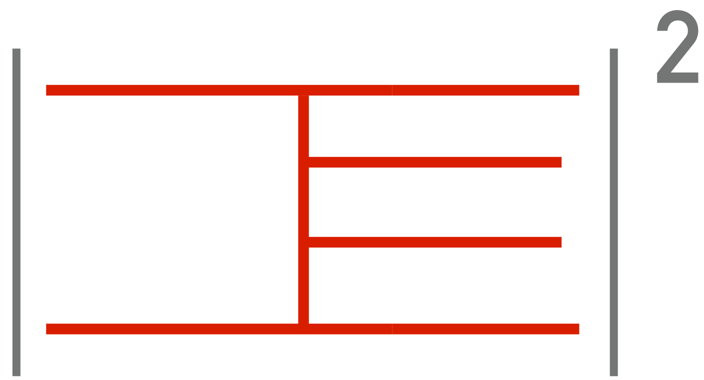


+ *complex conj.*

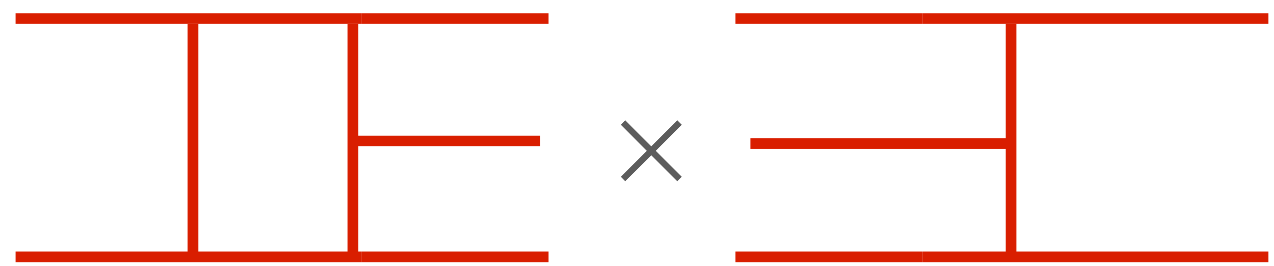
INGREDIENTS FOR A CALCULATION (generic 2→2 process)

NNLO

Tree
2→4

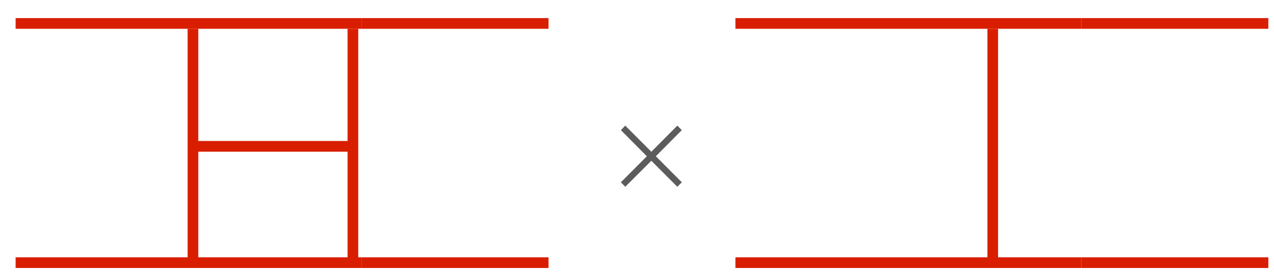


1-loop
2→3



+ *complex conj.*

2-loop
2→2



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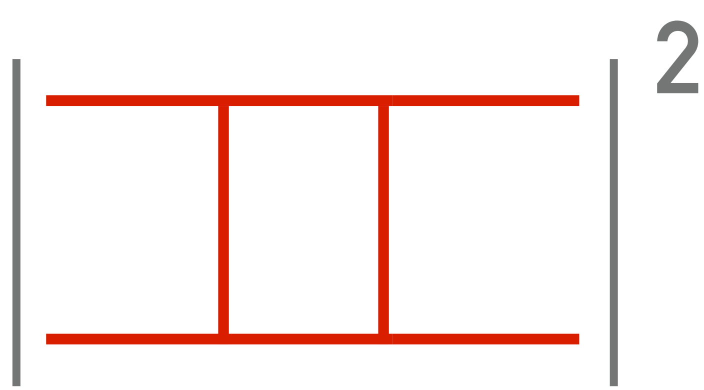
1-loop
2→2



EXAMPLE SERIES #1

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \qquad [\alpha_s \equiv \alpha_s(\sqrt{s_{e^+e^-}})]$$

$$= R_0 \left(1 + 0.32\alpha_s + 0.14\alpha_s^2 - 0.47\alpha_s^3 - 0.59316\alpha_s^4 + \cdots\right)$$

Baikov et al., 1206.1288
(numbers for γ-exchange only)

This is one of the few quantities calculated to N4LO

Good convergence of the series at every order
(at least for $\alpha_s(M_Z) = 0.118$)

EXAMPLE SERIES #2

$$\sigma(pp \to H) = (961\,\mathrm{pb})\times(\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \cdots)$$

$$\alpha_s \equiv \alpha_s(M_H/2)$$

$$\sqrt{s_{pp}} = 13\,\mathrm{TeV}$$

Anastasiou et al., 1602.00695 (ggF, hEFT)

pp→H (via gluon fusion) is one of only two hadron-collider processes known at N3LO

(the other is pp→H via weak-boson fusion)

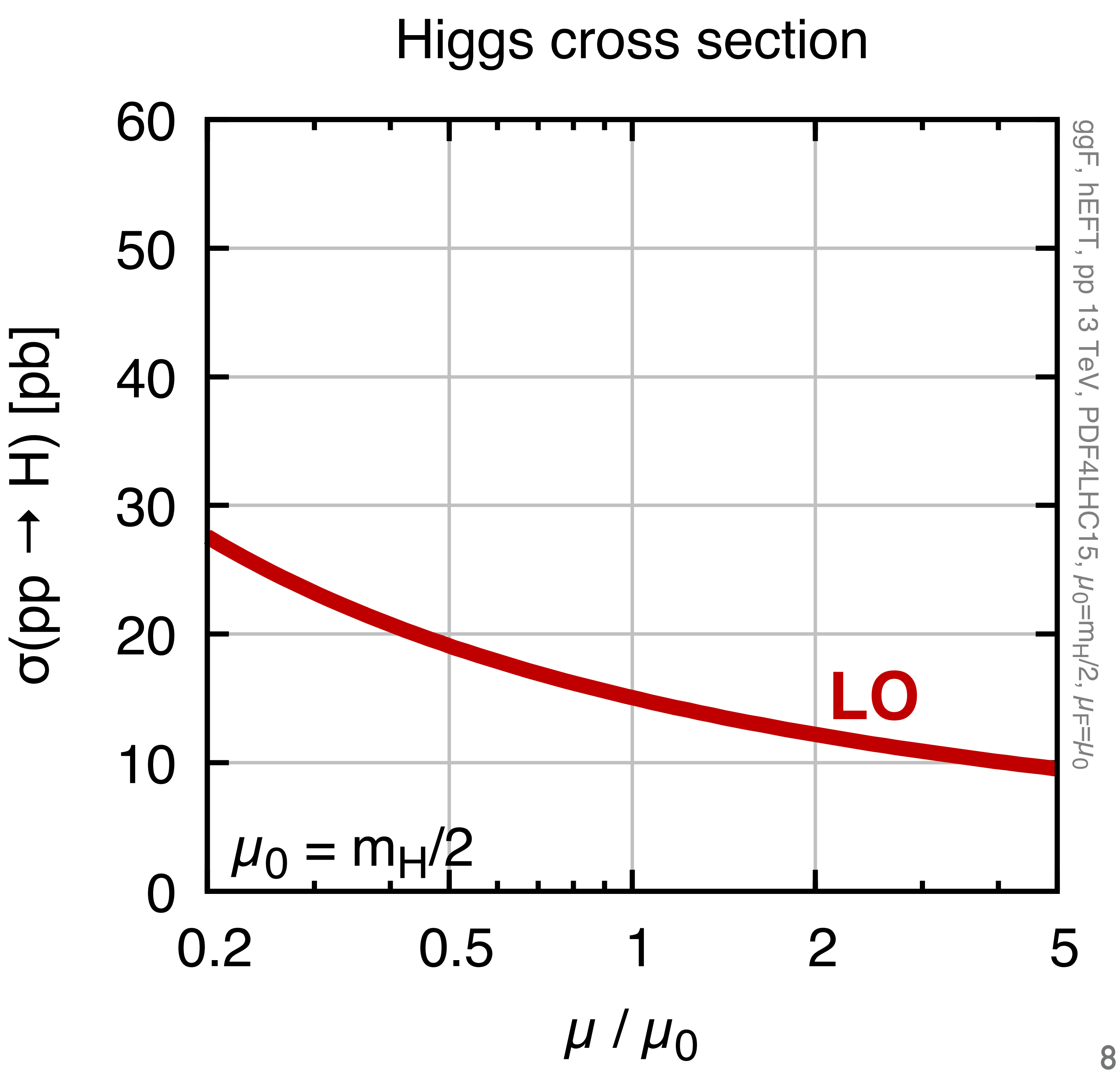
The series does not converge well

(explanations for why are only moderately convincing)

- On previous page, we wrote the series in terms of powers of $\alpha_s(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_s(\mu)$ for any choice of "renormalisation scale" μ.

LO

$$\sigma(pp \to H) = \sigma_0 \times \alpha_s^2(\mu)$$



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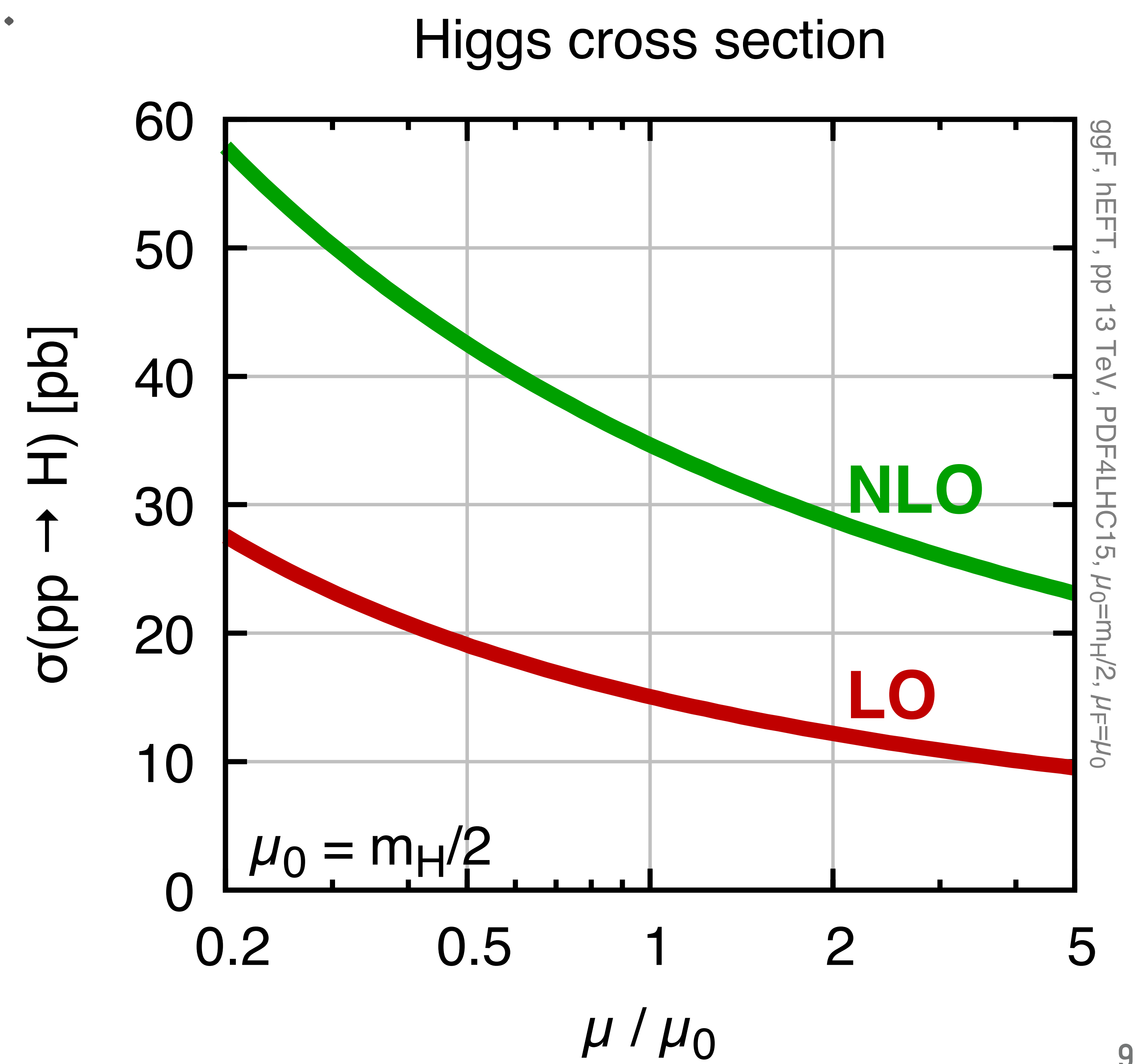
NLO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)\Big)$$



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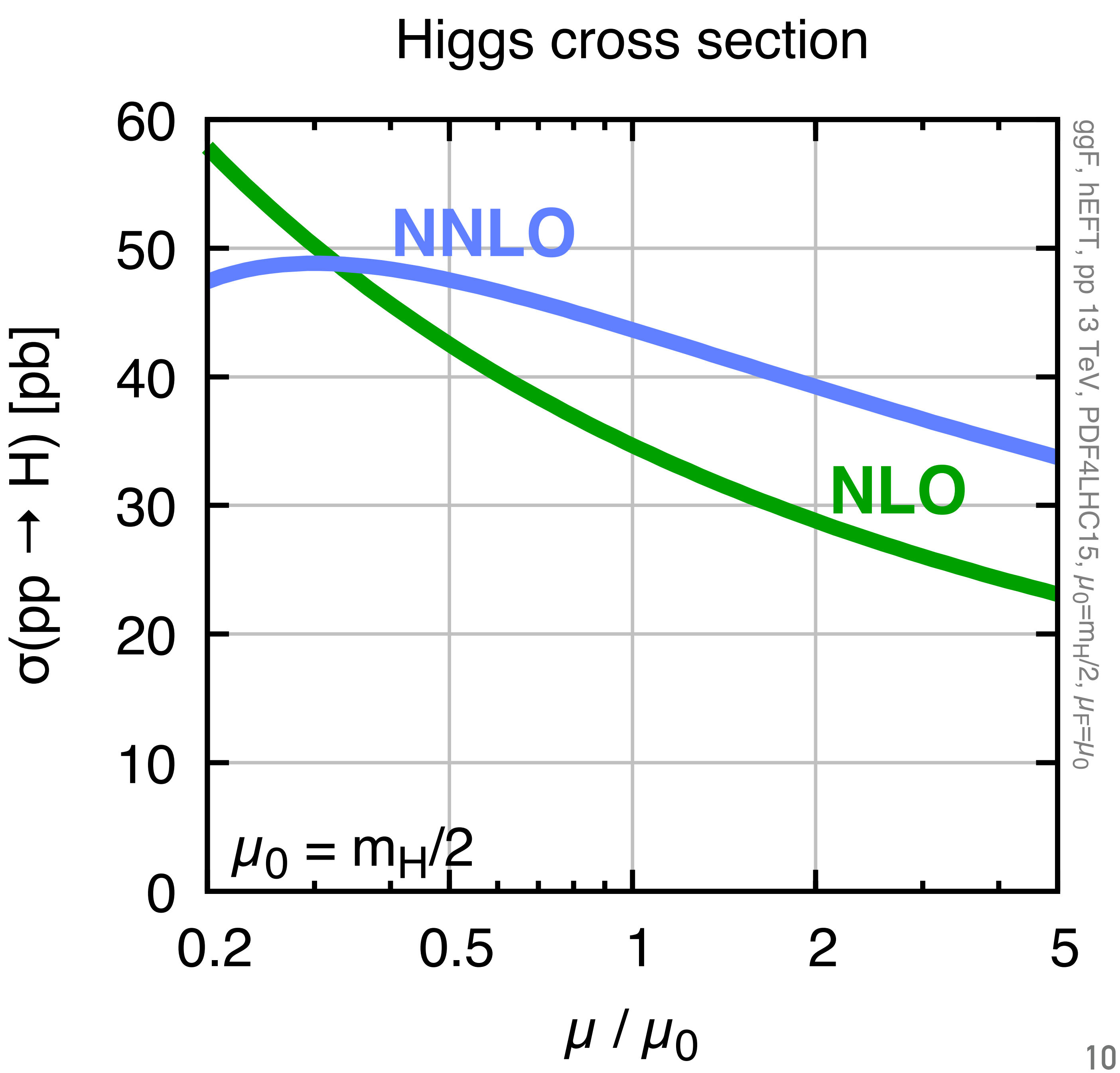
NNLO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln\frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu) + c_4(\mu)\alpha_s^4(\mu)\Big)$$



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N3LO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu)$$
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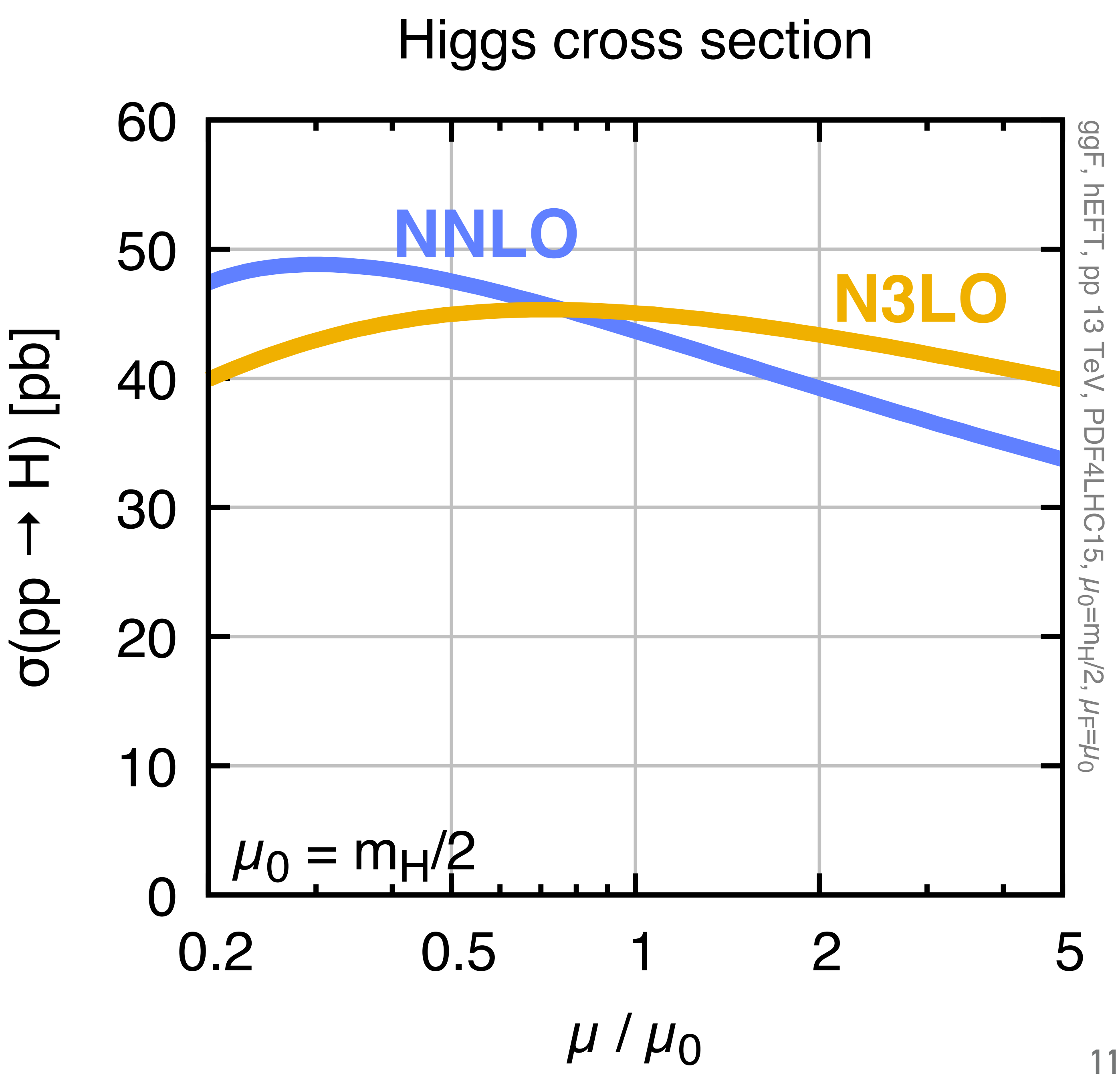

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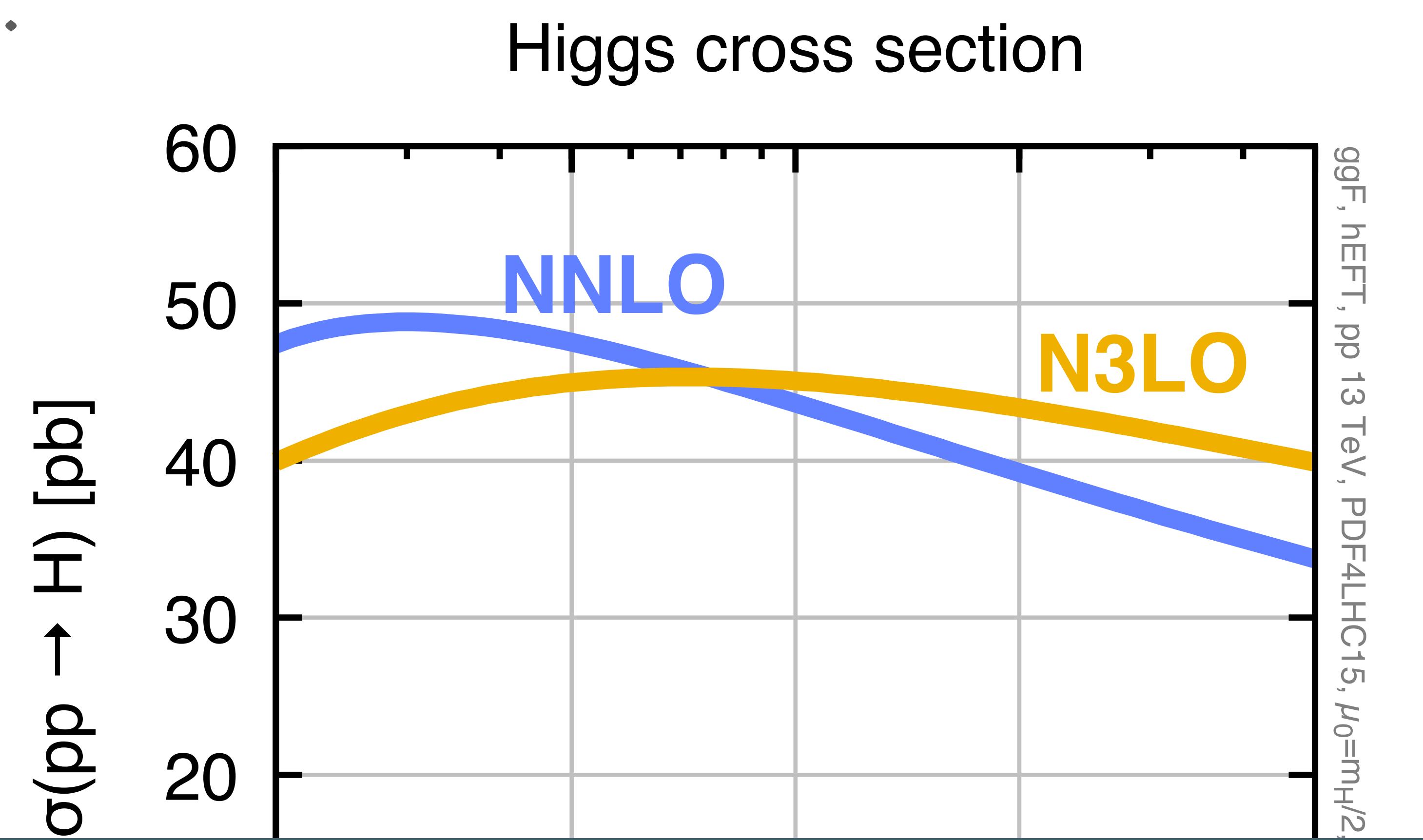
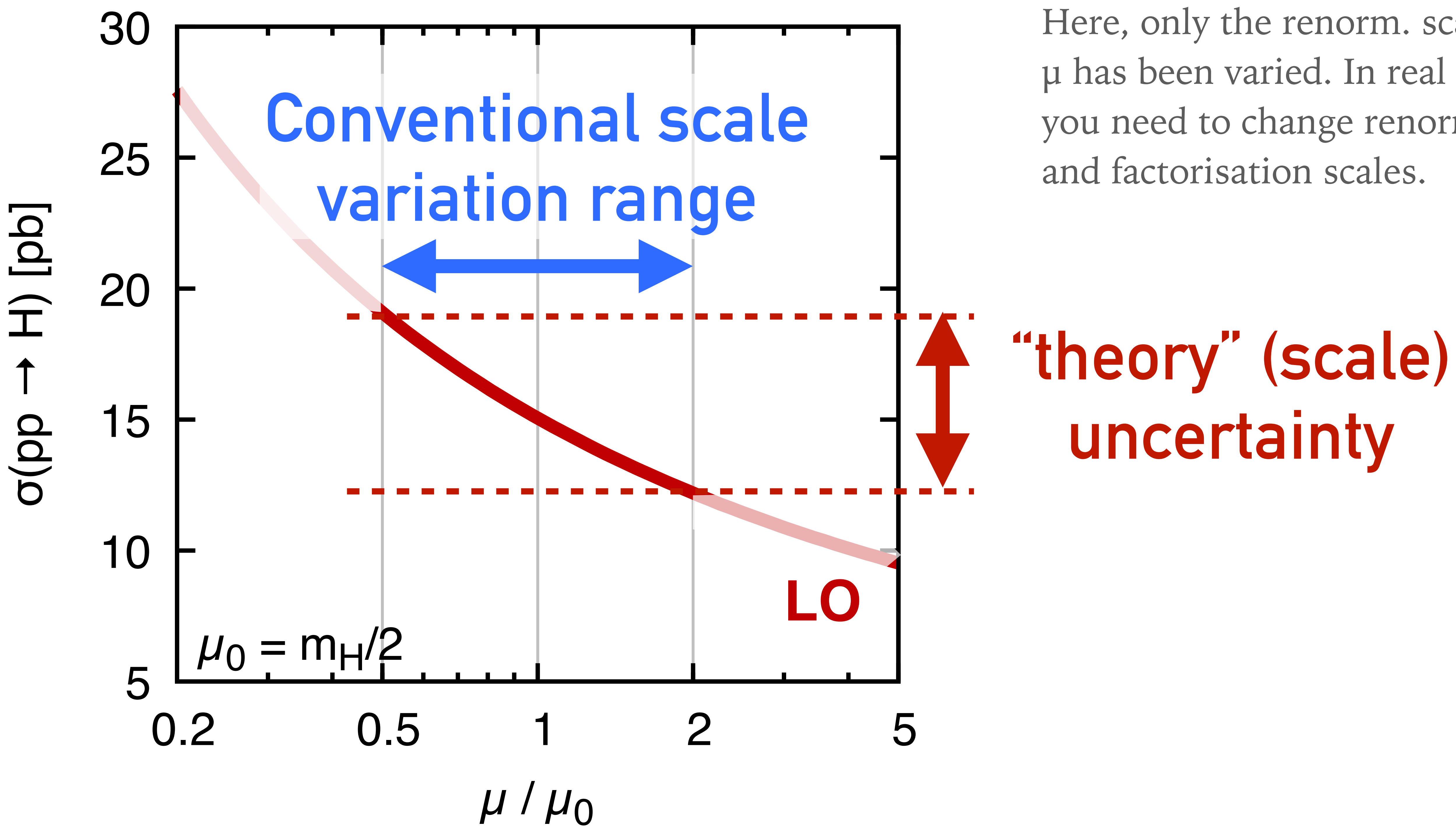


scale dependence (an intrinsic uncertainty) gets reduced as you go to higher order

Scale dependence as the "THEORY UNCERTAINTY"



Here, only the renorm. scale μ has been varied. In real life you need to change renorm. and factorisation scales.

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying μ in range 1/2 → 2 around central value

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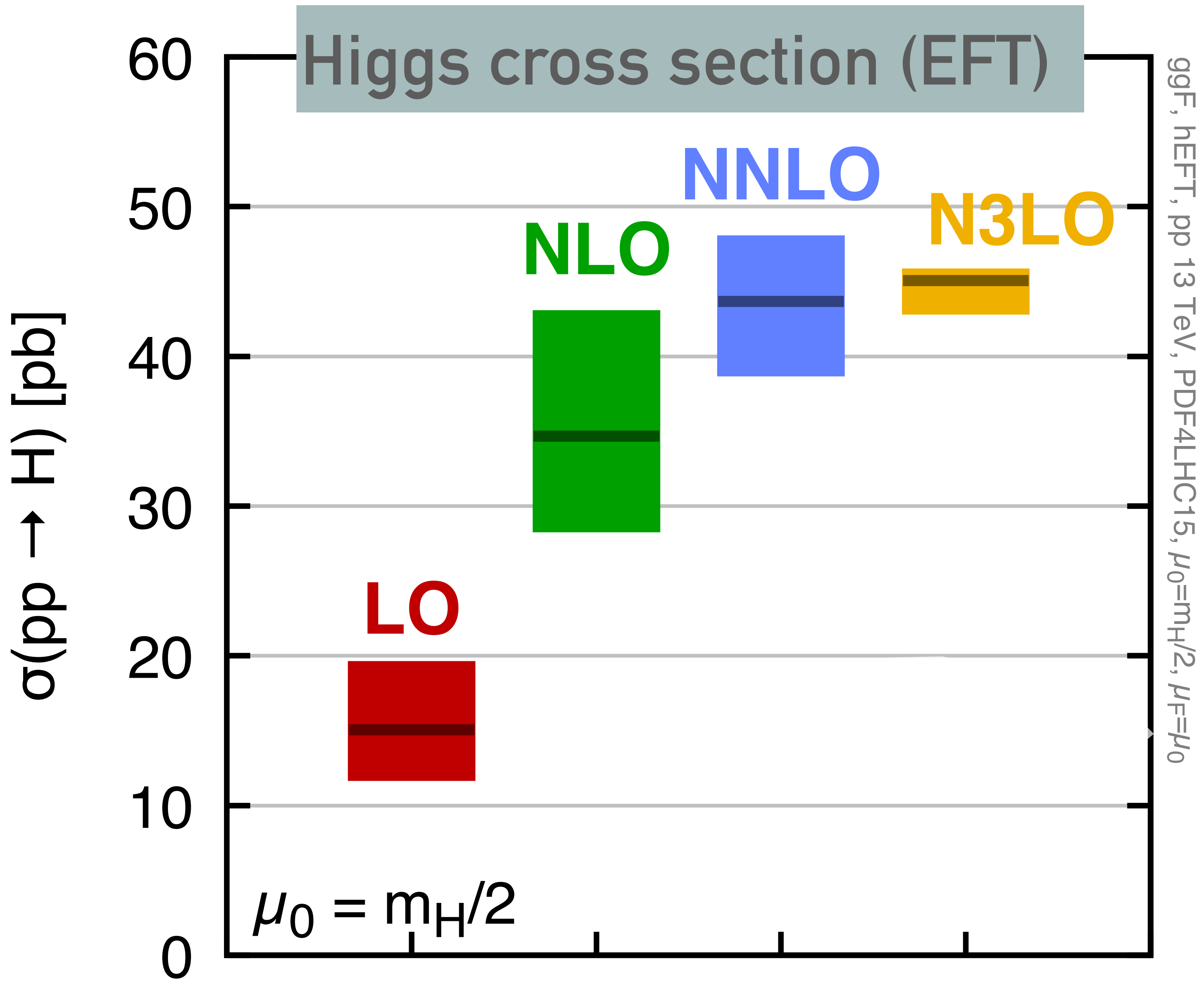

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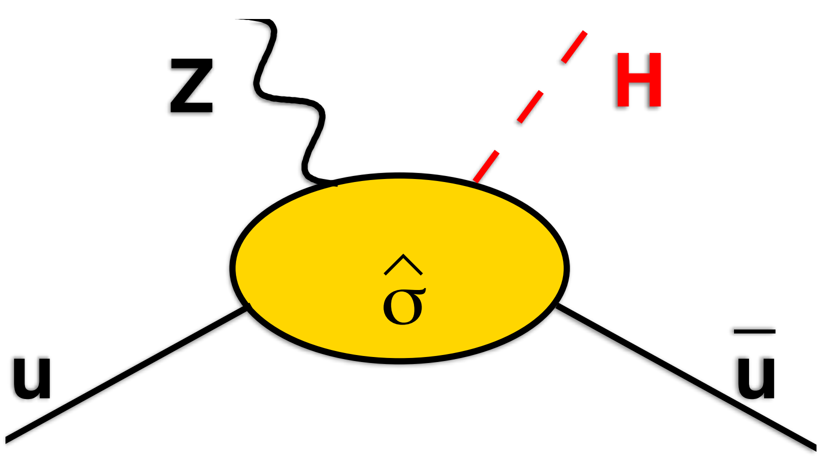
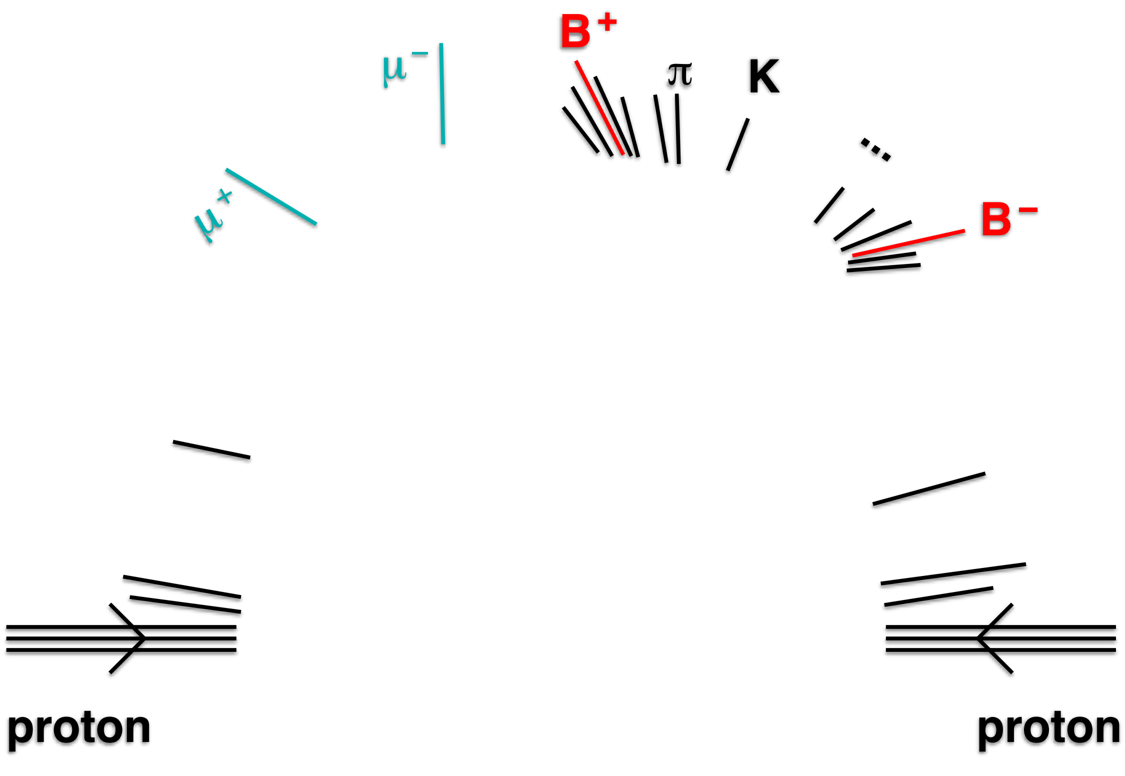
WHAT DO WE KNOW?

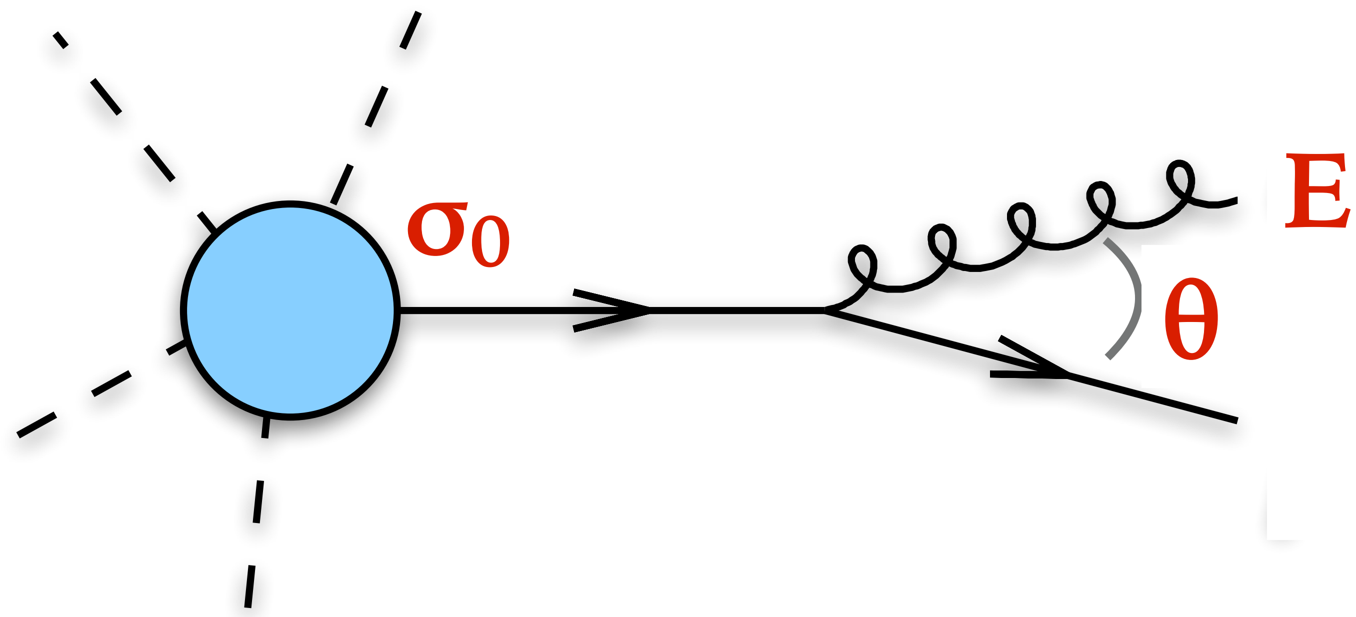
- LO: almost any process *(with MadGraph, ALPGEN, etc.)*
- NLO: most processes *(with MCFM, NLOJet++, MG5_aMC@NLO, Blackhat/NJet/Gosam/etc.+Sherpa)*
- NNLO: all 2→1 and many 2→2 (but not dijets) *(DY/HNNLO, FEWZ, MATRIX, MCFM & private codes)*
- N3LO: pp → Higgs via gluon fusion and weak-boson fusion *both in approximations (EFT, $QCD_1 \times QCD_2$)*
- NLO EW corrections, i.e. relative α_{EW} rather than α_s:
  most 2→1 and many 2→2

the real world?





GLUON EMISSION FROM A QUARK



Consider an emission with

- energy $\mathbf{E \ll \sqrt{s}}$ (“soft”)
- angle $\boldsymbol{\theta \ll 1}$ (“collinear” wrt quark)

Examine correction to some hard process with cross section $\boldsymbol{\sigma_0}$

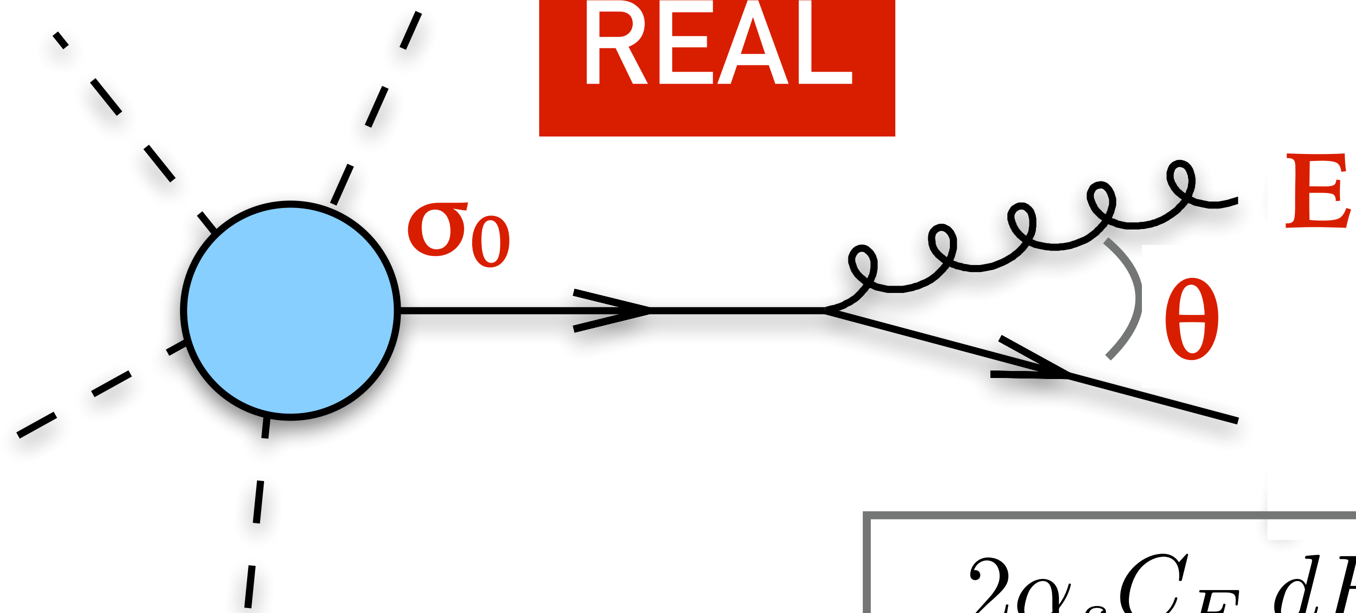
$$d\sigma \simeq \sigma_0 \times \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

This has a divergence when E→0 or θ→0

[in some sense because of quark propagator going on-shell]

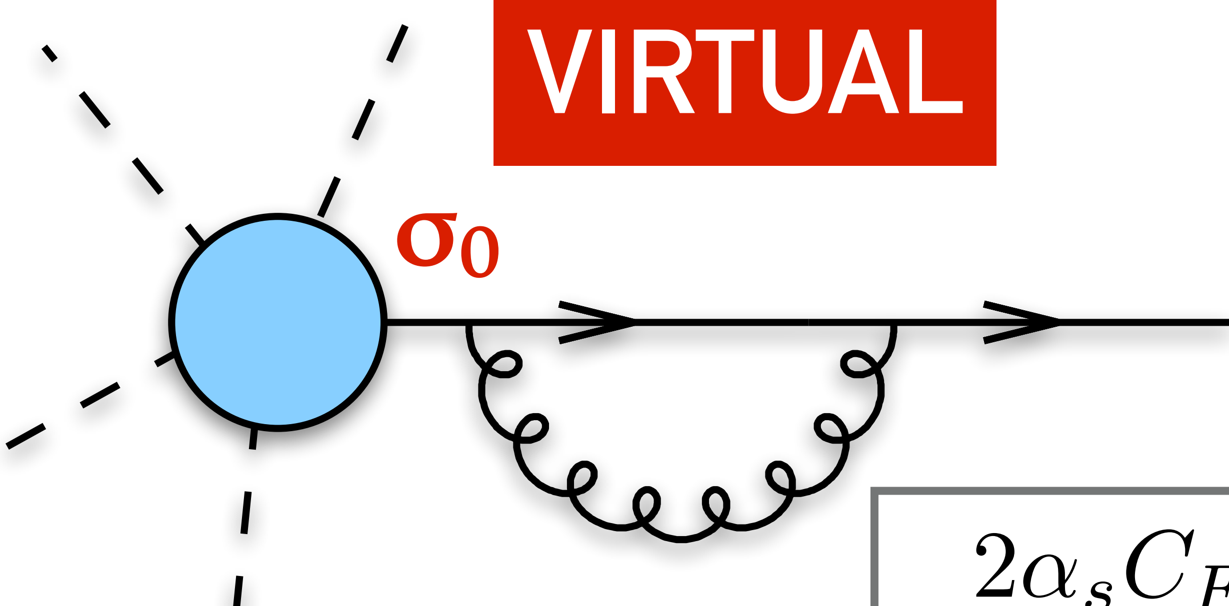
How come we get finite cross sections?

REAL



$$+\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

VIRTUAL



$$-\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

Divergences are present in both real and virtual diagrams.

If you are "**inclusive**", i.e. your measurement doesn't care whether a soft/collinear gluon has been emitted then the **real and virtual divergences cancel.**

Suppose we're not inclusive — e.g. calculate probability of emitting a gluon

Probability P_g of emitting gluon from a quark with energy Q:

$$P_g \simeq \frac{2\alpha_s C_F}{\pi} \int^Q \frac{dE}{E} \int^1 \frac{d\theta}{\theta} \Theta(E\theta > Q_0)$$

This diverges unless we cut off the integral for transverse momenta ($p_T \simeq E\,\theta$) below some non-perturbative threshold Q_0.

On the grounds that perturbation theory doesn't apply for $p_T \sim \Lambda_{QCD}$
language of quarks and gluons becomes meaningless

With this cutoff, the result is

$$P_g \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} + \mathcal{O}\left(\alpha_s \ln Q\right)$$

this is called a "double logarithm"

[it crops up all over the place in QCD]

Suppose we're not inclusive — e.g. calculate probability of emitting a gluon

Suppose we take $Q_0 \sim \Lambda_{QCD}$, what do we get?

Let's use $\alpha_s = \alpha_s(Q) = 1/(2b \ln Q/\Lambda)$
[Actually over most of integration range this is optimistically small]

$$P_g \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} \to \frac{C_F}{2b\pi} \ln \frac{Q}{\Lambda_{QCD}} \to \frac{C_F}{4b^2\pi\, \alpha_s}$$

Put in some numbers: $Q = 100$ GeV, $\Lambda_{QCD} \simeq 0.2$ GeV, $C_F=4/3$, $b \simeq 0.6$

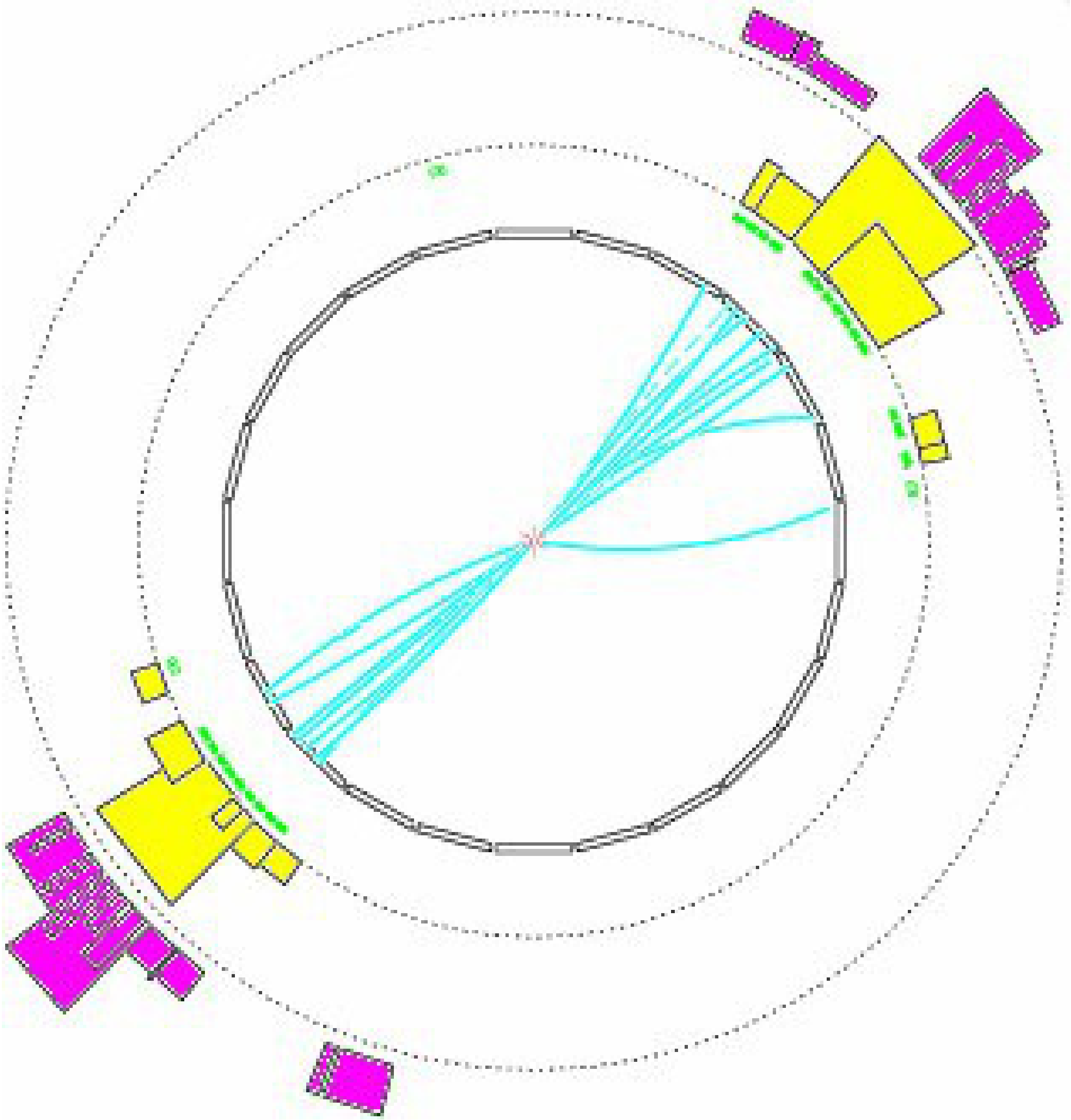
$$P_g \simeq 2.2$$

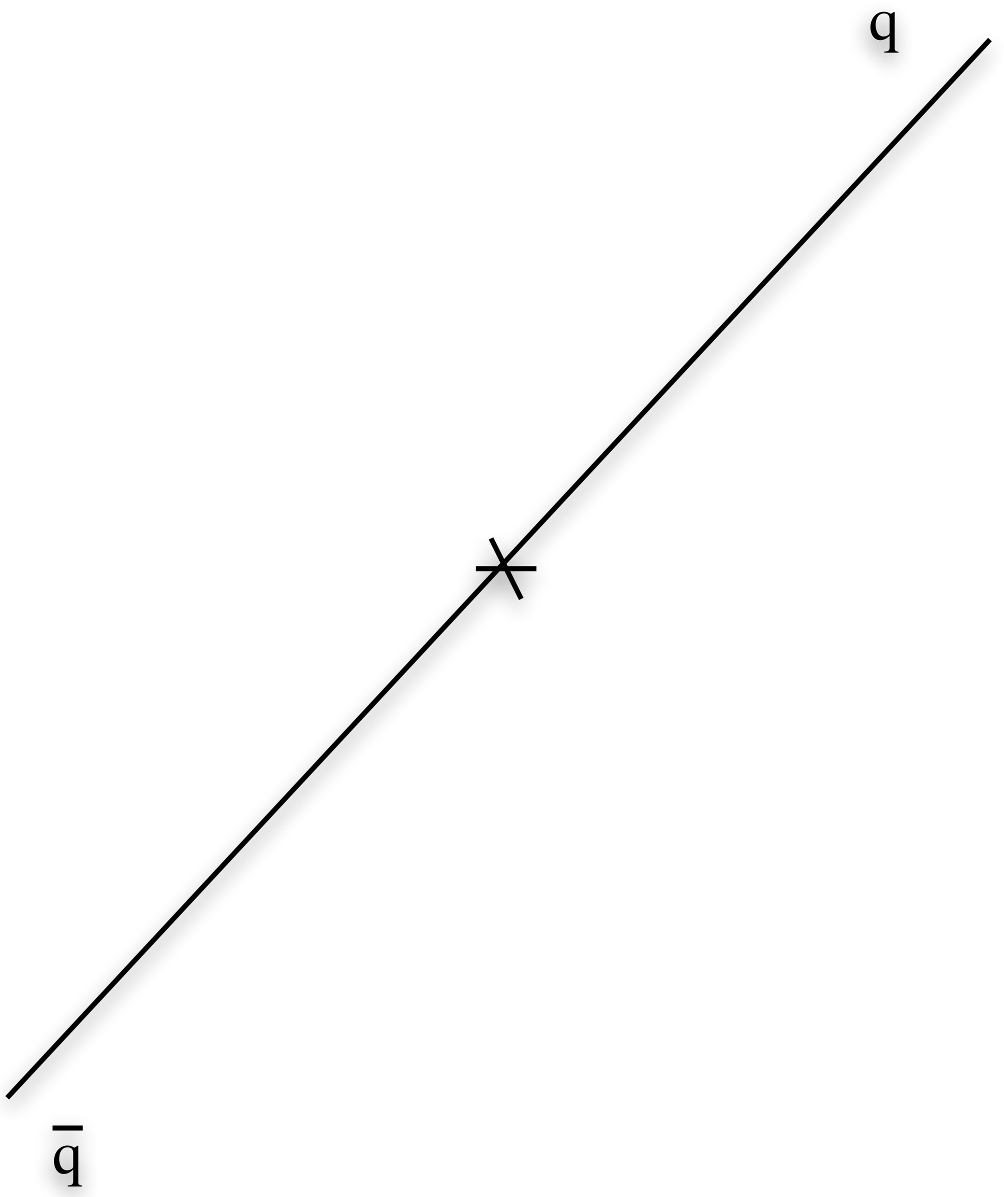
This is supposed to be an $O(\alpha_s)$ correction.

But the final result ~ $1/\alpha_s$

QCD hates to not emit gluons!

Picturing a QCD event





Start off with a qqbar system

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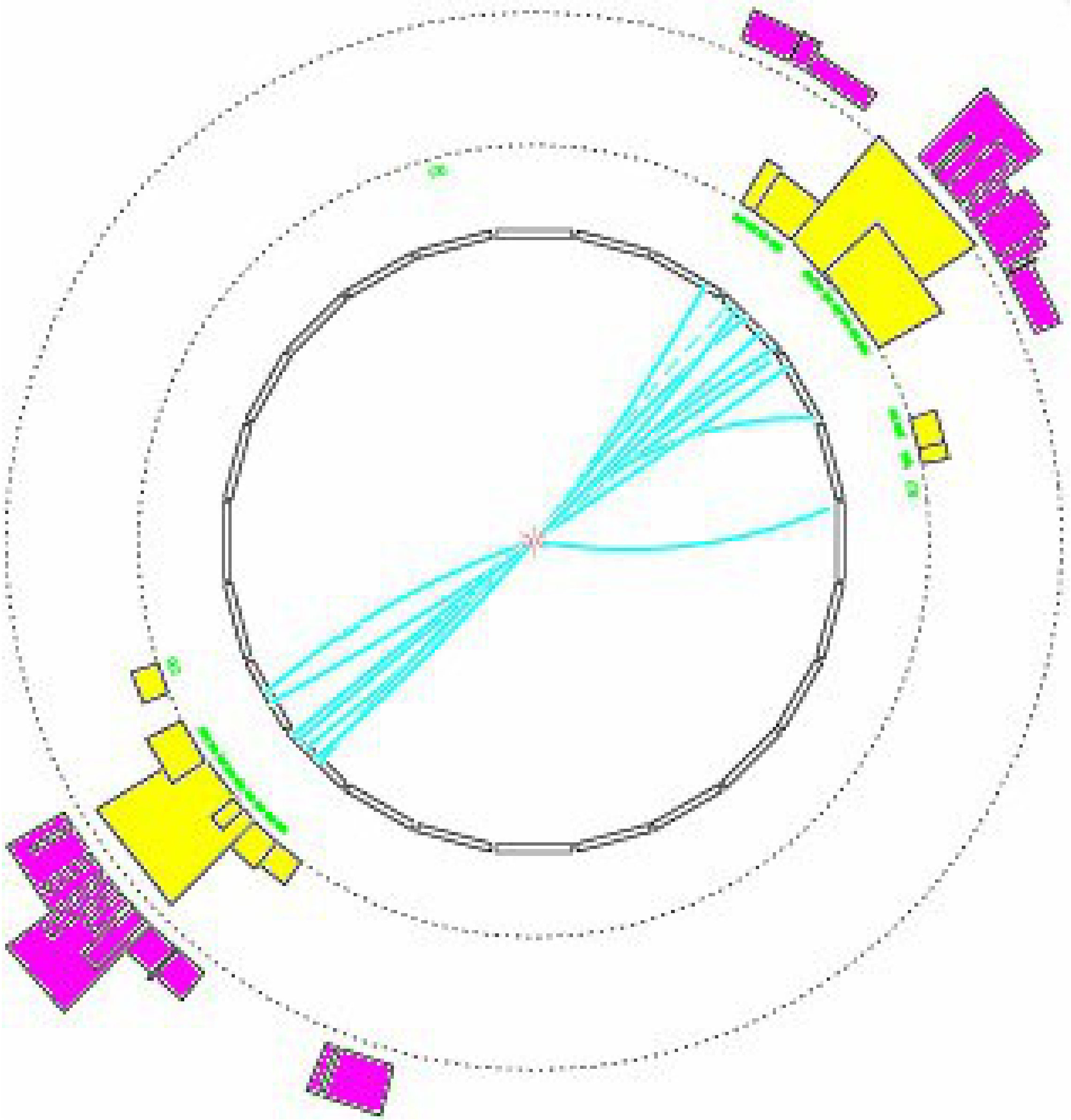
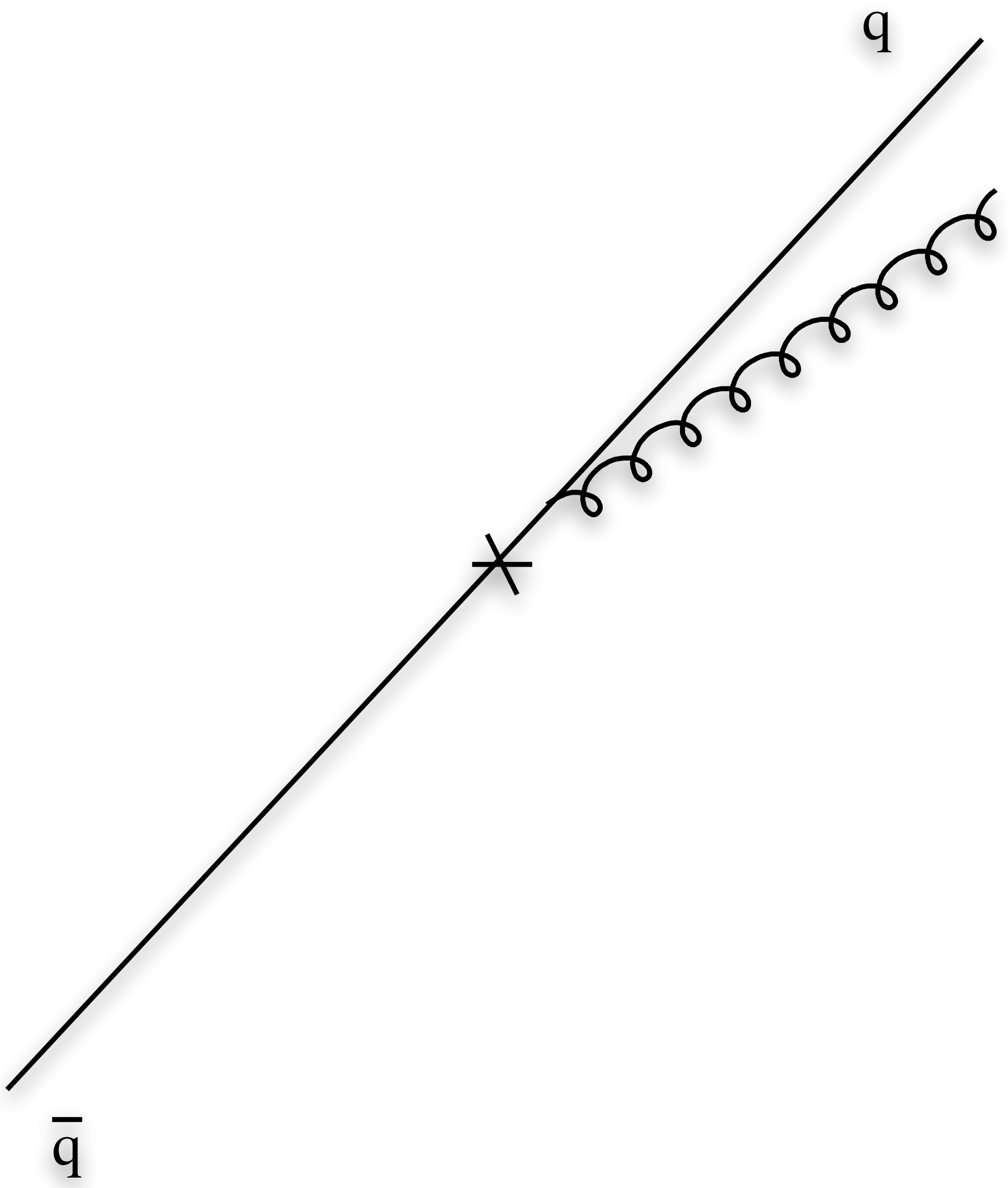
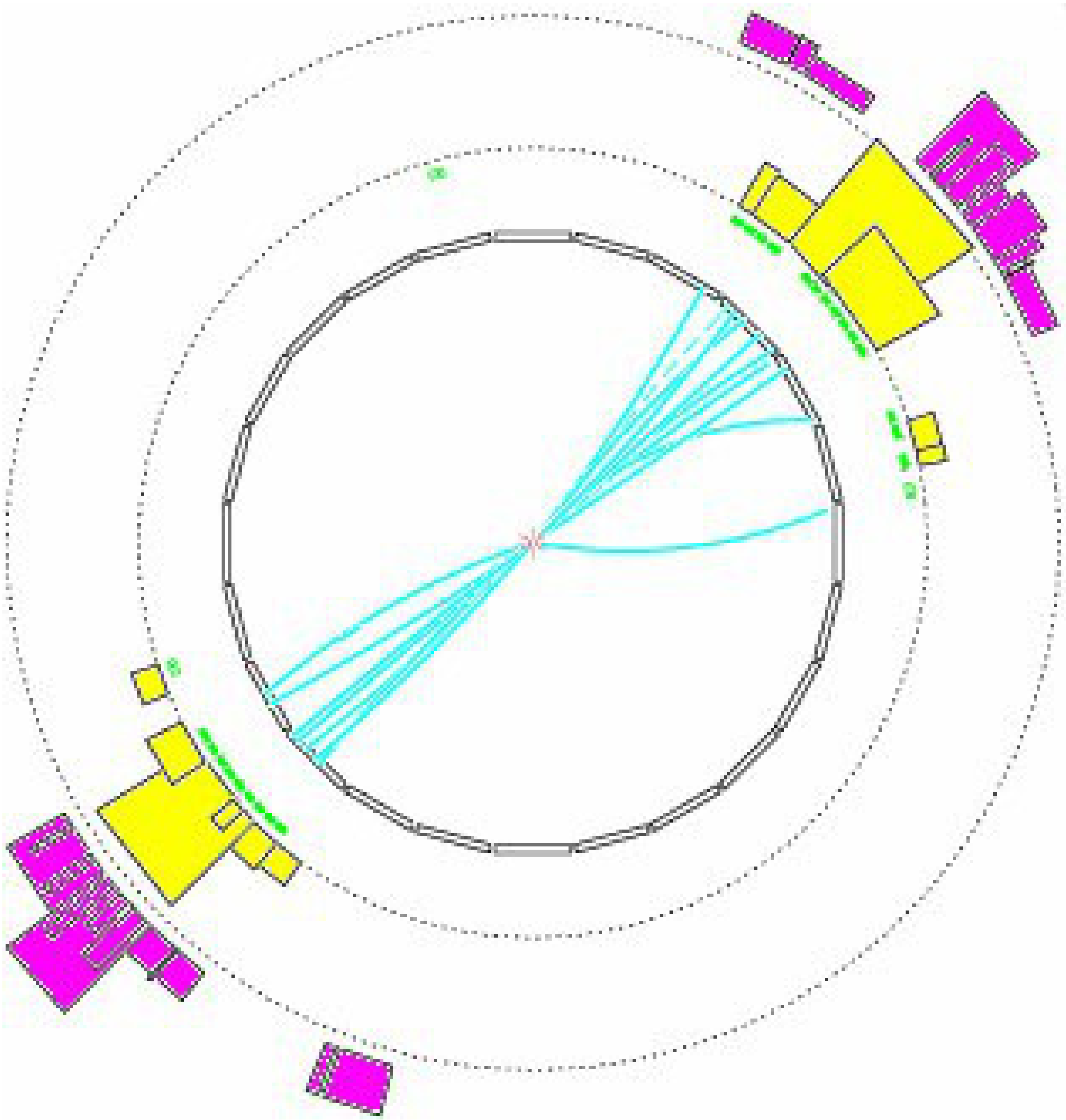
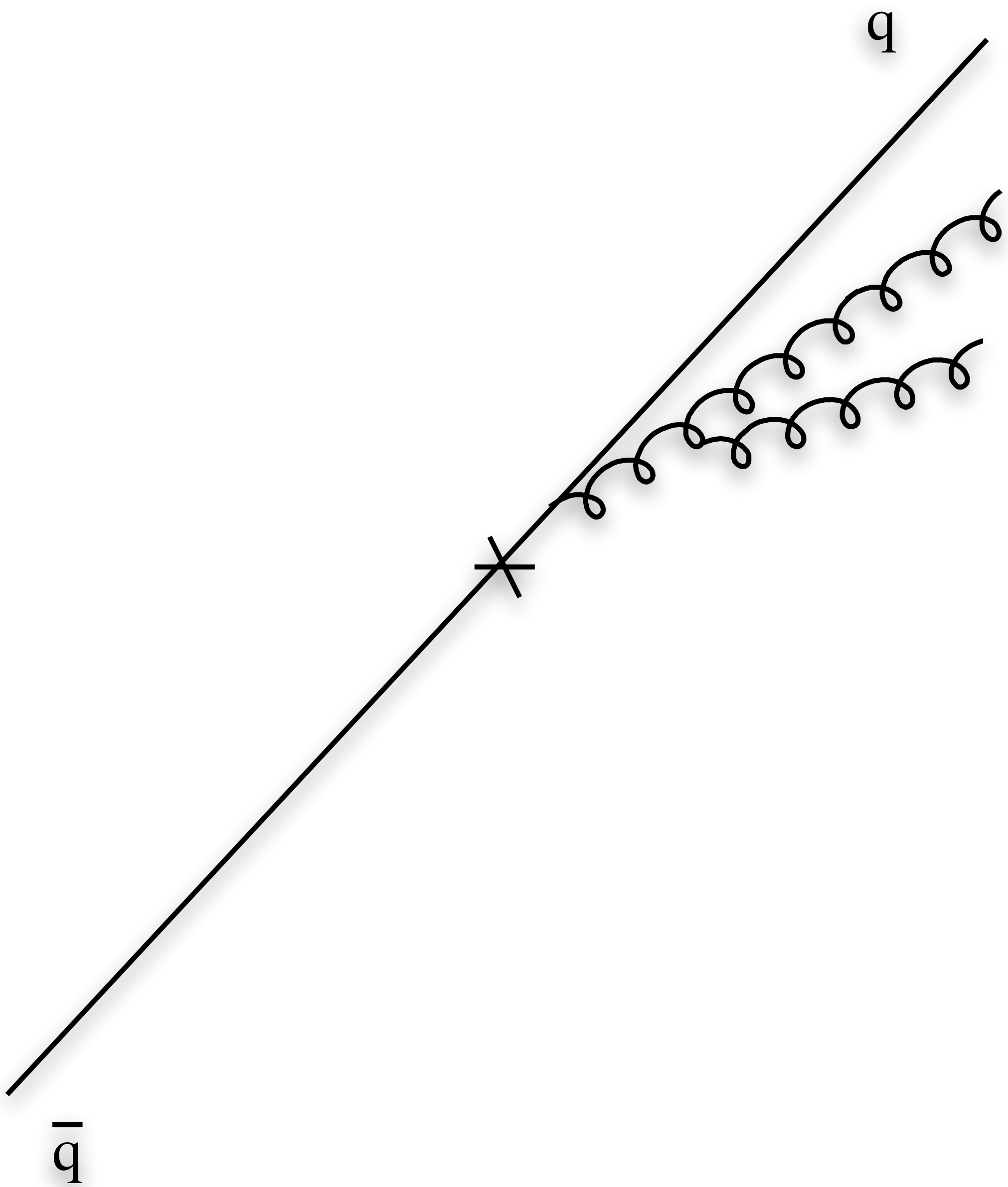




a gluon gets emitted at small angles

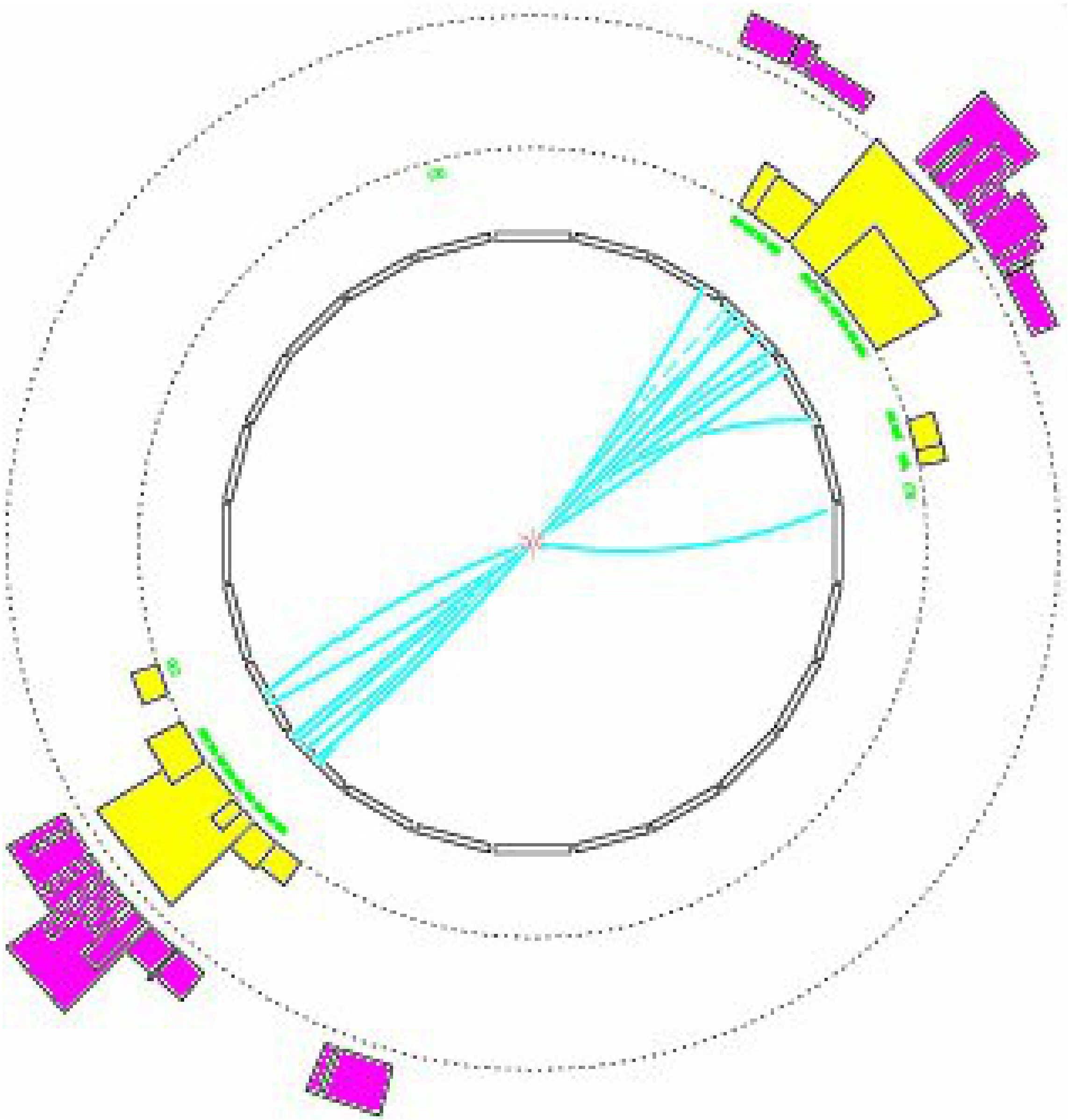
Picturing a QCD event

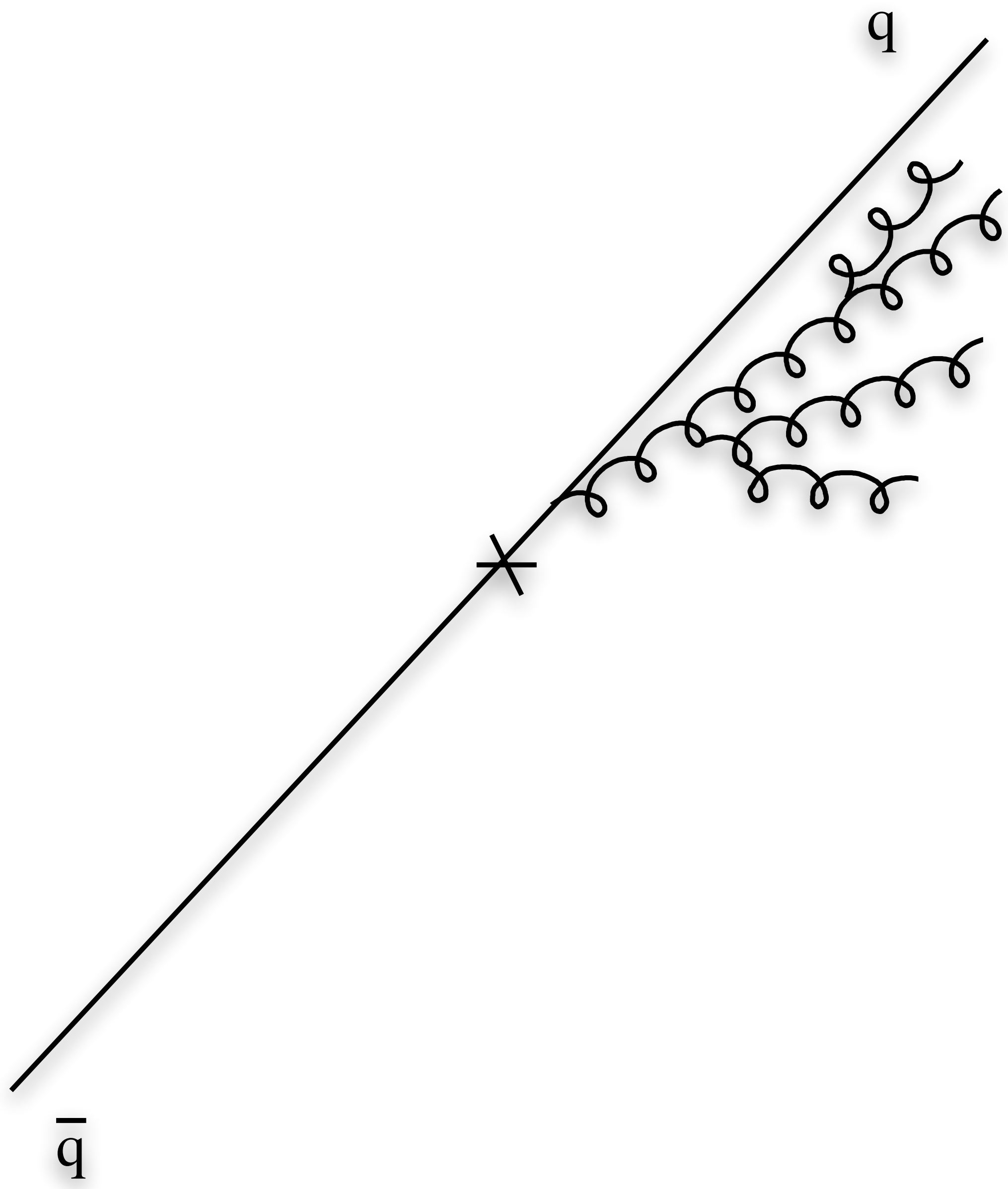




it radiates a further gluon

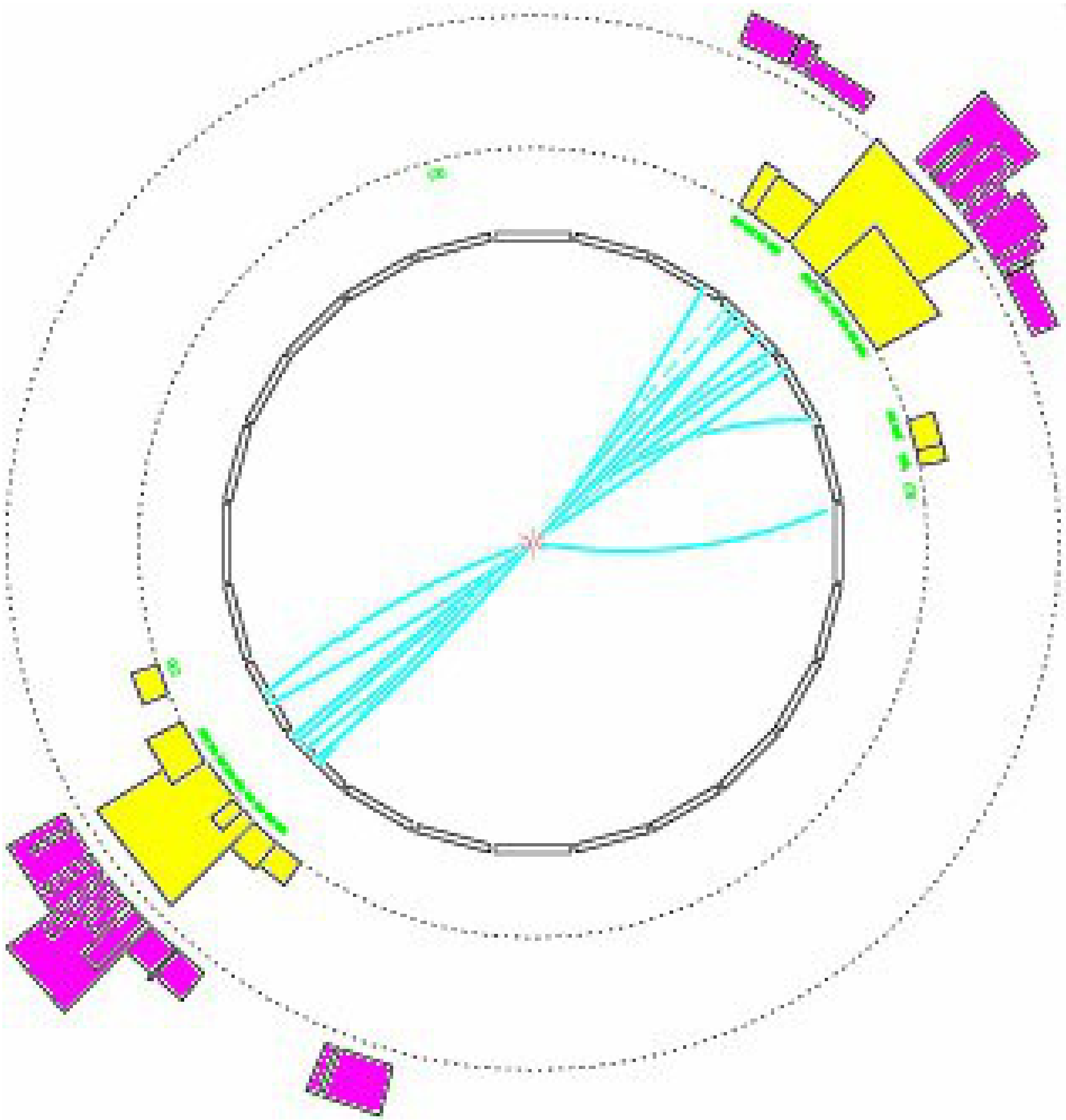
Picturing a QCD event

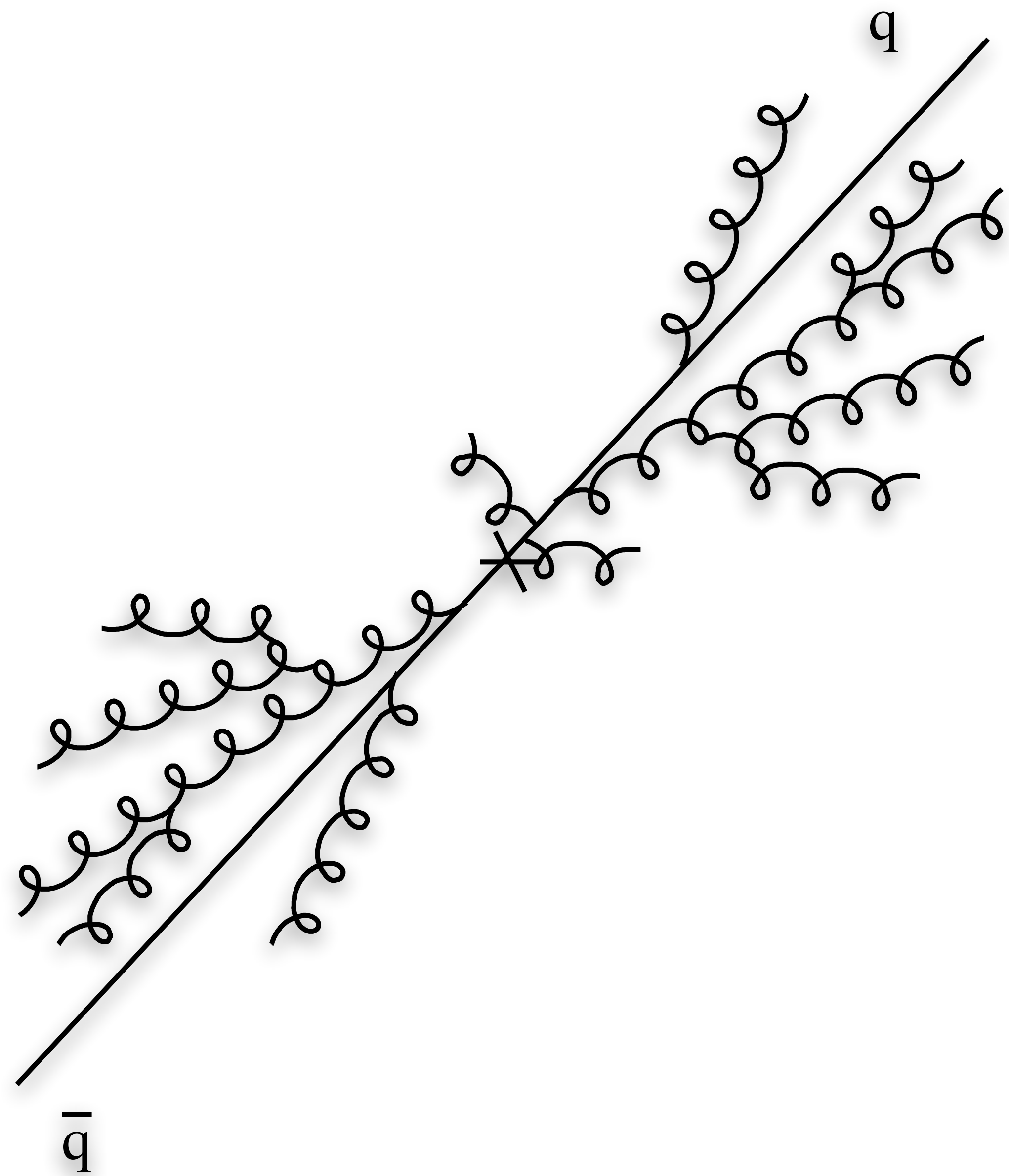




and so forth

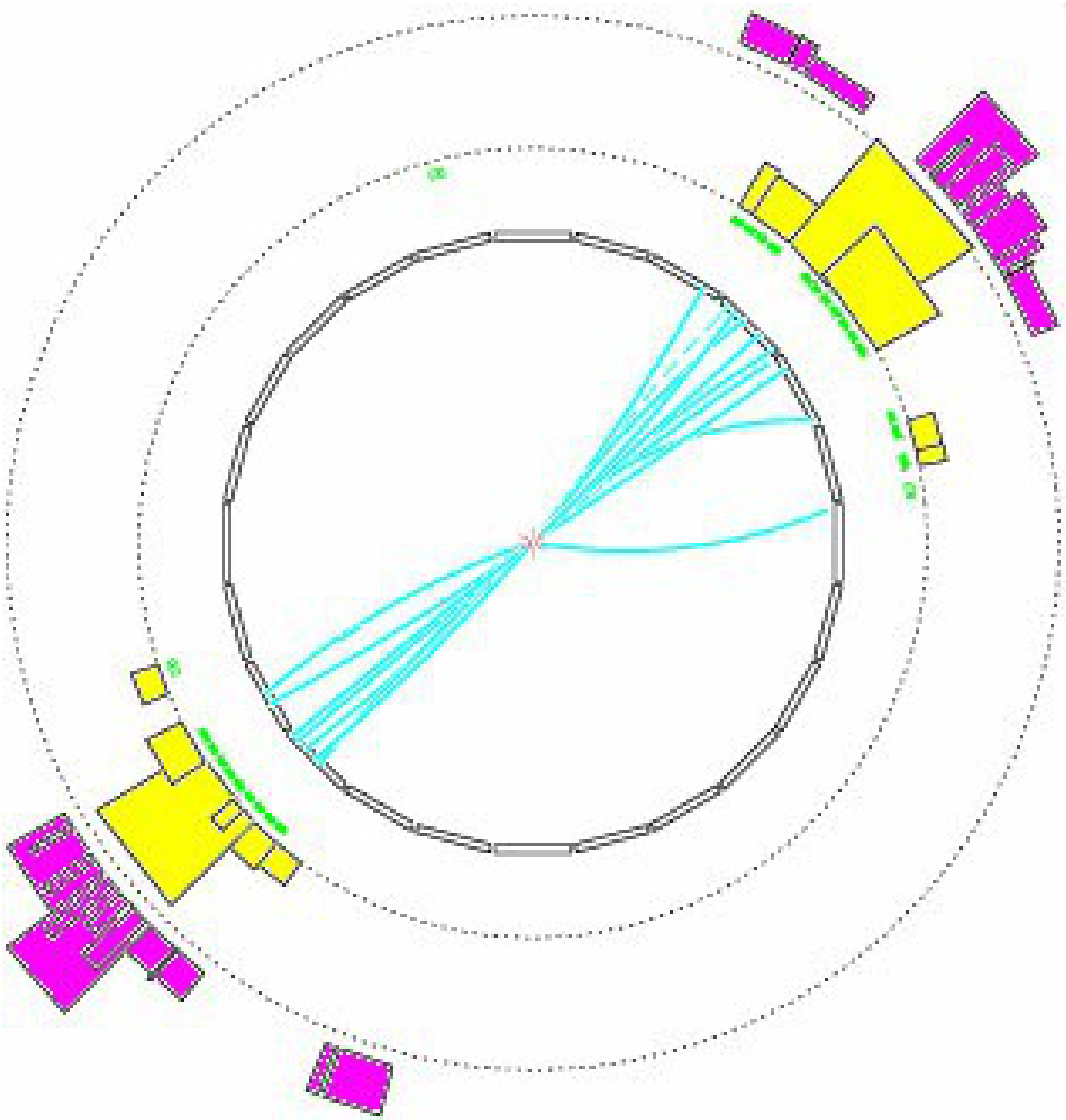
Picturing a QCD event

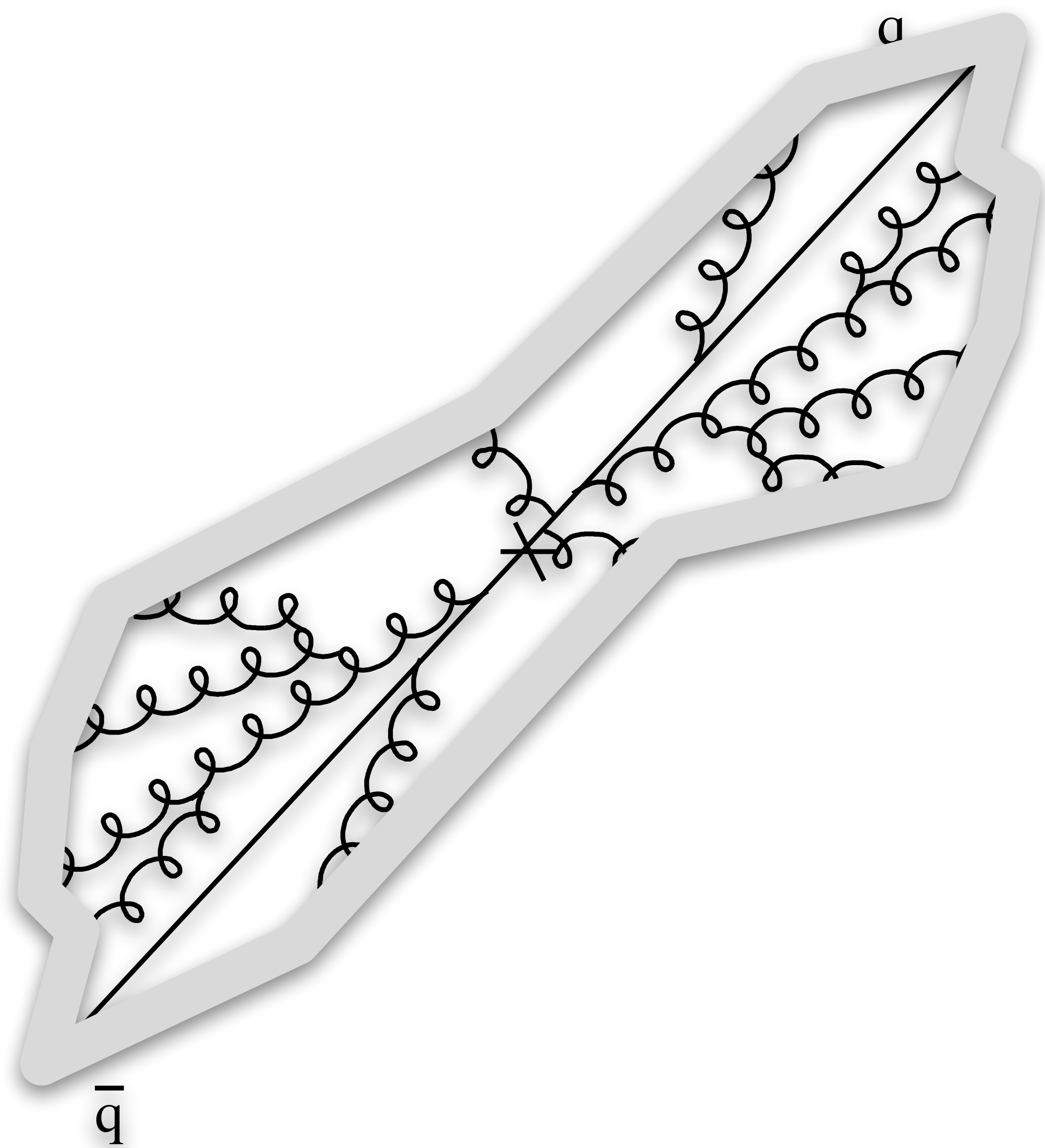




meanwhile the same happened on the other side

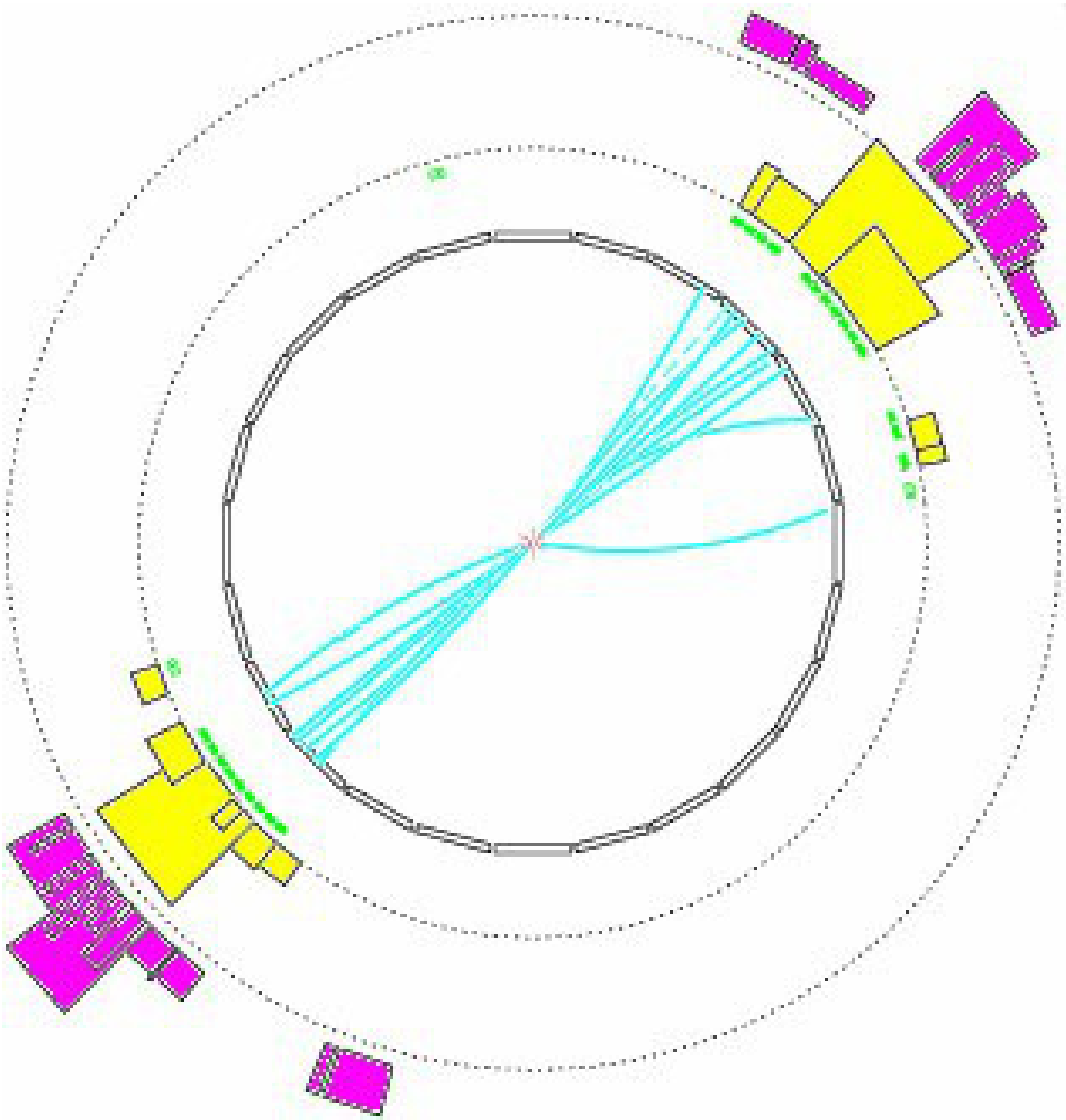
Picturing a QCD event

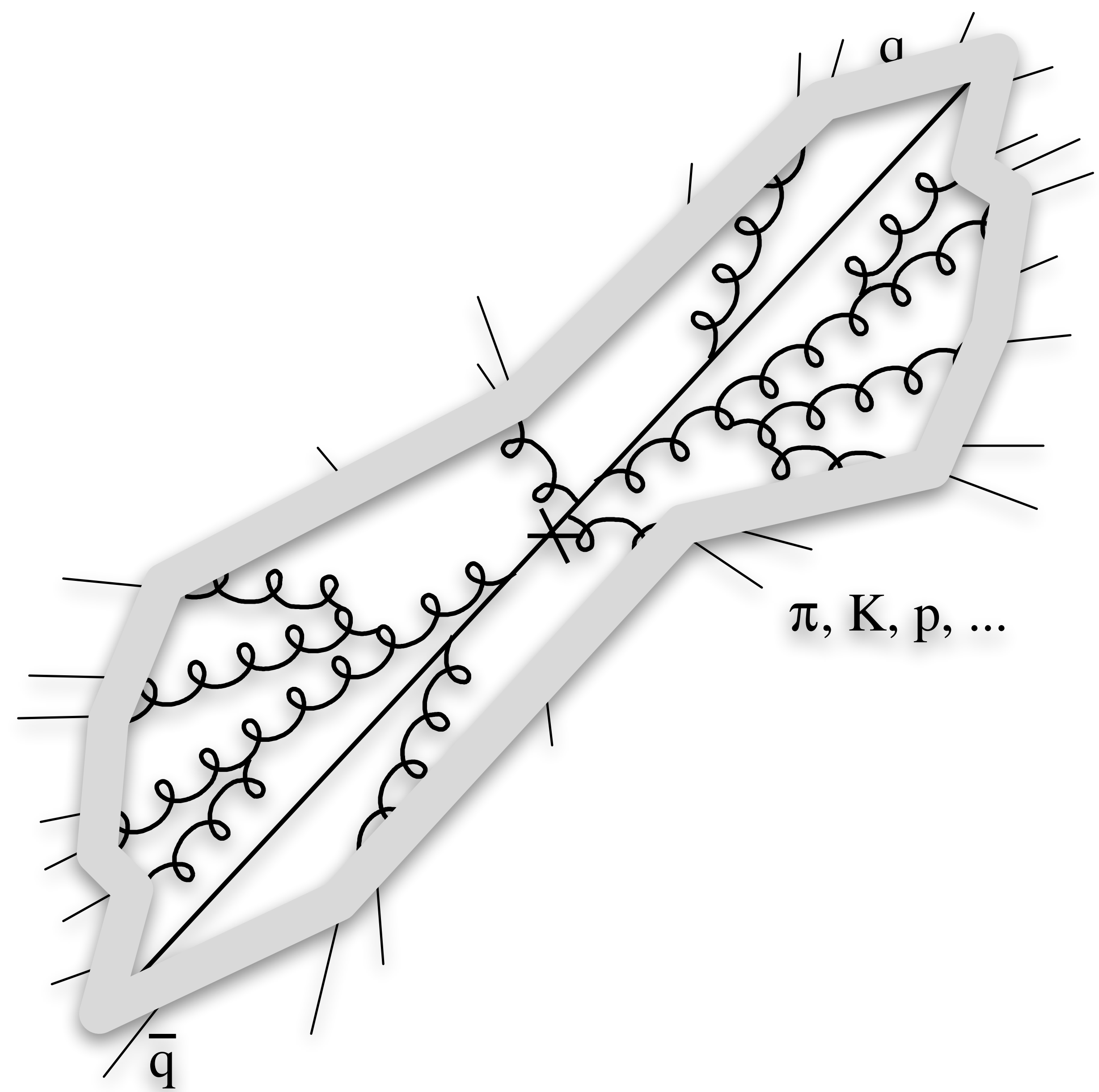




then a non-perturbative transition occurs

Picturing a QCD event





giving a pattern of hadrons that "remembers" the gluon branching

(hadrons mostly produced at small angles wrt qqbar directions — two "jets")

resummation and parton showers

the previous slides applied in practice

- It's common to ask questions like "*what is the probability that a Higgs boson is produced with transverse momentum < p_T*"
- Answer is given (~) by a "**Sudakov form factor**", i.e. the probability of not emitting any gluons with transverse momentum > p_T.

$$P(\text{Higgs trans.mom.} < p_T) \simeq \exp\left[-\frac{2\alpha_s C_A}{\pi}\ln^2\frac{M_H}{p_T}\right]$$

- when p_T is small, the logarithm is large and compensates for the smallness of α_s — so you need to resum log-enhanced terms to all orders in α_s.

What do we know about resummation?

- You'll sometimes see mention of "NNLL" or similar
- This means next-next-to-leading logarithmic
- Leading logarithmic (LL) means you sum all terms with p=n+1 (for n=1…∞) in

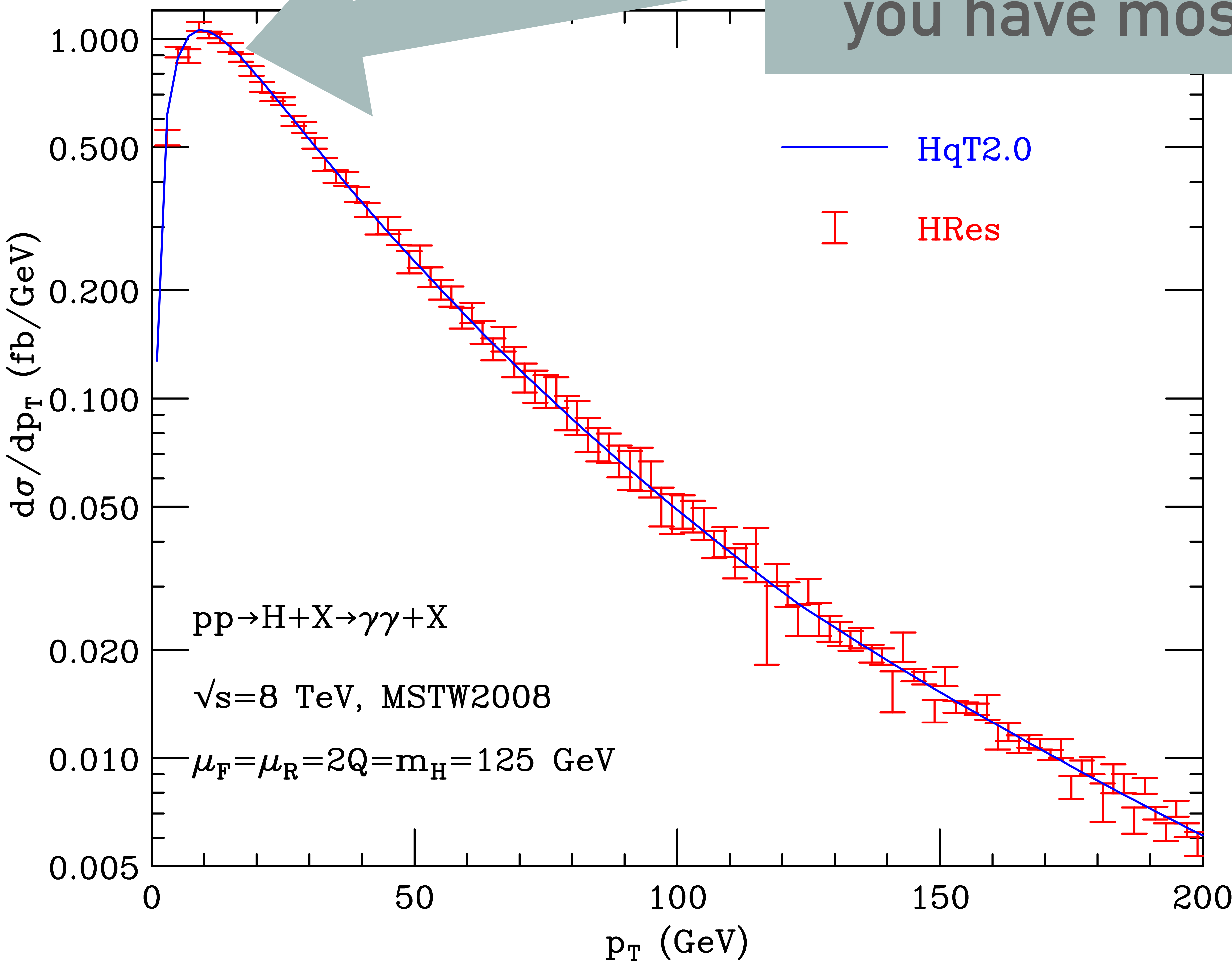
$$\exp\left[-\sum_{n,p} \alpha_s^n \ln^p \frac{M_H}{p_T}\right]$$

- NLL: all terms with p=n (for n=1…∞)
- NNLL: all terms with p=n–1 (for n=1…∞)

In real life, the function that appears in the resummation is sometimes instead a Fourier or Mellin transform of an exponential

Resummation of Higgs p_T spectrum

Resummation is essential to predict small-p_T region (where you have most of the events)



This kind of resummation is an input to nearly all LHC Higgs studies

de Florian et al 1203.6321

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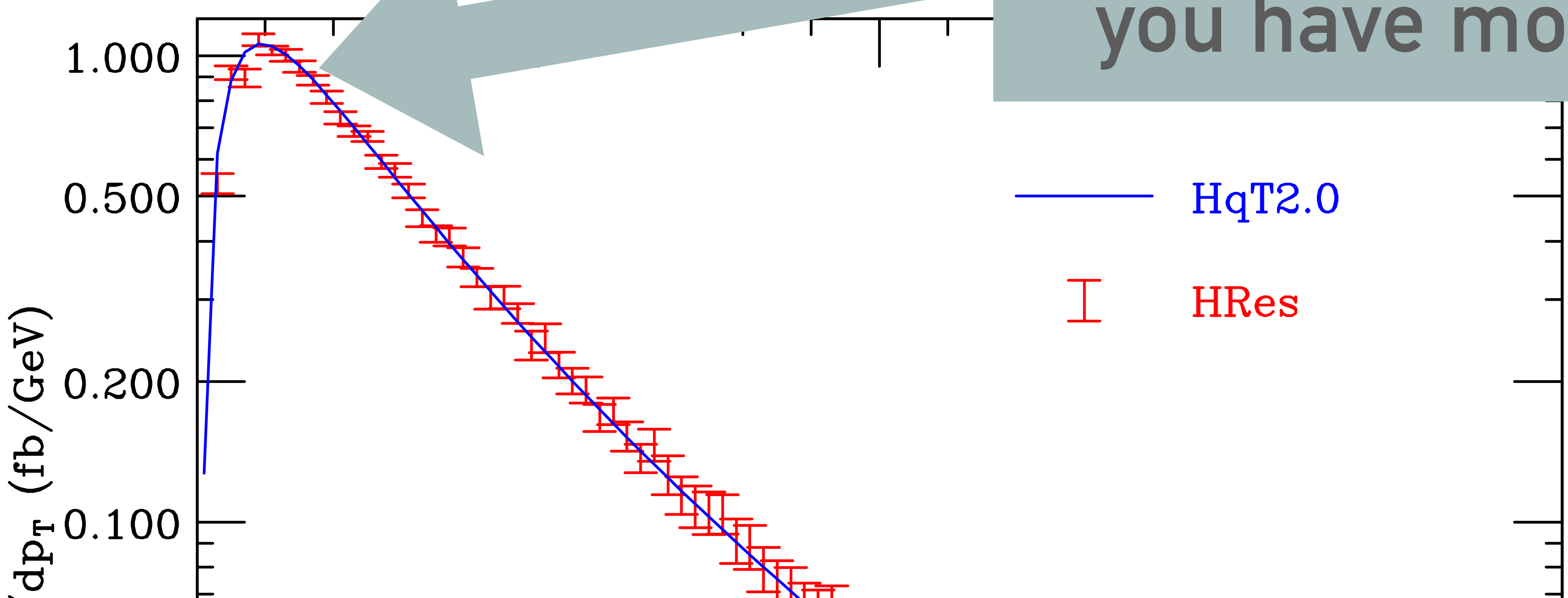

This kind of resummation is an input to nearly all

This is resummation of a kinematic variable — can usually be made robust by examining region with $p_T \ll m_H$

Another kind of resummation is **threshold resummation**, of logs of $\tau = (1 - M^2/s)$. For many applications (ttbar, Higgs) it's debated whether τ is small enough for resummation to bring genuine information

resummation v. parton showers (the basic idea)

- a resummation predicts **one observable** to high accuracy
- a parton shower takes the same idea of a Sudakov form factor and uses it to generate emissions
- from probability of not emitting gluons above a certain p_T, you can deduce p_T distribution of first emission

1. use a random number generator (r) to sample that p_T distribution

$$\text{deduce } p_T \text{ by solving } r = \exp\left[-\frac{2\alpha_s C_A}{\pi} \ln^2 \frac{p_{T,\max}^2}{p_T^2}\right]$$

2. repeat for next emission, etc., until p_T falls below some non-perturbative cutoff

very similar to radioactive decay, with time ~ $1/p_T$ and a decay rate ~ $p_T \log 1/p_T$

A toy shower

https://github.com/gavinsalam/zuoz2016-toy-shower

(fixed coupling, primary branching only, only p_T, no energy conservation, no PDFs, etc.)

```python
#!/usr/bin/env python
# an oversimplified (QED-like) parton shower
# for Zuoz lectures (2016) by Gavin P. Salam
from random import random
from math import pi, exp, log, sqrt

ptHigh = 100.0
ptCut  = 1.0
alphas = 0.12
CA=3

def main():
    for iev in range(0,10):
        print "\nEvent", iev
        event()

def event():
    # start with maximum possible value of Sudakov
    sudakov  = 1
    while (True):
        # scale it by a random number
        sudakov *= random()
        # deduce the corresponding pt
        pt = ptFromSudakov(sudakov)
        # if pt falls below the cutoff, event is finished
        if (pt < ptCut): break
        print "  primary emission with pt = ", pt

def ptFromSudakov(sudakovValue):
    """Returns the pt value that solves the relation
       Sudakov = sudakovValue (for 0 < sudakovValue < 1)
    """
    norm = (2*CA/pi)
    # r = Sudakov = exp(-alphas * norm * L^2)
    # --> log(r) = -alphas * norm * L^2
    # --> L^2 = log(r)/(-alphas*norm)
    L2 = log(sudakovValue)/(-alphas * norm)
    pt = ptHigh * exp(-sqrt(L2))
    return pt

main()
```

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    L2 = log(sudakovValue)/(-alphas * norm)
    pt = ptHigh * exp(-sqrt(L2))
    return pt

main()
```

```
% python ./toy-shower.py

Event 0
  primary emission with pt =  58.4041962726
  primary emission with pt =  3.61999582015
  primary emission with pt =  2.31198814996

Event 1
  primary emission with pt =  32.1881228375
  primary emission with pt =  10.1818306204
  primary emission with pt =  10.1383134201
  primary emission with pt =  7.24482350383
  primary emission with pt =  2.35709074796
  primary emission with pt =  1.0829758034

Event 2
  primary emission with pt =  64.934992001
  primary emission with pt =  16.4122436094
  primary emission with pt =  2.53473253194

Event 3
  primary emission with pt =  37.6281171491
  primary emission with pt =  22.7262873764
  primary emission with pt =  12.0255817868
  primary emission with pt =  4.73678636215
  primary emission with pt =  3.92257832288

Event 4
  primary emission with pt =  21.5359449851
  primary emission with pt =  4.01438733798
  primary emission with pt =  3.33902663941
  primary emission with pt =  2.02771620824
  primary emission with pt =  1.05944759028

. . .
```

A toy shower

https://github.com/gavinsalam/zuoz2016-toy-shower

(fixed coupling, primary branching only, only p_T, no energy conservation, no PDFs, etc.)

```
#!/usr/bin/env python
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Event 3
  primary emission with pt =  37.6281171491
  primary emission with pt =  22.7262873764
  primary emission with pt =  12.0255817868
  primary emission with pt =  4.73678636215
```

Exercise: replace C_A=3 (emission from gluons) with C_F=4/3 (emission from quarks) and see how pattern of emissions changes (multiplicity, p_T of hardest emission, etc.)

A real-world shower (Herwig)

---PARTON SHOWERS---

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
9	UQRK	94	141	4	6	11	16	2.64	-9.83	592.2	590.2	-49.07
10	CONE	0	100	4	5	0	0	-0.27	0.96	0.1	1.0	0.00
11	GLUON	21	2	9	12	32	33	-1.02	3.59	5.6	6.7	0.75-
12	GLUON	21	2	9	13	34	35	0.25	1.46	3.6	4.0	0.75-
13	GLUON	21	2	9	14	36	37	-0.87	1.62	4.7	5.1	0.75-
14	GLUON	21	2	9	15	38	39	-0.81	4.17	3611.7	3611.7	0.75-
15	GLUON	21	2	9	16	40	41	-0.19	-1.01	1727.7	1727.7	0.75-
16	UD	2101	2	9	25	42	41	0.00	0.00	1054.6	1054.6	0.32-
17	GLUON	94	142	5	6	19	21	-2.23	0.44	-233.5	232.8	-18.36
18	CONE	0	100	5	8	0	0	0.77	0.64	0.2	1.0	0.00
19	GLUON	21	2	17	20	43	44	1.60	0.58	-2.1	2.8	0.75
20	UD	2101	2	17	21	45	44	0.00	0.00	-2687.6	2687.6	0.32
21	UQRK	2	2	17	32	46	45	0.63	-1.02	-4076.9	4076.9	0.32
22	Z0/GAMA*	23	195	7	22	251	252	-257.66	-219.68	324.8	477.5	88.56
23	UQRK	94	144	8	6	25	31	258.06	210.29	33.9	345.5	86.10
24	CONE	0	100	8	5	0	0	0.21	0.17	-1.0	1.0	0.00
25	UQRK	2	2	23	26	47	42	26.82	24.33	23.7	43.3	0.32
26	GLUON	21	2	23	27	48	49	8.50	8.18	6.0	13.3	0.75
27	GLUON	21	2	23	28	50	51	73.27	61.24	12.0	96.2	0.75
28	GLUON	21	2	23	29	52	53	73.66	58.54	-6.3	94.3	0.75
29	GLUON	21	2	23	30	54	55	67.58	52.13	-7.3	85.7	0.75
30	GLUON	21	2	23	31	56	57	6.98	4.60	2.3	8.7	0.75
31	GLUON	21	2	23	43	58	59	1.24	1.26	3.6	4.1	0.75

INITIAL STATE SHOWER

FINAL STATE SHOWER

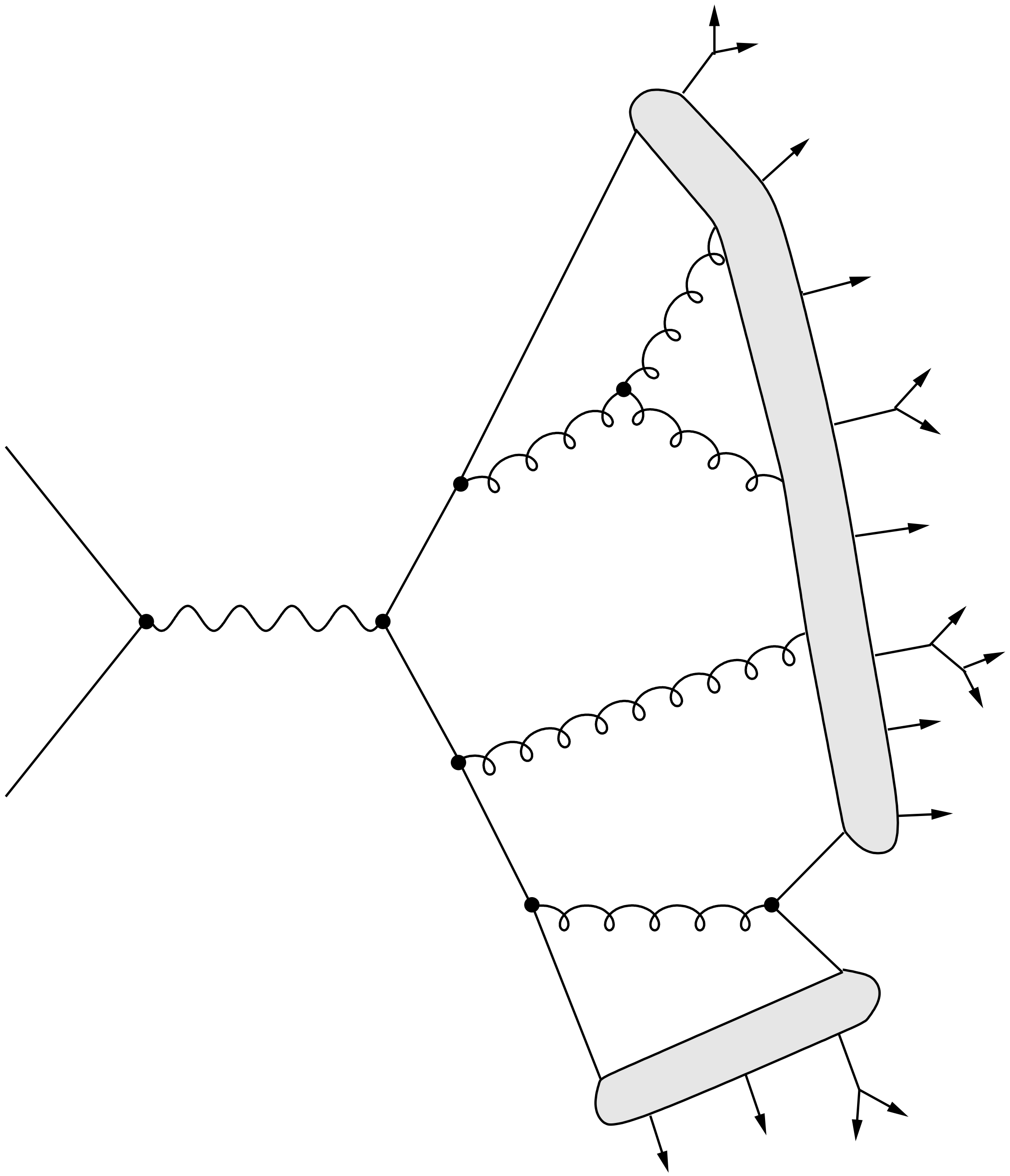
real-world Monte Carlo parton shower programs

- Pythia, Herwig, Sherpa
(each has one or more formulations of a parton shower)
- Sudakov approximation is not accurate for high-p_T emissions, and intrinsic accuracy of cross sections is LO
  - showers combined with NLO through tools like MC@NLO or POWHEG
(NNLO matching is a research topic with first tools available)
  - Full matrix elements for hard emissions included through methods like MLM, CKKW, FxFx, Sherpa "merging" or through Vincia or MiNLO techniques

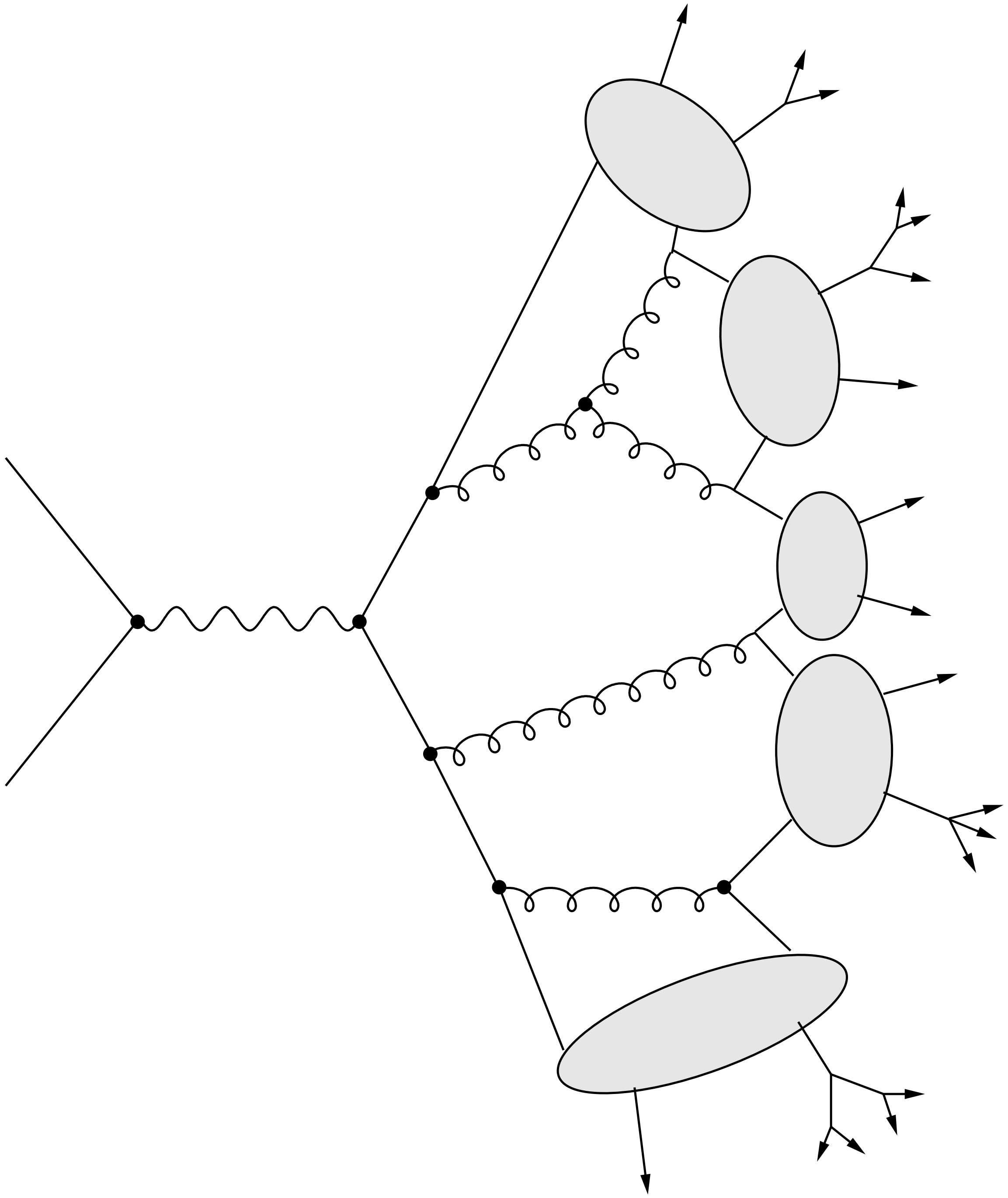
hadronisation & MPI

essential models for realistic events

two main models for the parton–hadron transition ("hadronisation")



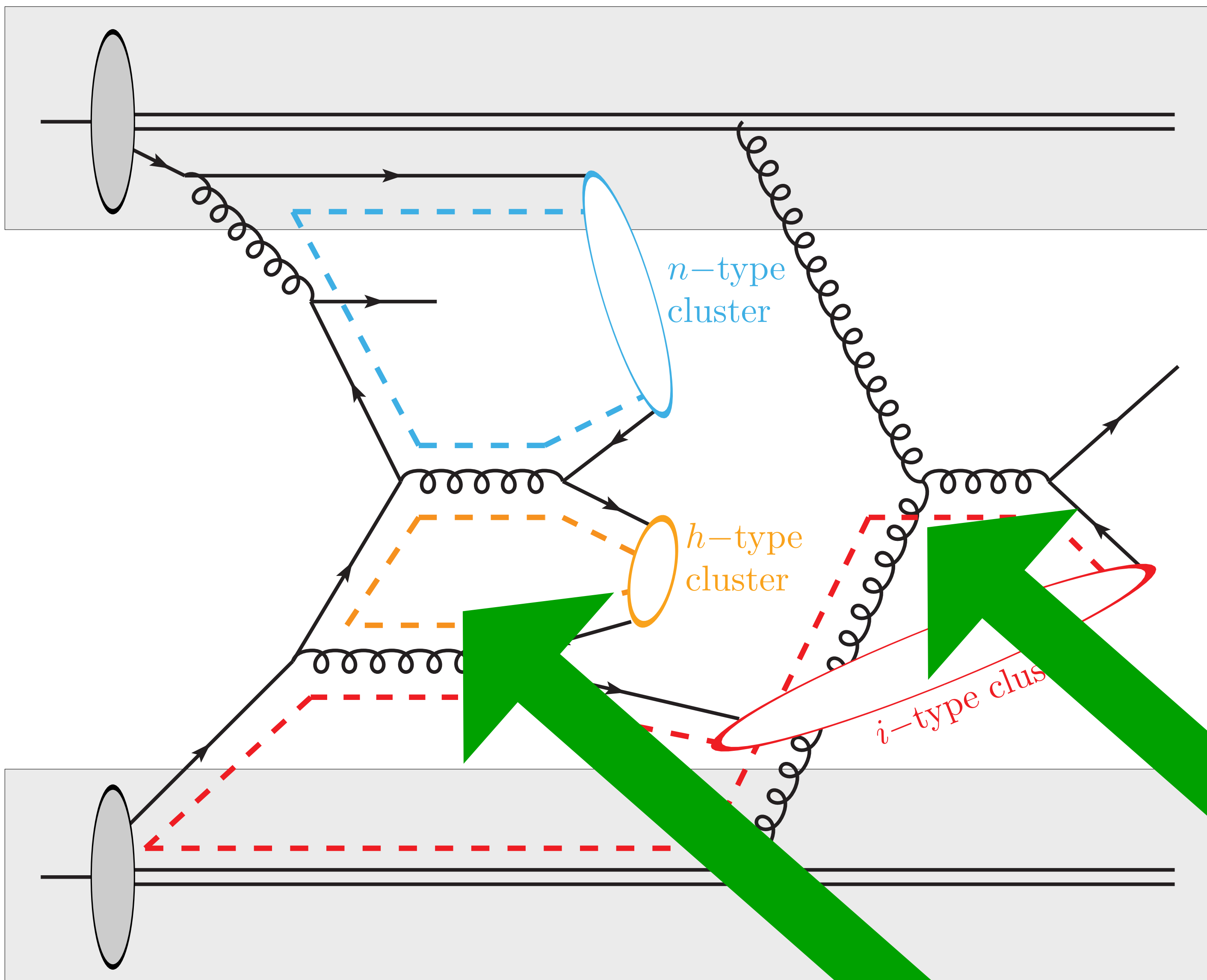
String Fragmentation
(Pythia and friends)



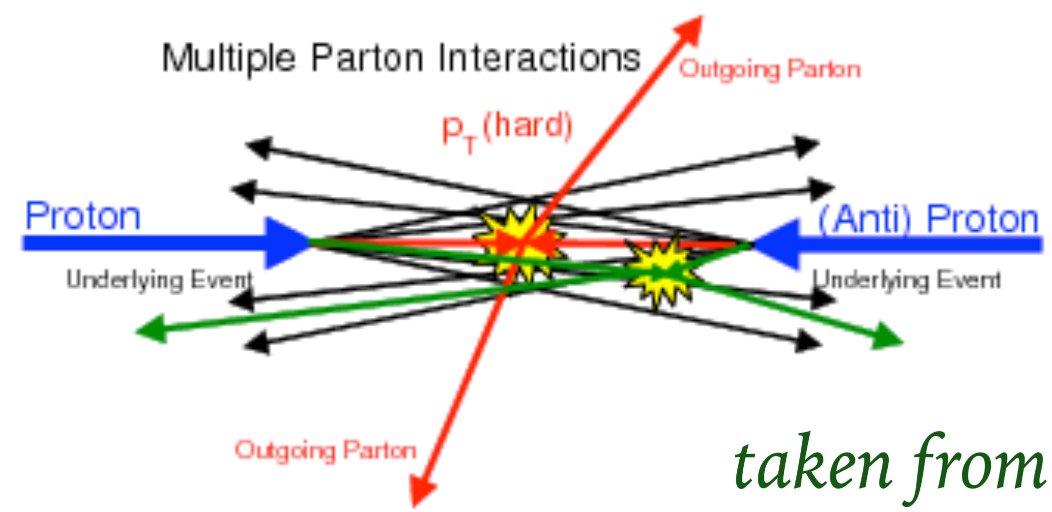
Cluster Fragmentation
(Herwig) (& Sherpa)

Pictures from ESW book

multi-parton interactions (MPI, a.k.a. underlying event)

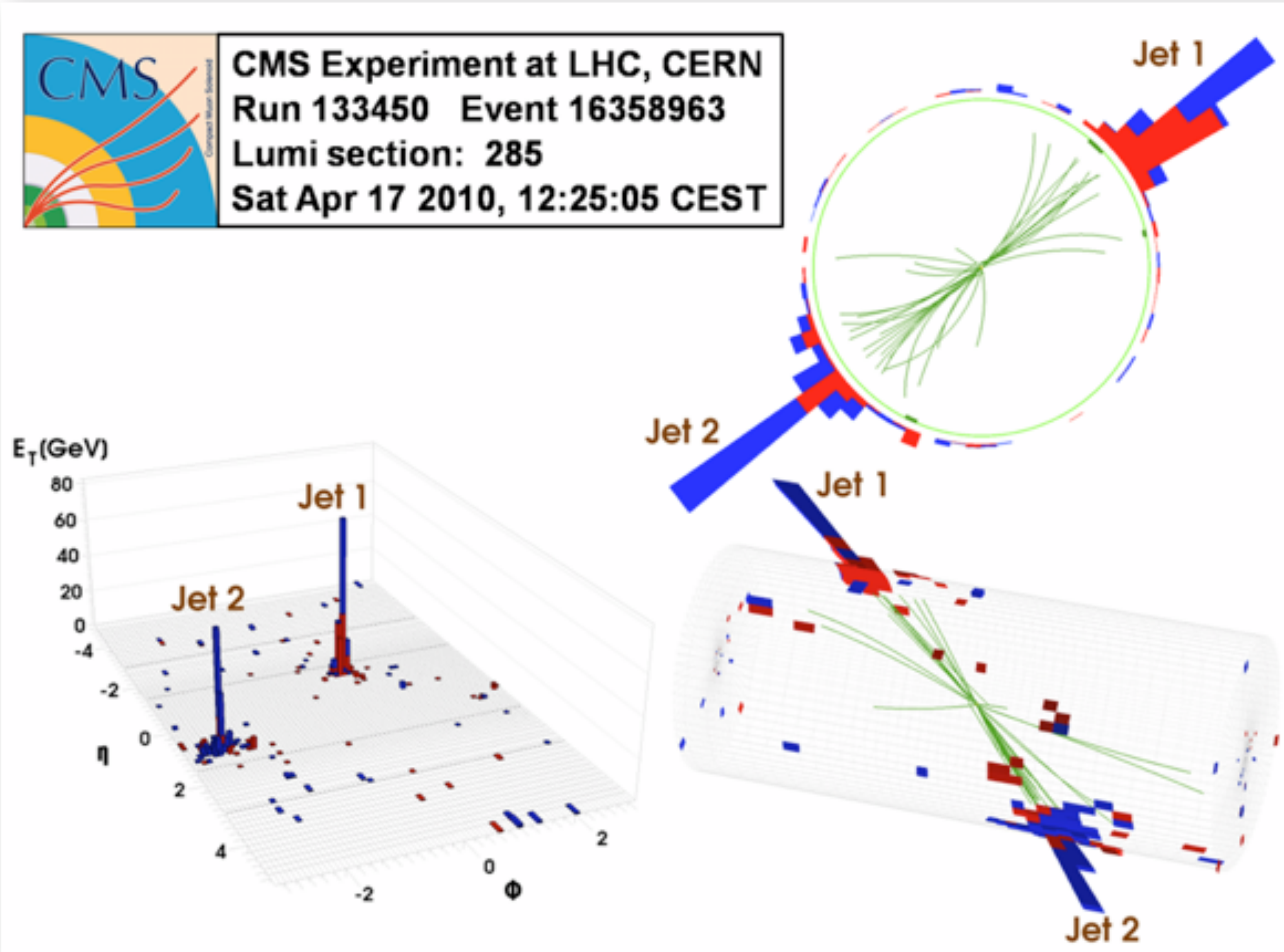


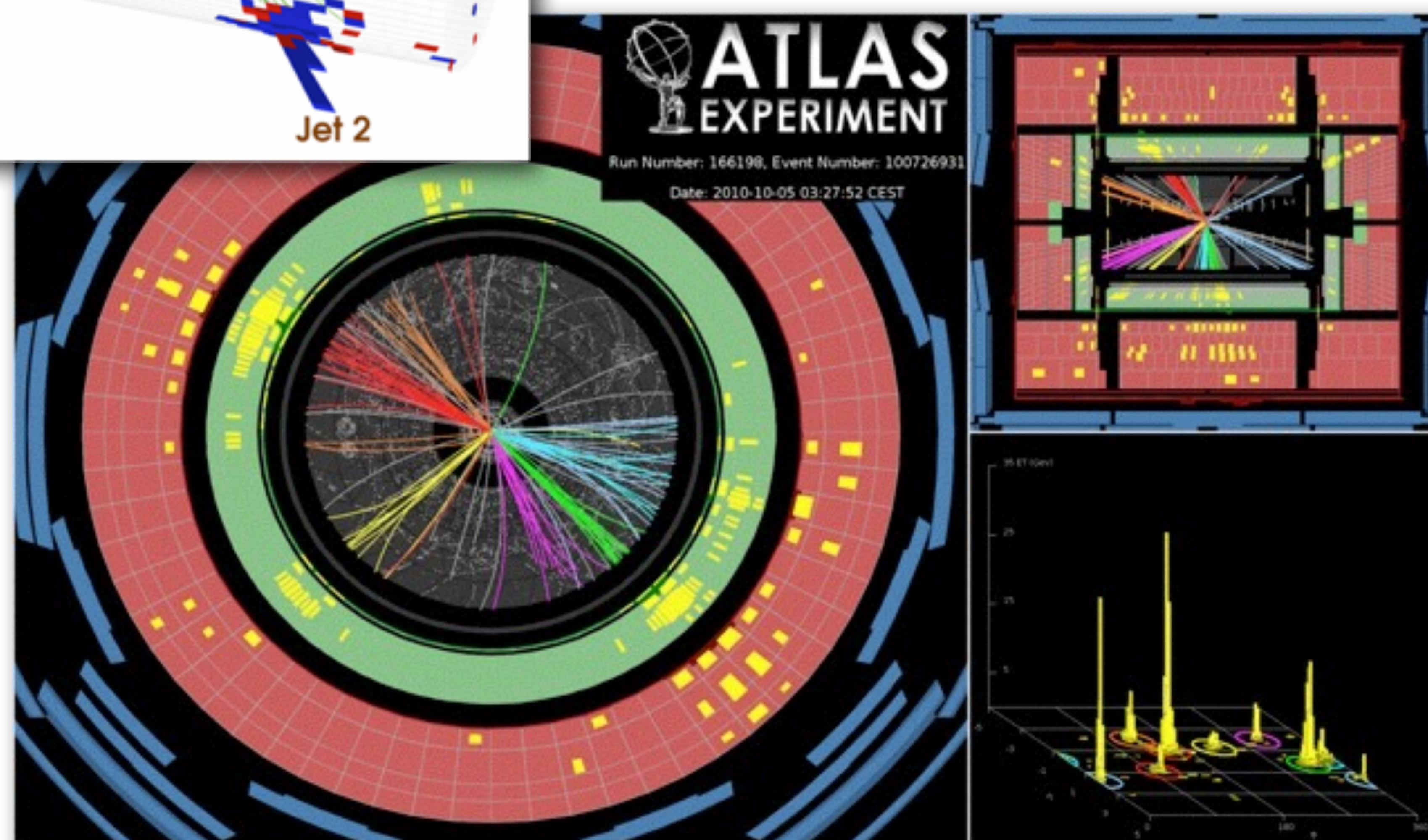
taken from
1206.2205



taken from
R. Field

Allow 2→2 scatterings of multiple other partons in the incoming protons

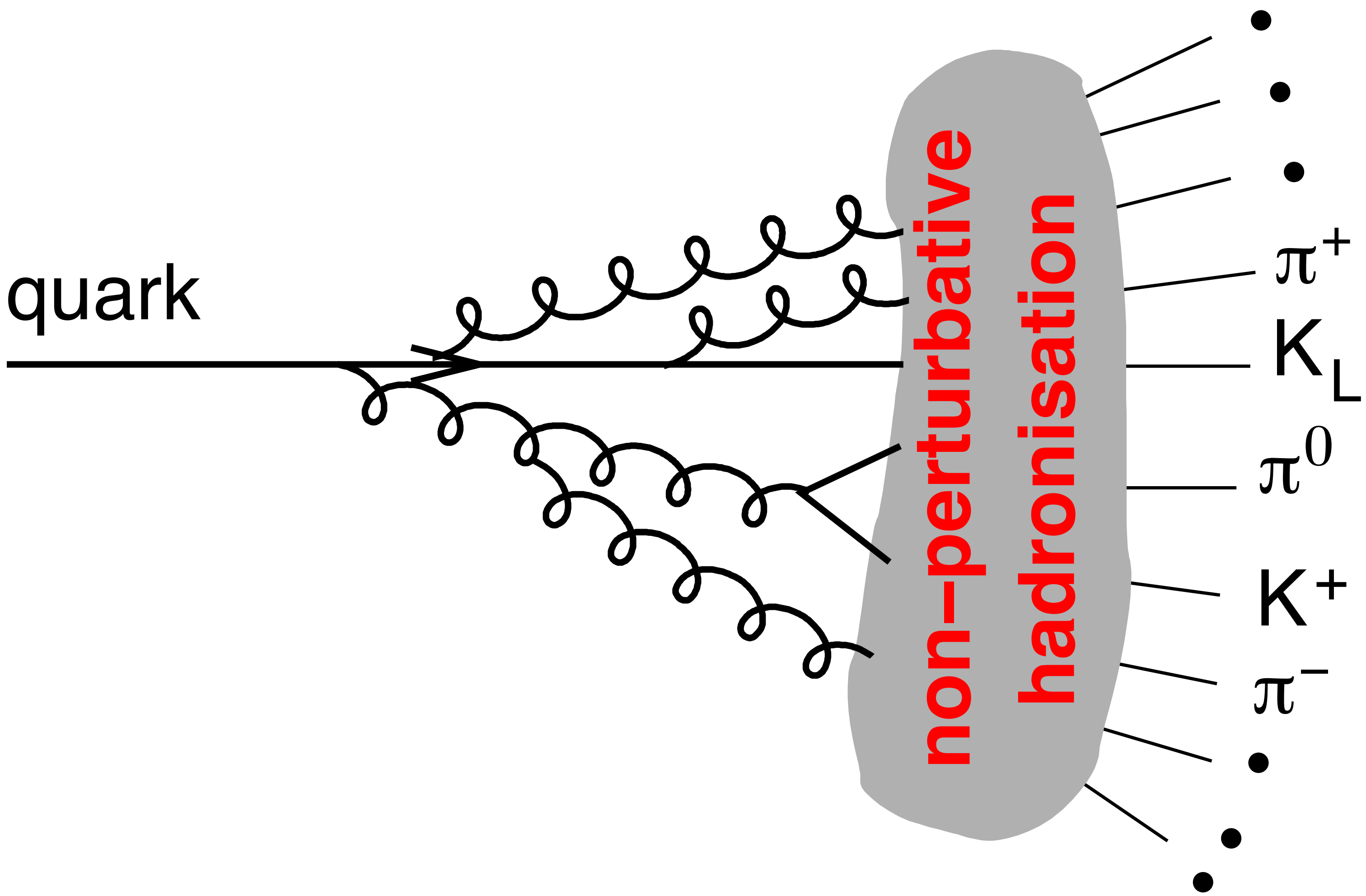




jets

i.e. how we make sense of the hadronic part of events

WHY DO WE SEE JETS?



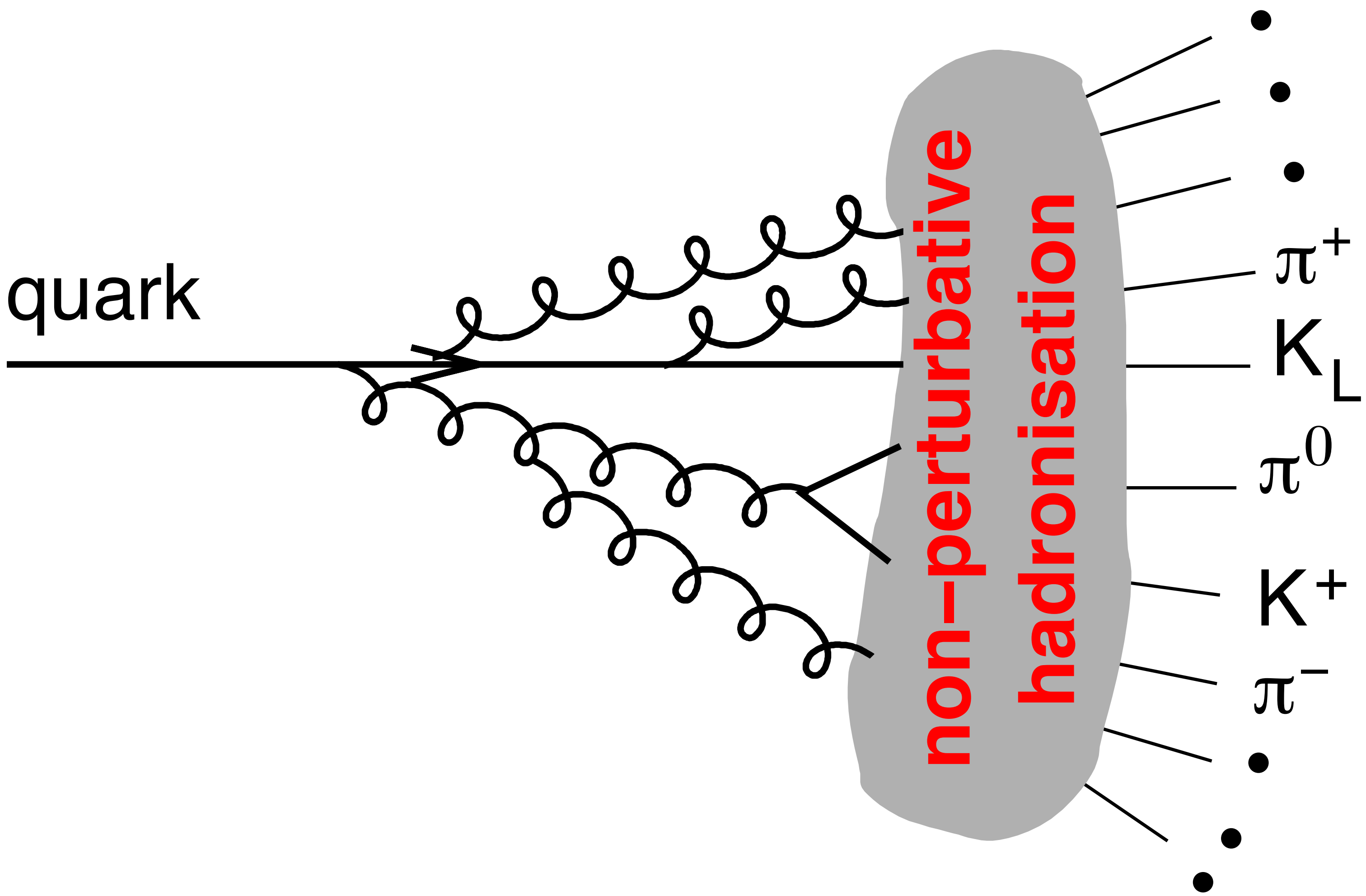
Gluon emission

$$\int \alpha_s \frac{dE}{E}\frac{d\theta}{\theta} \gg 1$$

Non-perturbative physics

$$\alpha_s \sim 1$$

WHY DO WE SEE JETS?



Gluon emission

$$\int \alpha_s \frac{dE}{E}\frac{d\theta}{\theta} \gg 1$$

Non-perturbative physics

$$\alpha_s \sim 1$$

While you can see jets with your eyes, **to do quantitative physics**, you need an algorithmic procedure that **defines what exactly a jet is**

make a choice, specify a Jet Definition

$$\{p_i\} \xrightarrow{\text{jet definition}} \{j_k\}$$

particles,
4-momenta,
calorimeter towers, ...

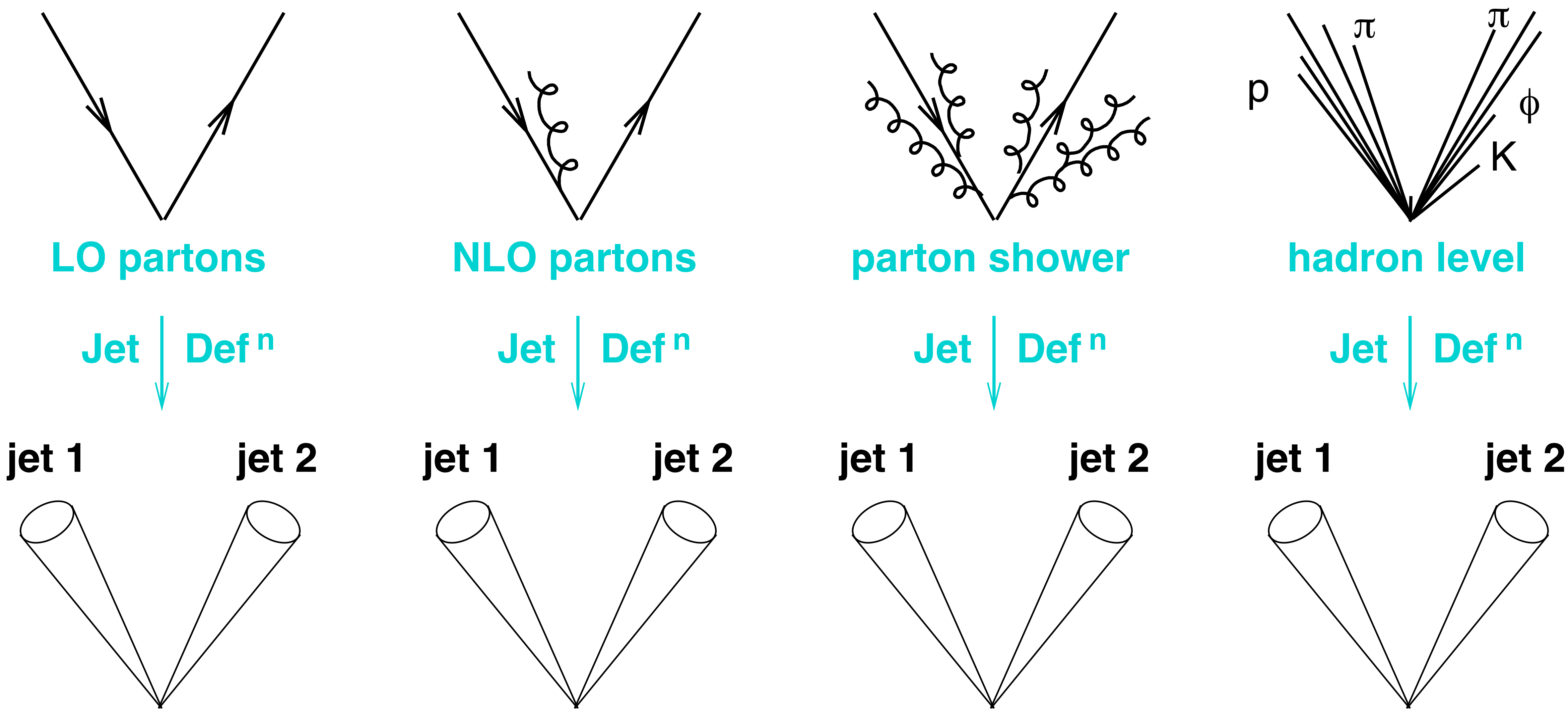
jets

- Which particles do you put together into a same jet?
- How do you recombine their momenta
(4-momentum sum is the obvious choice, right?)

"Jet [definitions] are legal contracts between theorists and experimentalists"
-- MJ Tannenbaum

They're also a way of organising the information in an event
1000's of particles per events, up to 20.000,000 events per second

what should a jet definition achieve?



projection to jets should be resilient to QCD effects

Two parameters, R and $p_{t,min}$

(These are the two parameters in essentially every widely used hadron-collider jet algorithm)

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

Sequential recombination algorithm

1. Find smallest of d_{ij}, d_{iB}
2. If ij, recombine them
3. If iB, call i a jet and remove from list of particles
4. repeat from step 1 until no particles left

Only use jets with $p_t > p_{t,min}$

anti-k_t algorithm

Cacciari, GPS & Soyez, 0802.1189

anti-k_t in action

Clustering grows around hard cores

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

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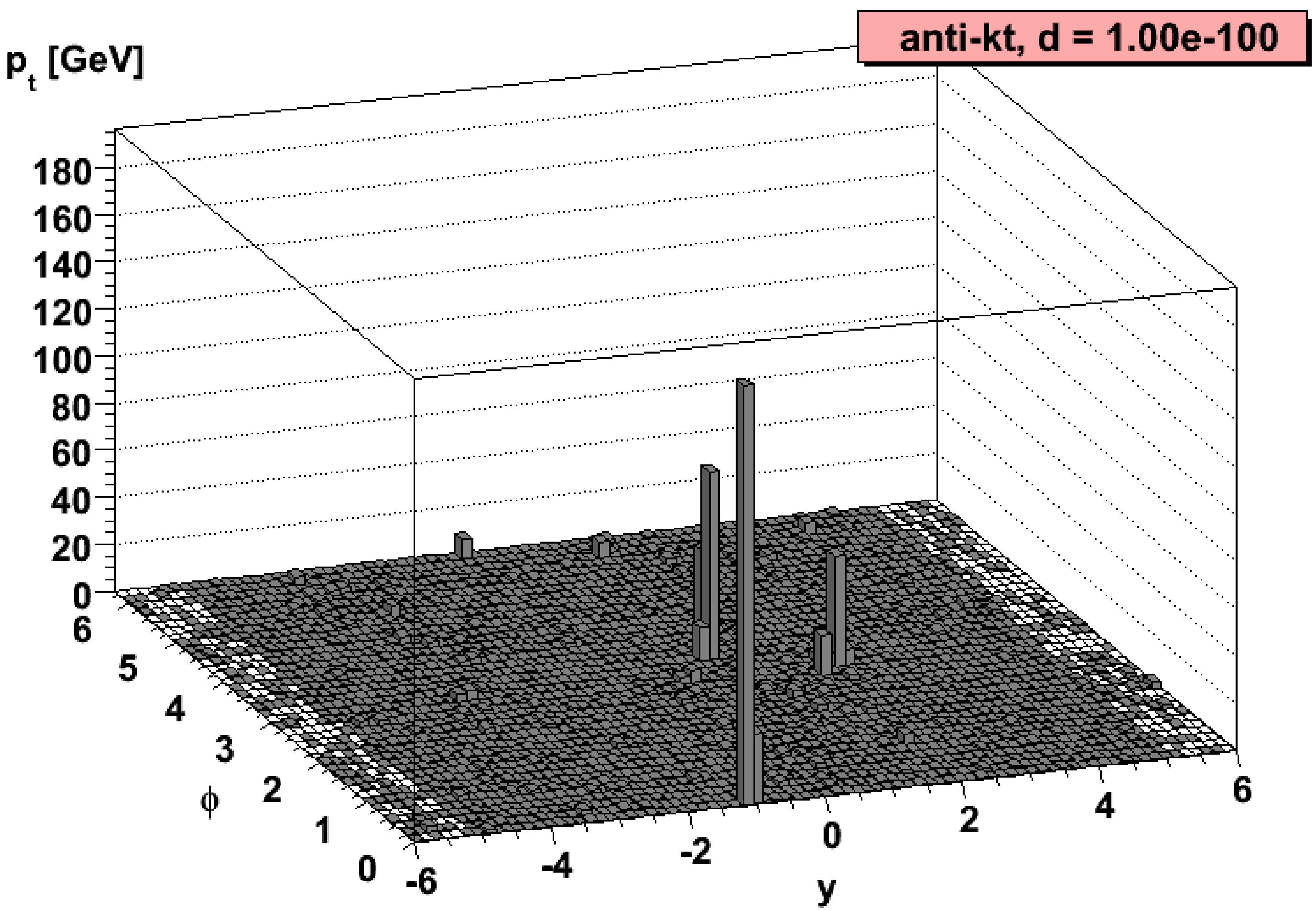

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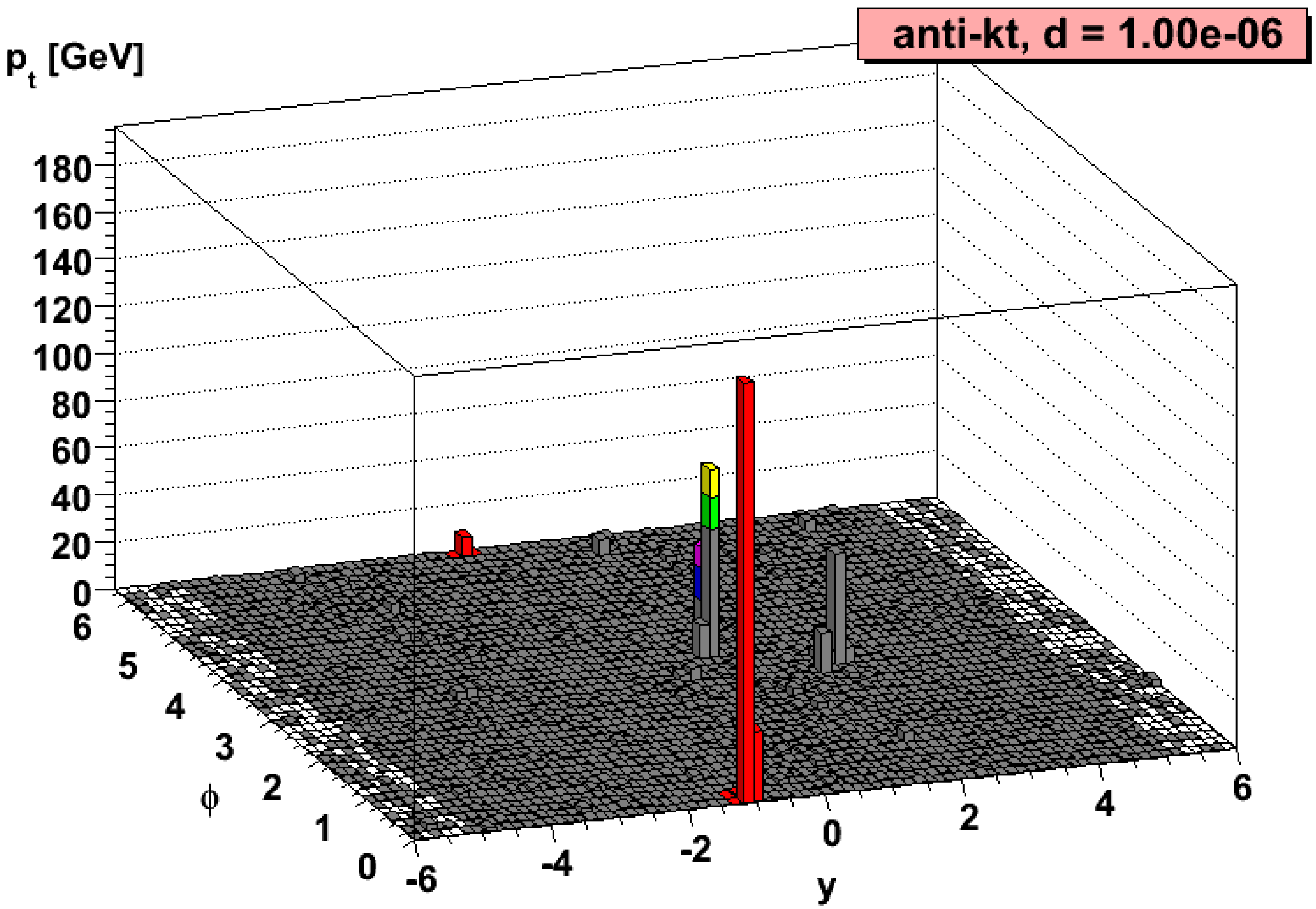

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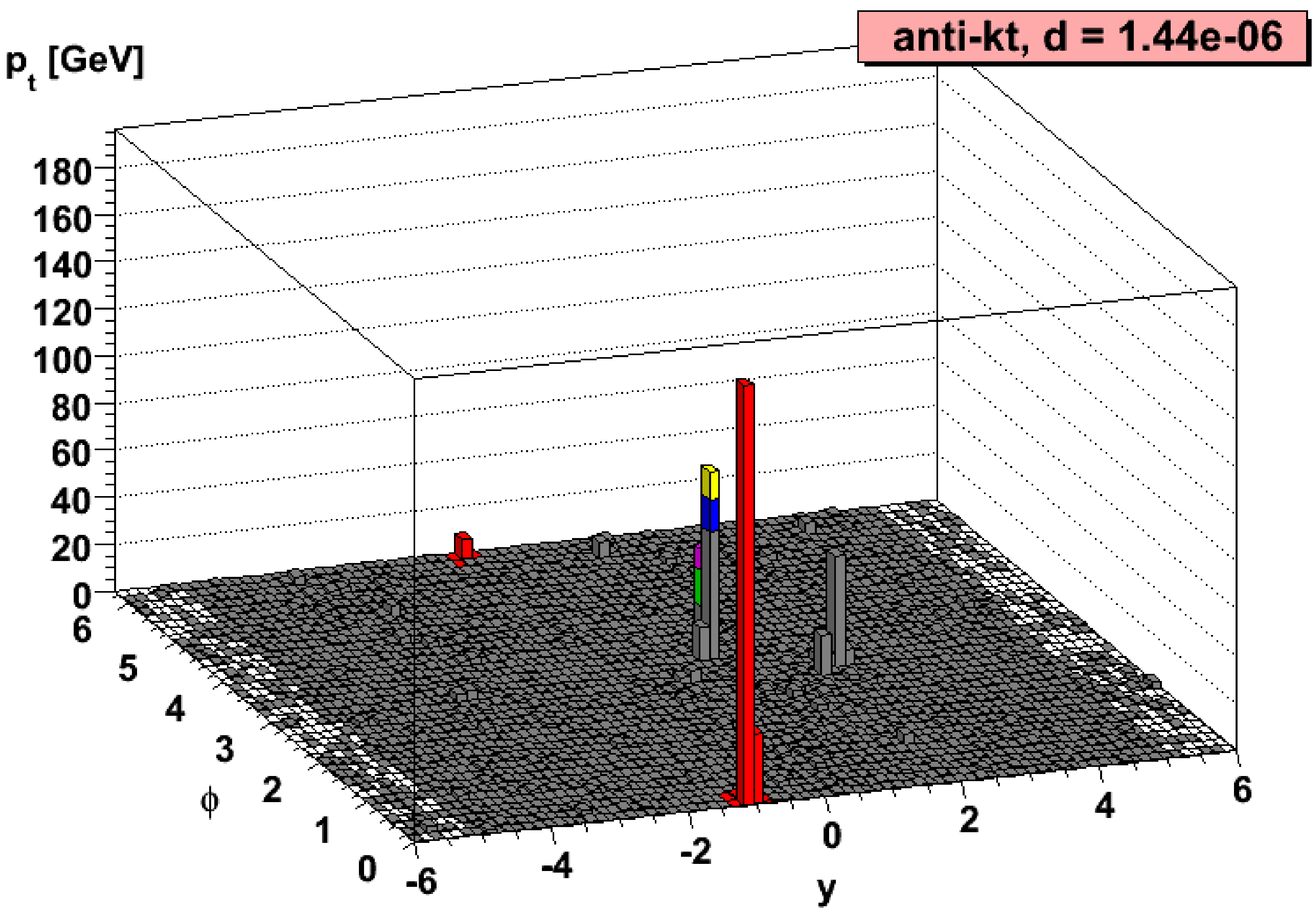

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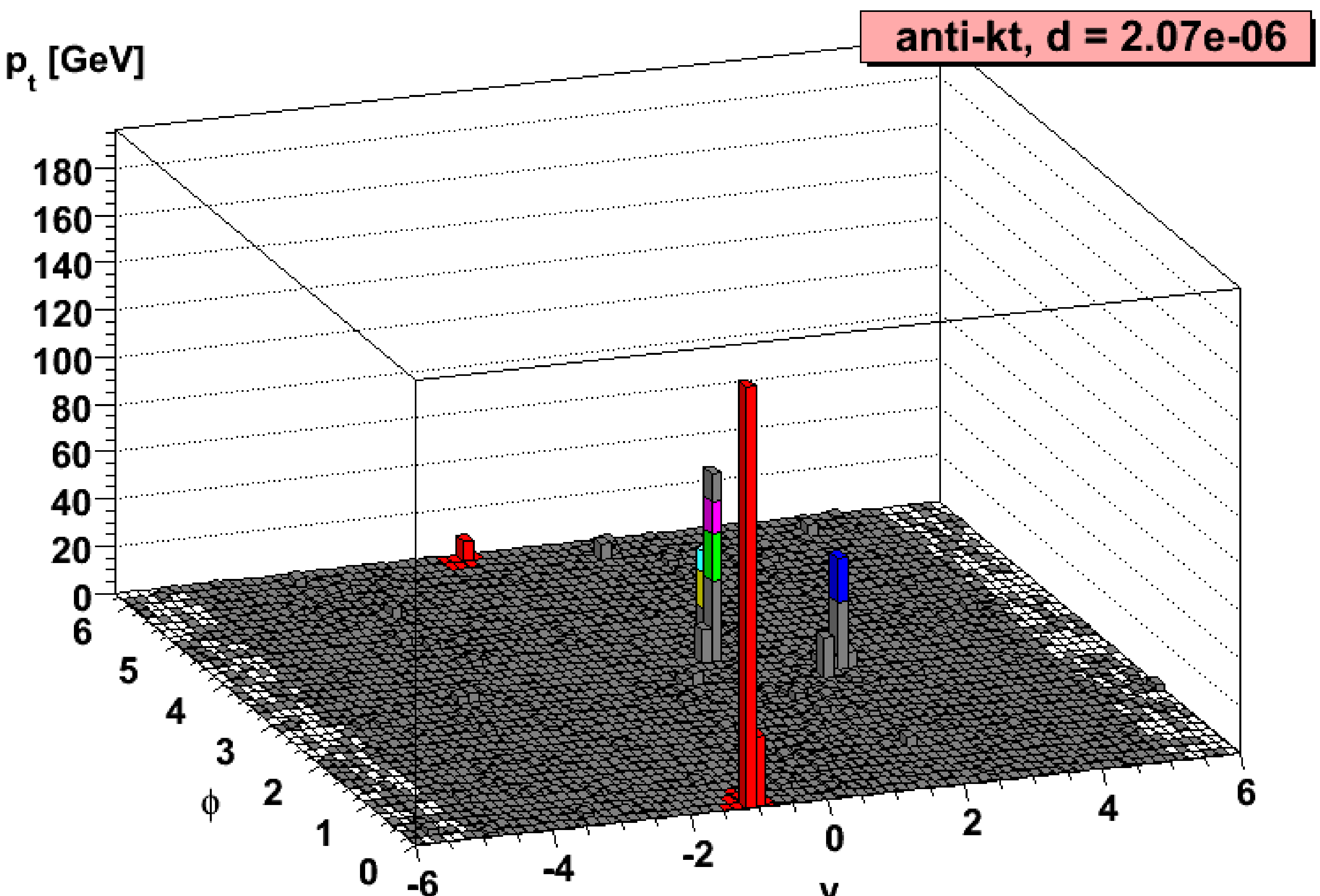

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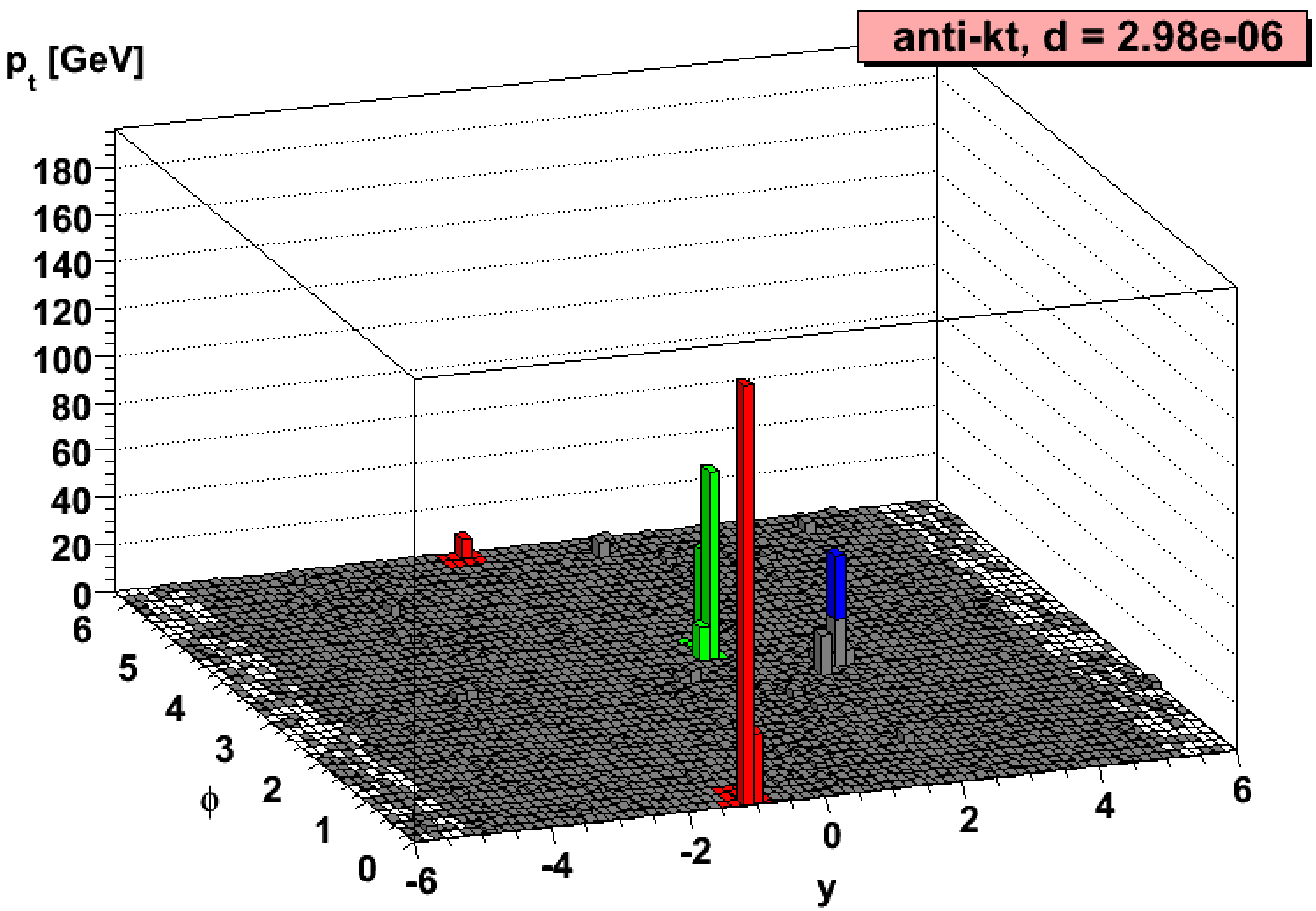

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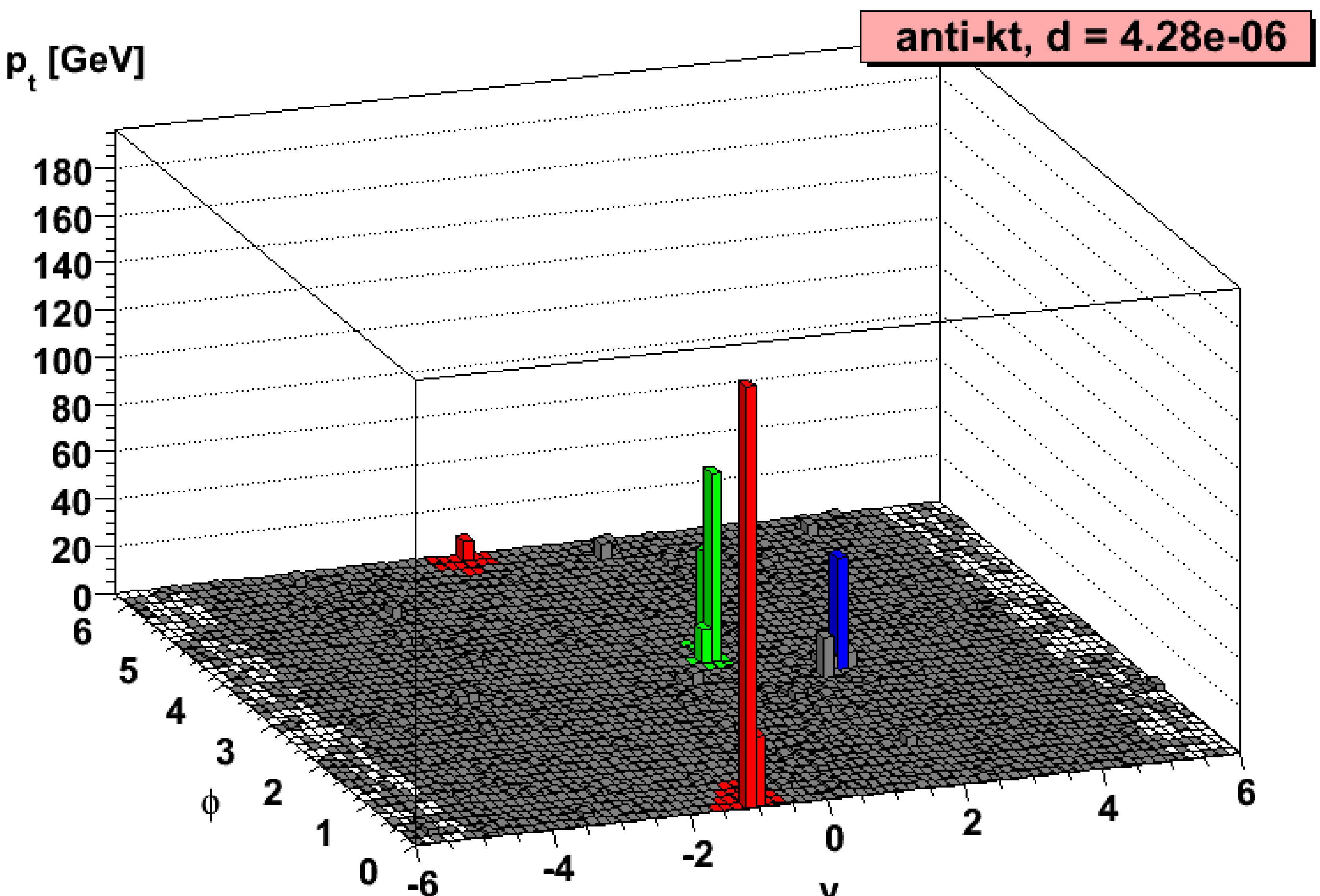

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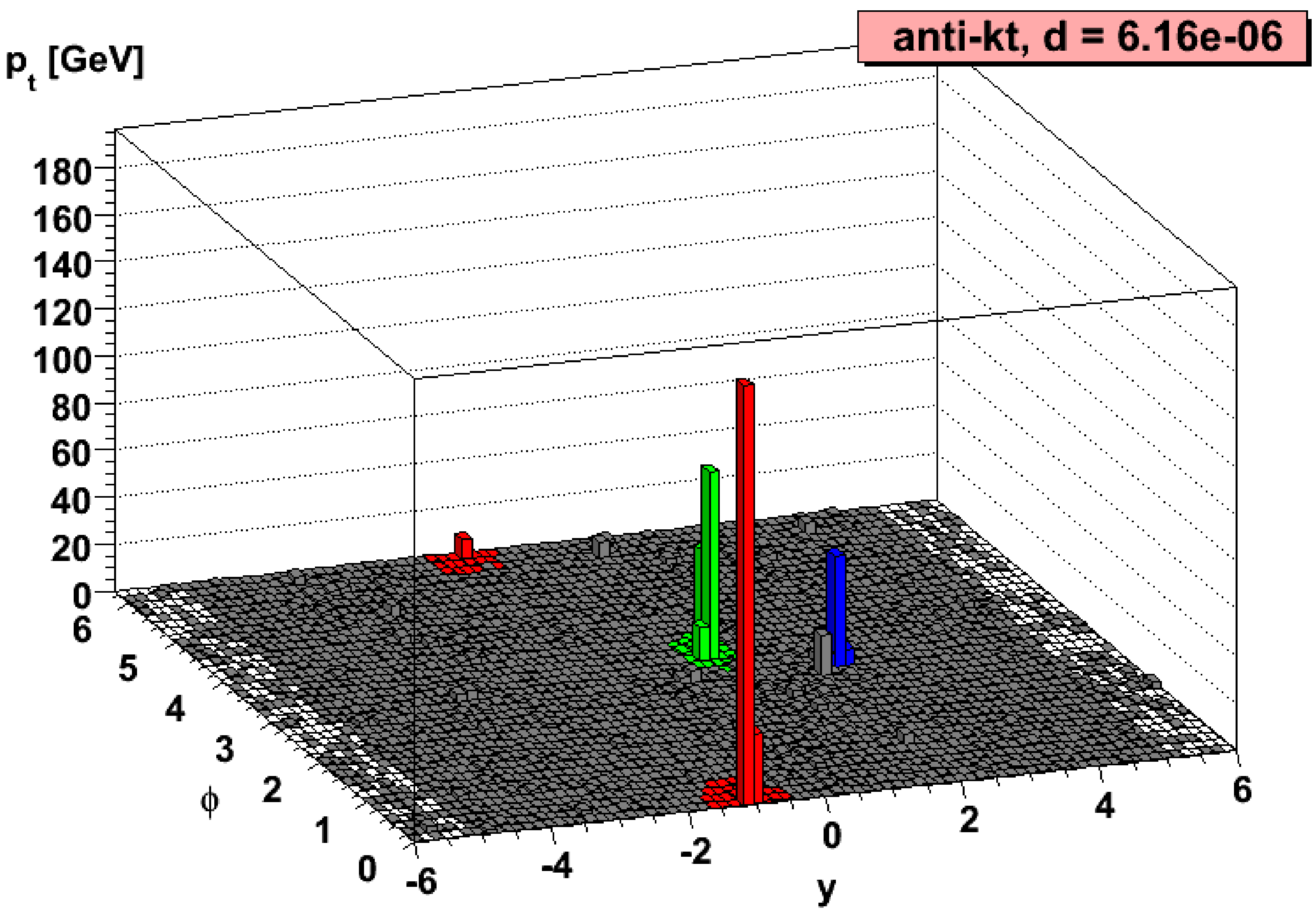

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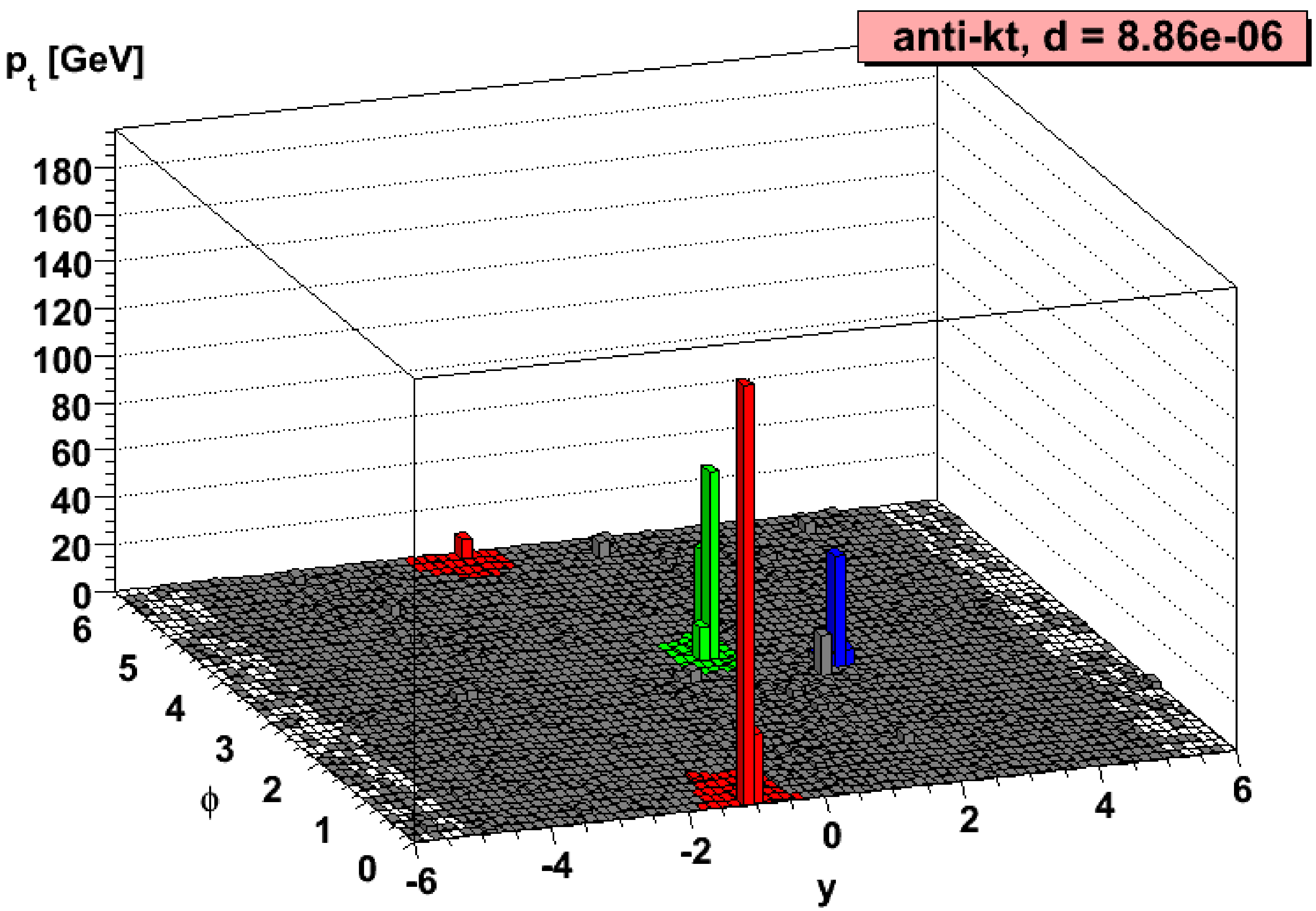

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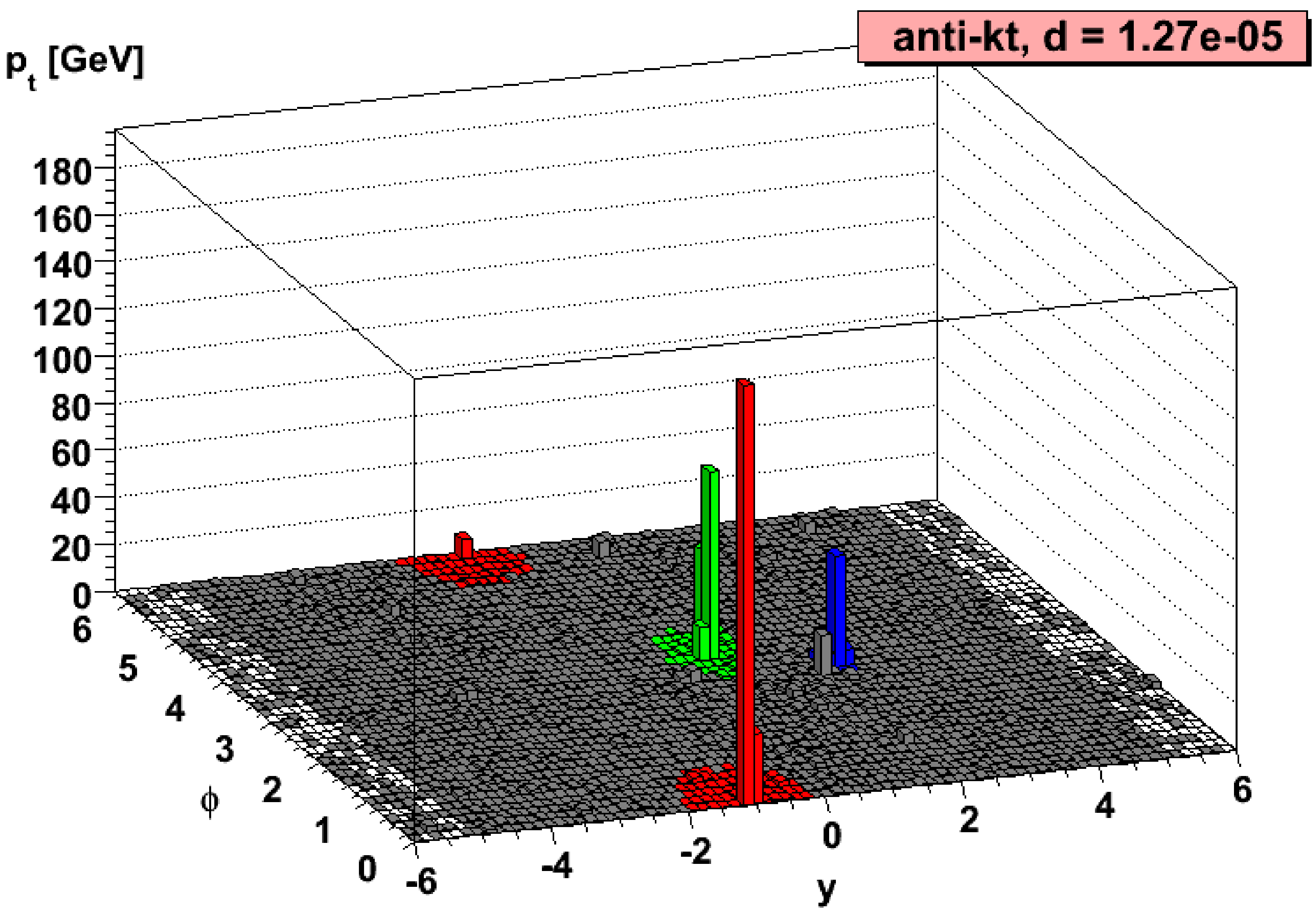

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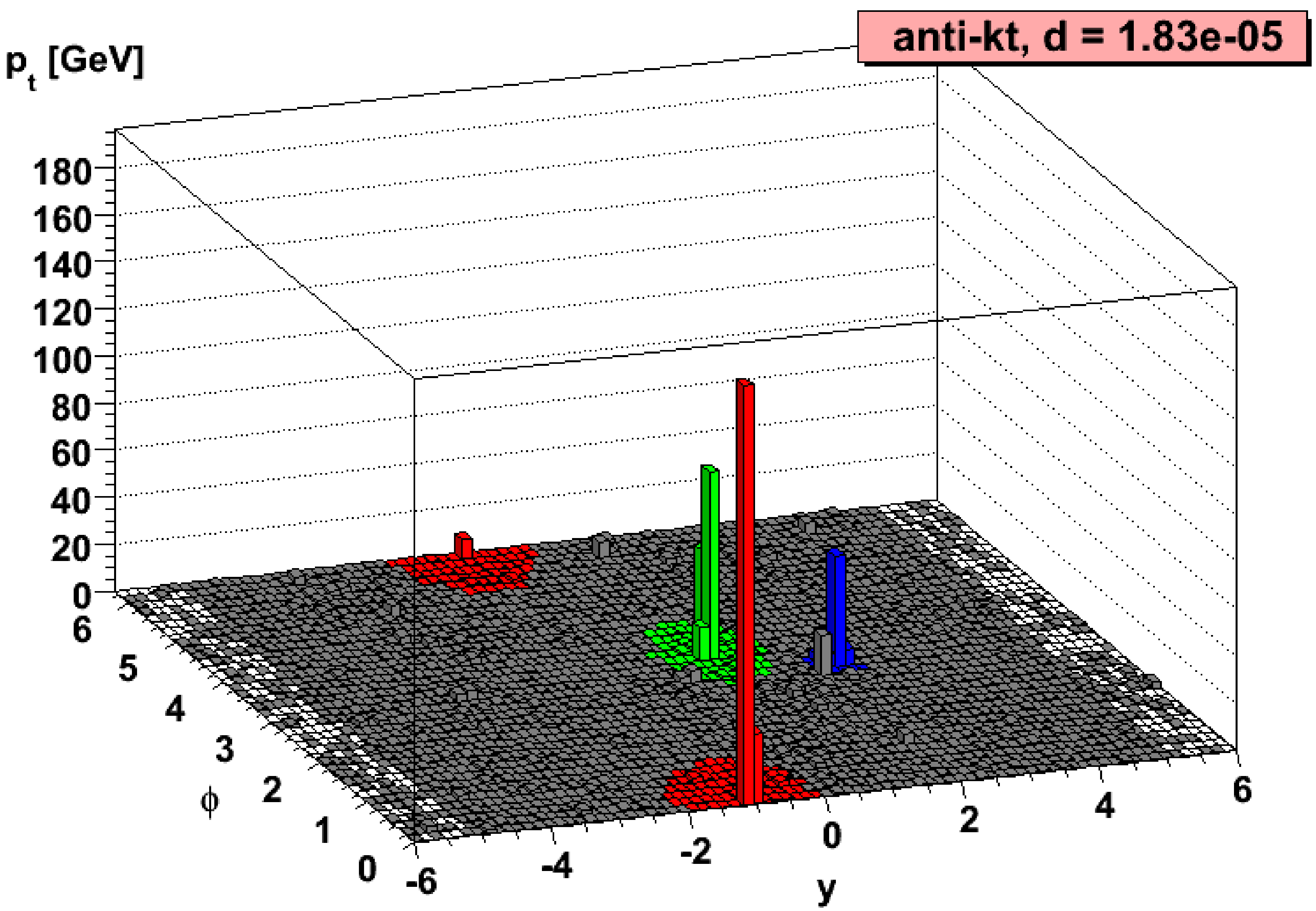

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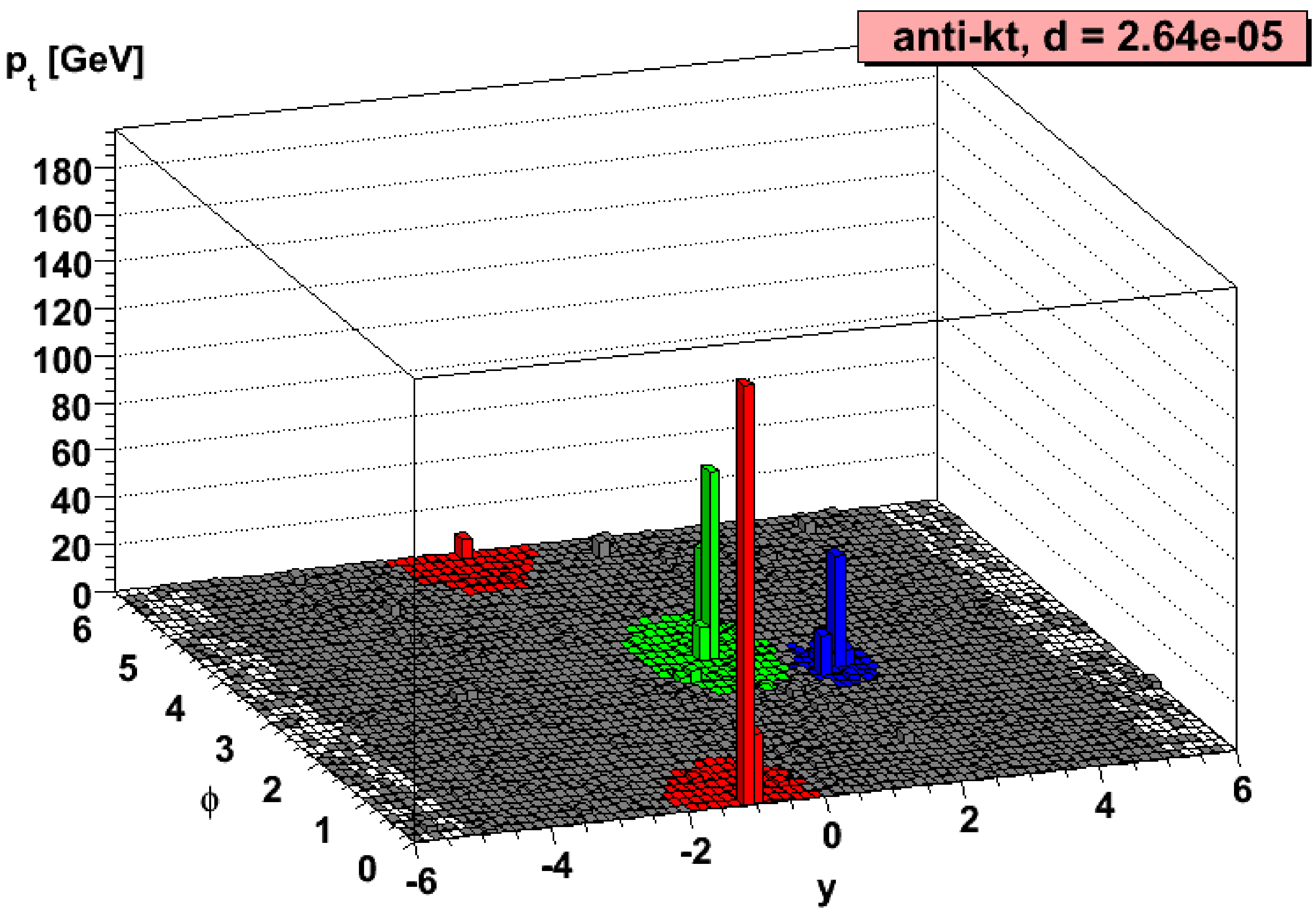

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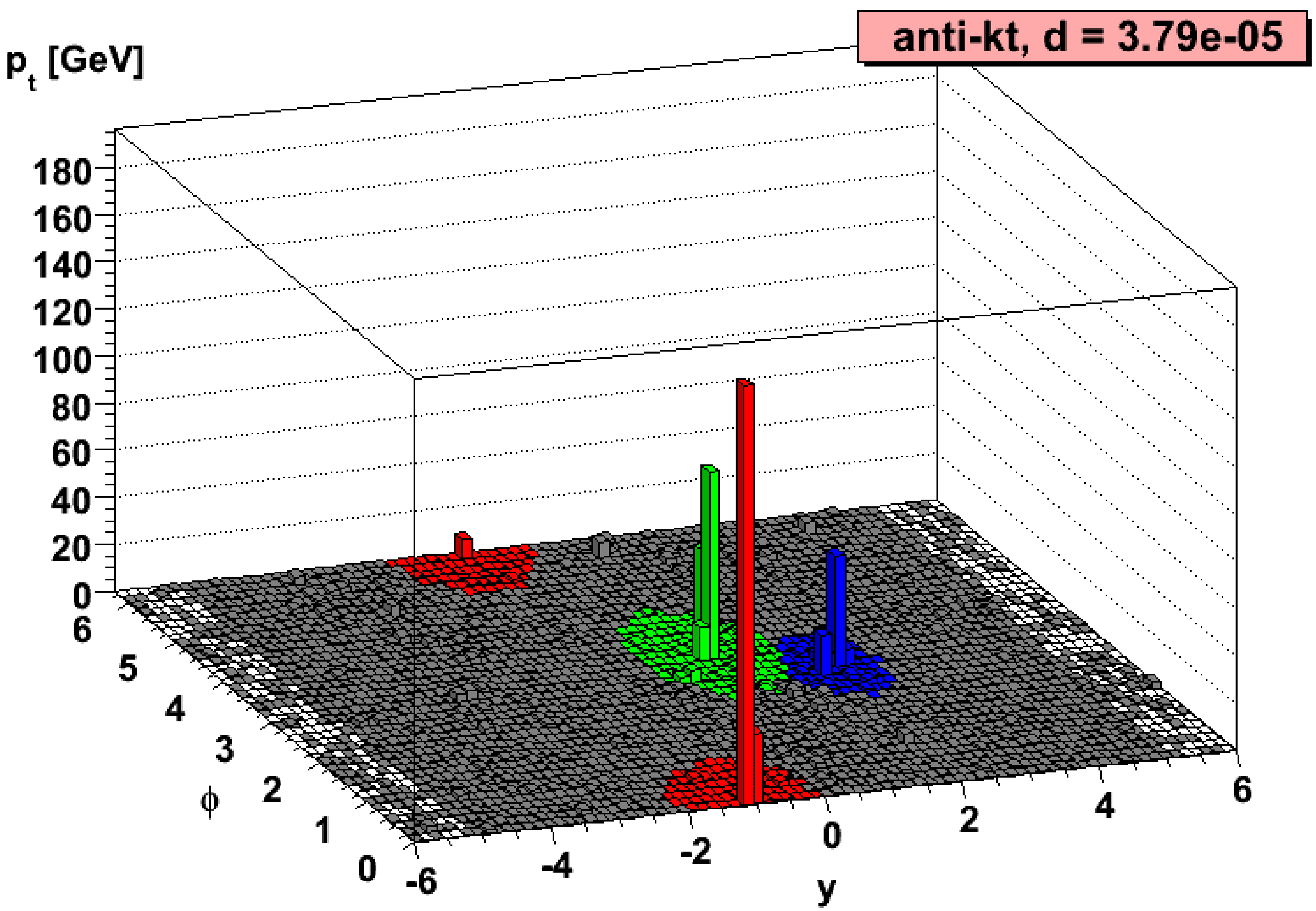

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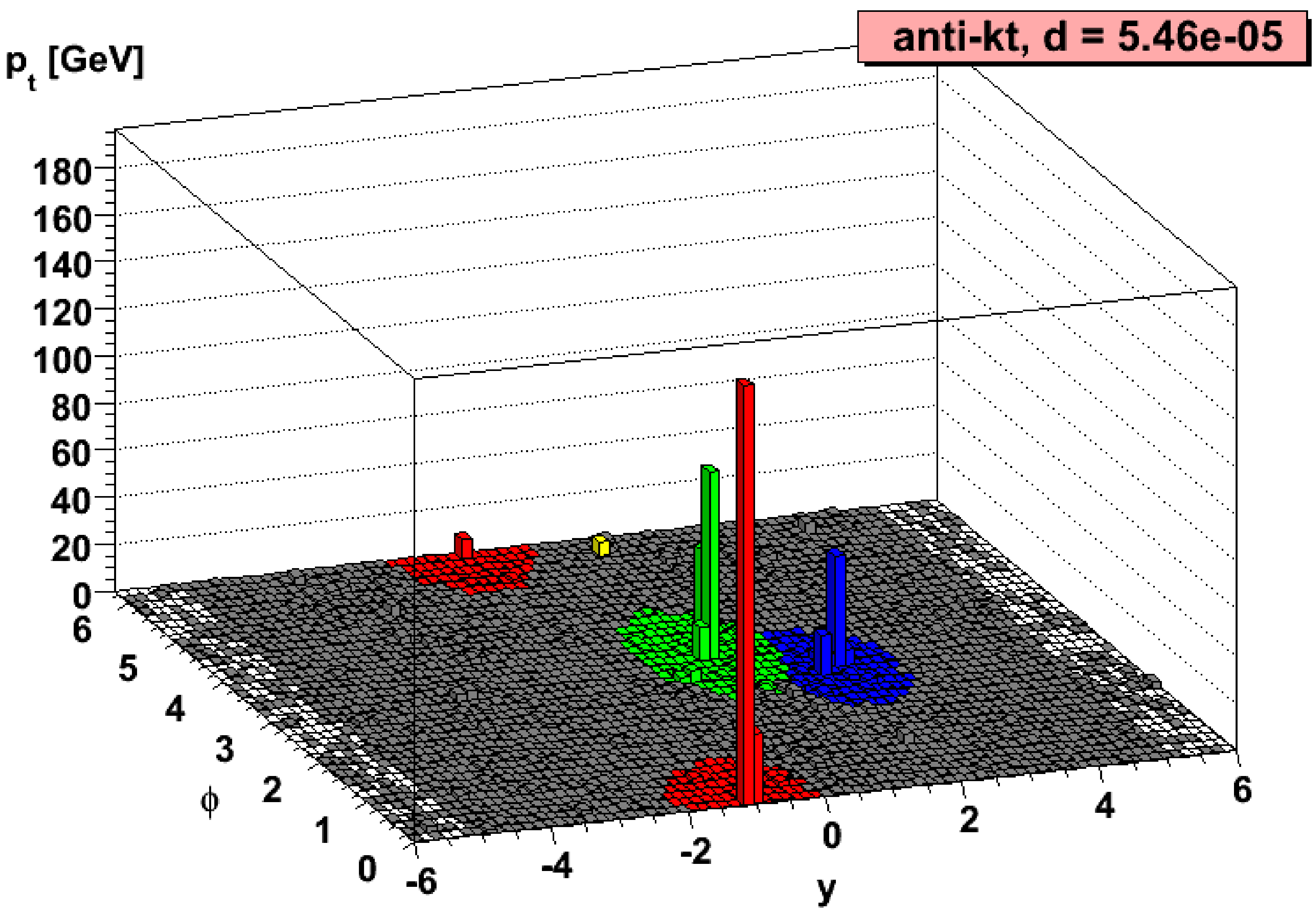

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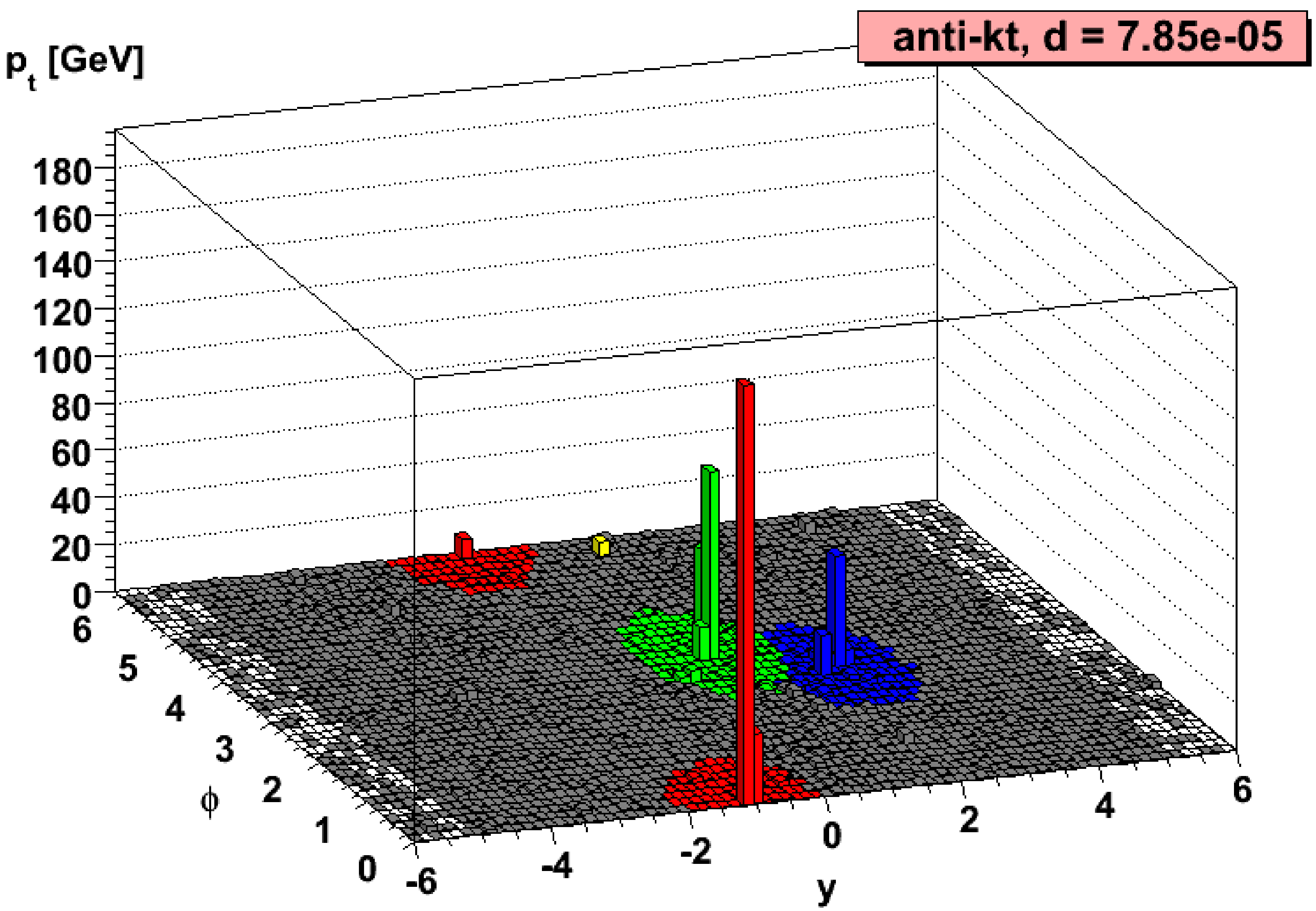

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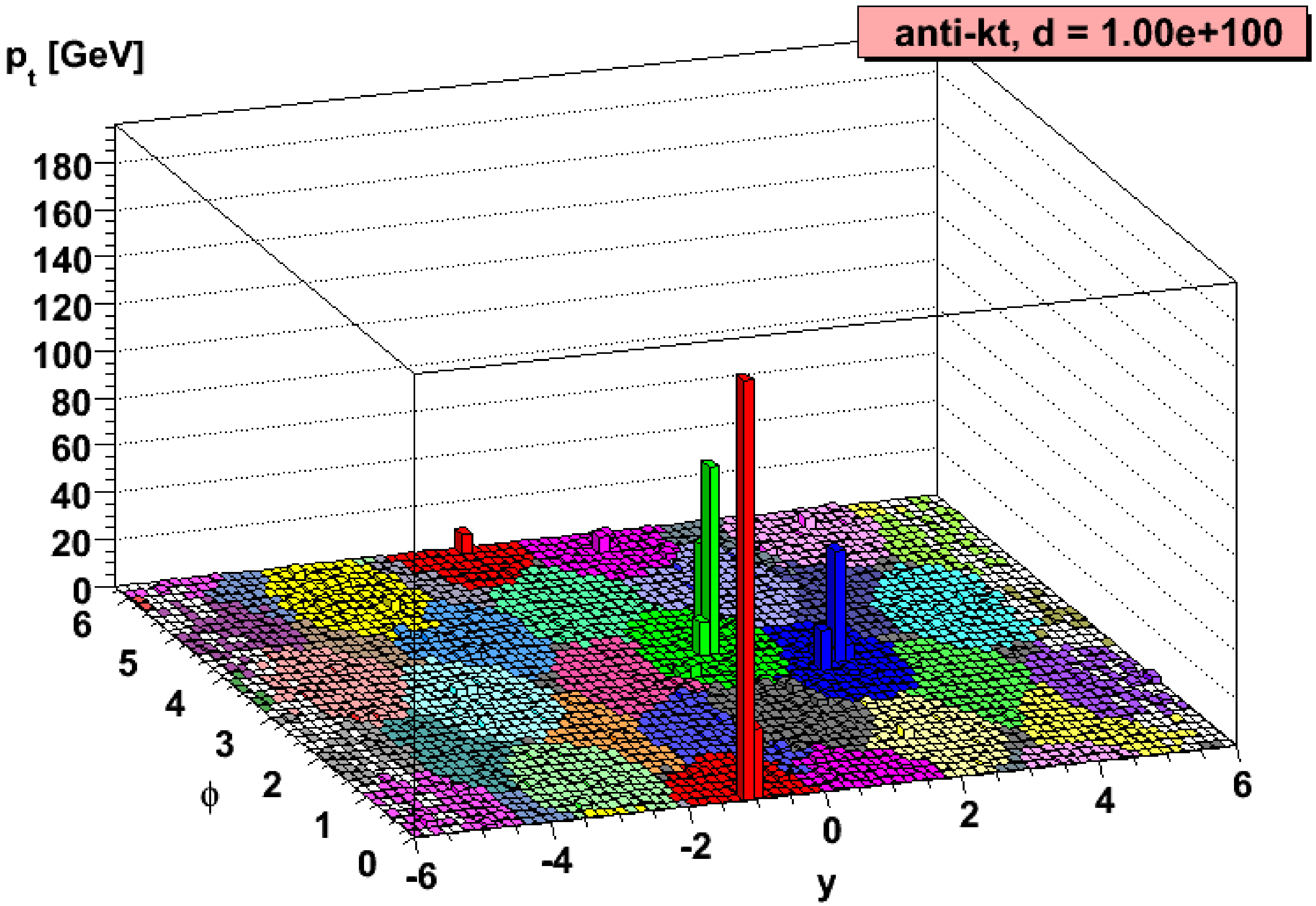


Anti-k_t gives circular jets ("cone-like") in a way that's infrared safe

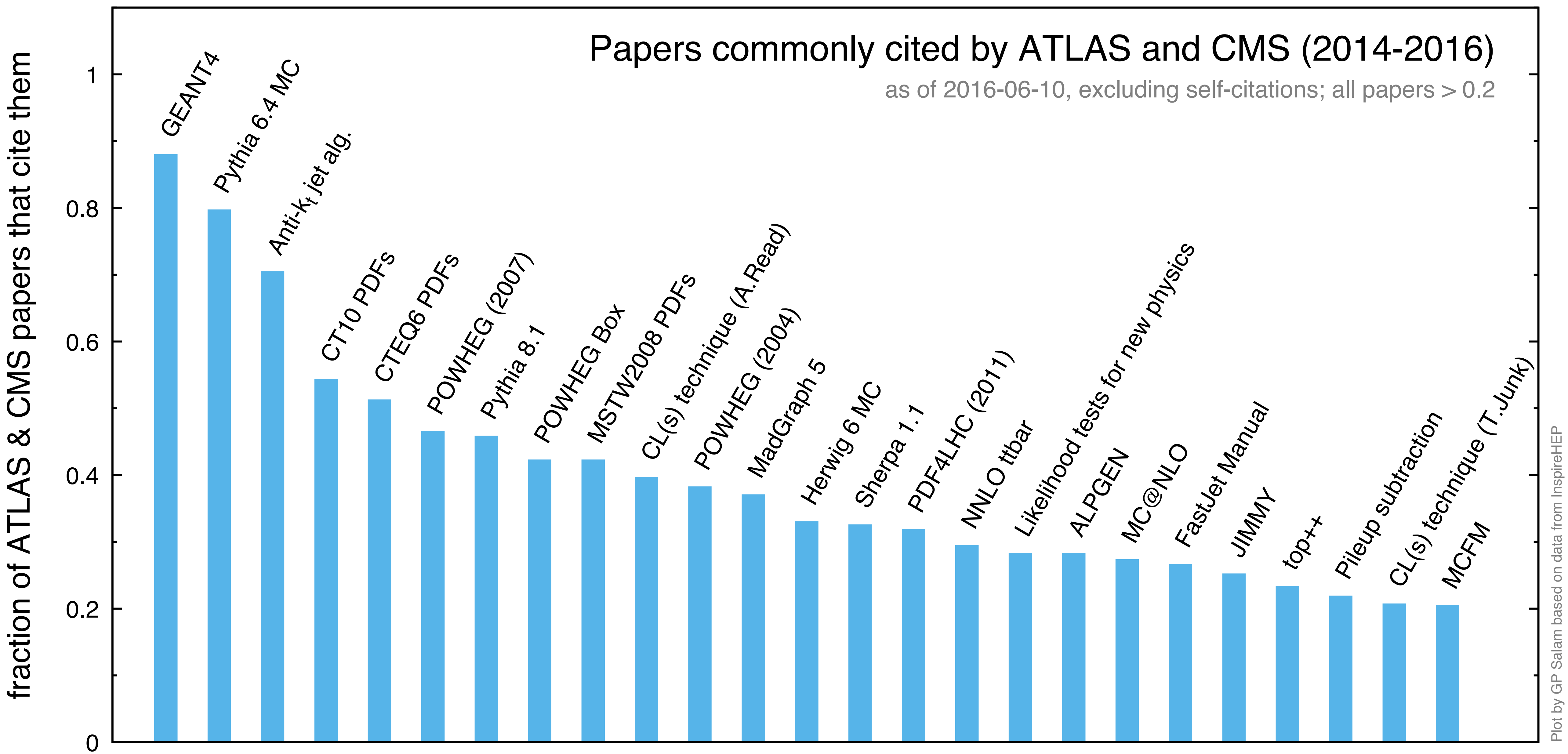
conclusions

3 Signal and background models

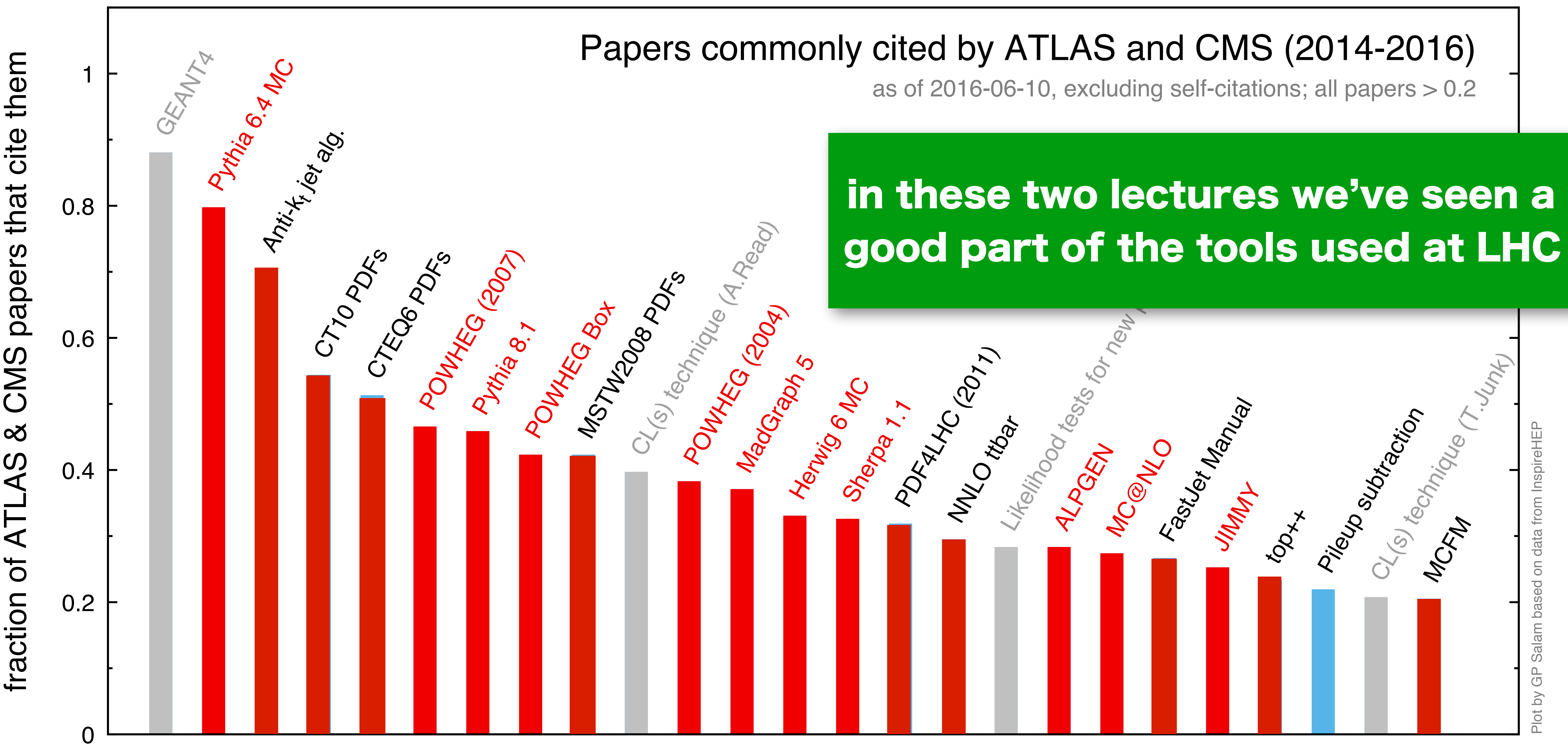
The ggF and VBF production modes for $H \to WW^*$ are modelled at next-to-leading order (NLO) in the strong coupling α_S with the POWHEG MC generator [22–25], interfaced with PYTHIA8 [26] (version 8.165) for the parton shower, hadronisation, and underlying event. The CT10 [27] PDF set is used and the parameters of the PYTHIA8 generator controlling the modelling of the parton shower and the underlying event are those corresponding to the AU2 set [28]. The Higgs boson mass set in the generation is 125.0 GeV, which is close to the measured value. The POWHEG ggF model takes into account finite quark masses and a running-width Breit–Wigner distribution that includes electroweak corrections at NLO [29]. To improve the modelling of the Higgs boson p_T distribution, a reweighting scheme is applied to reproduce the prediction of the next-to-next-to-leading-order (NNLO) and next-to-next-to-leading-logarithm (NNLL) dynamic-scale calculation given by the HRES 2.1 program [30]. Events with ≥ 2 jets are further reweighted to reproduce the p_T^H spectrum predicted by the NLO POWHEG simulation of Higgs boson production in association with two jets (H + 2 jets) [31]. Interference with continuum WW production [32, 33] has a negligible impact on this analysis due to the transverse-mass selection criteria described in Section 4 and is not included in the signal model.

Jets are reconstructed from topological clusters of calorimeter cells [50–52] using the anti-k_t algorithm with a radius parameter of $R = 0.4$ [53]. Jet energies are corrected for the effects of calorimeter non-

WHAT DO ATLAS & CMS USE MOST FREQUENTLY?



WHAT DO ATLAS & CMS USE MOST FREQUENTLY?



CONCLUSIONS

- A huge number of ingredients goes into hadron-collider predictions and studies (α_s, PDFs, matrix elements, resummation, parton showers, non-perturbative models, jet algorithms, etc.)
- a key idea is the separation of time scales ("factorisation")
  - short timescales: the hard process
  - long timescales: hadronic physics
  - in between: parton showers, resummation, DGLAP
- as long as you ask the right questions (e.g. look at jets, not individual hadrons), you can exploit this separation for quantitative, accurate, collider physics

EXTRA SLIDES

It turns out you can calculate the gluon multiplicity analytically, by summing all orders (n) of perturbation theory:

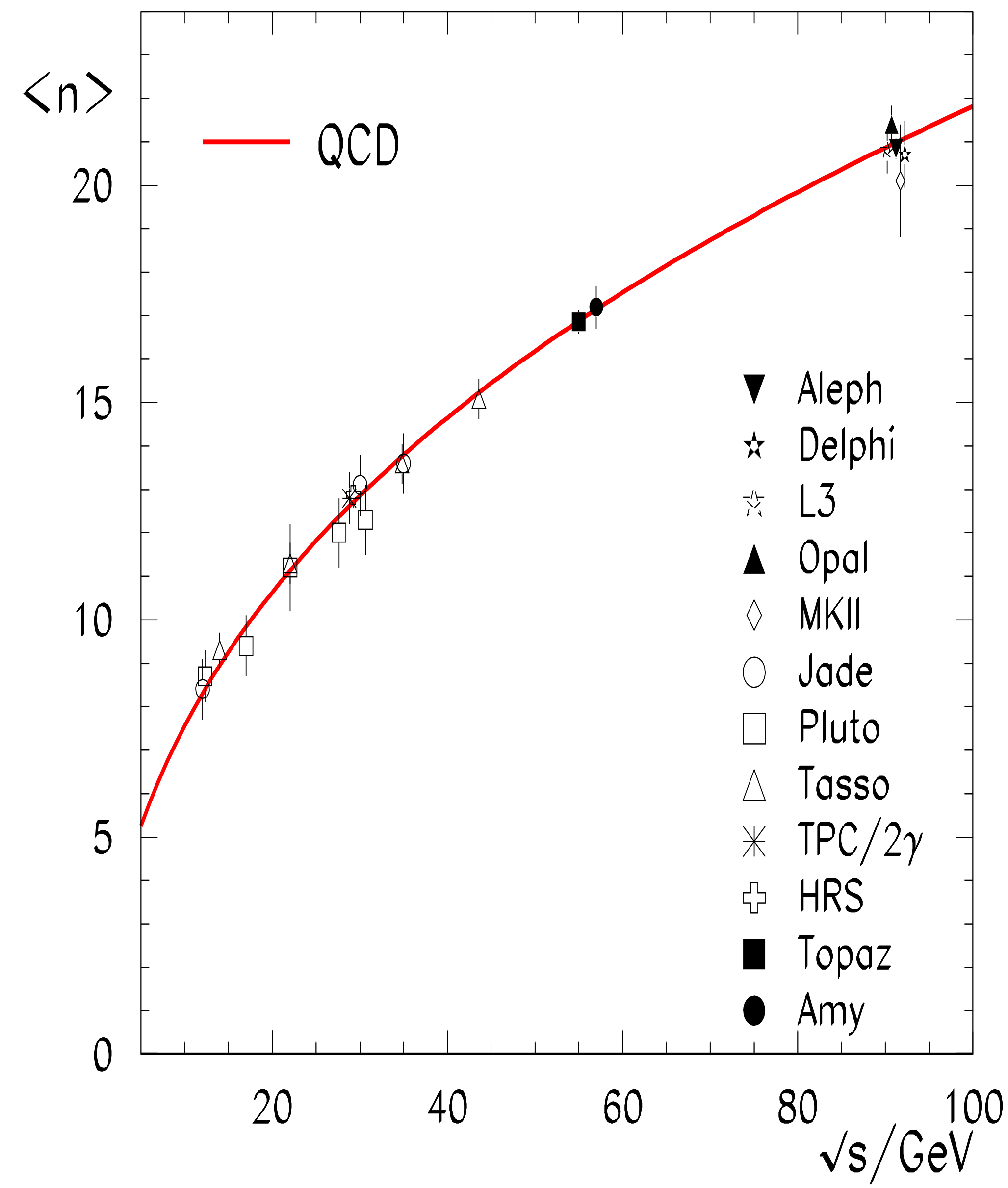
$$\langle N_g \rangle \sim \sum_n \frac{1}{(n!)^2} \left(\frac{C_A}{\pi b} \ln \frac{Q}{\Lambda} \right)^n$$

$$\sim \exp \sqrt{\frac{4 C_A}{\pi b} \ln \frac{Q}{\Lambda}}$$

Compare to data for **hadron** multiplicity ($Q \equiv \sqrt{s}$)

Including some other higher-order terms and fitting overall normalisation

Agreement is amazing!

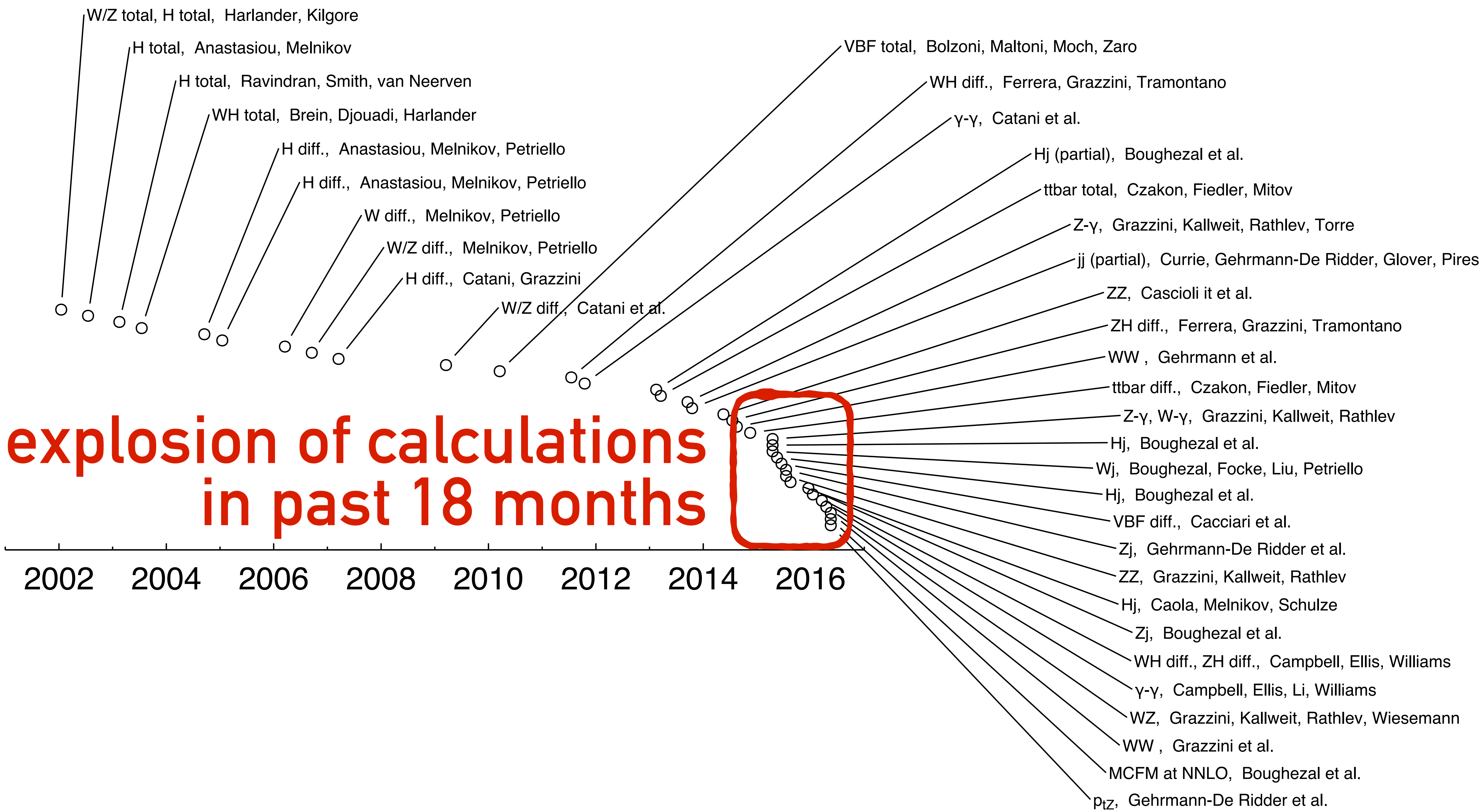


charged hadron multiplicity in e^+e^- events adapted from ESW

nnlo

NNLO hadron-collider calculations v. time

let me know of any significant omissions



Combining 2-loops / 1-loop / tree

$f(z)$ is some function with finite limit for $z \to 0$

"SLICING"

$$\sigma = \left(c - \ln\frac{1}{\text{cut}}\right) \cdot f(0) + \int_{\text{cut}}^{1} dz \frac{f(z)}{z}$$

virtual & counterterm:
get from soft-collinear resummation

real part:
use MC integration (cut has to be small, but not too small)

qT-subtraction: Catani, Grazzini
N-jettiness subtraction: Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

$f(z)$ is some function with finite limit for $z \to 0$

LOCAL SUBTRACTION

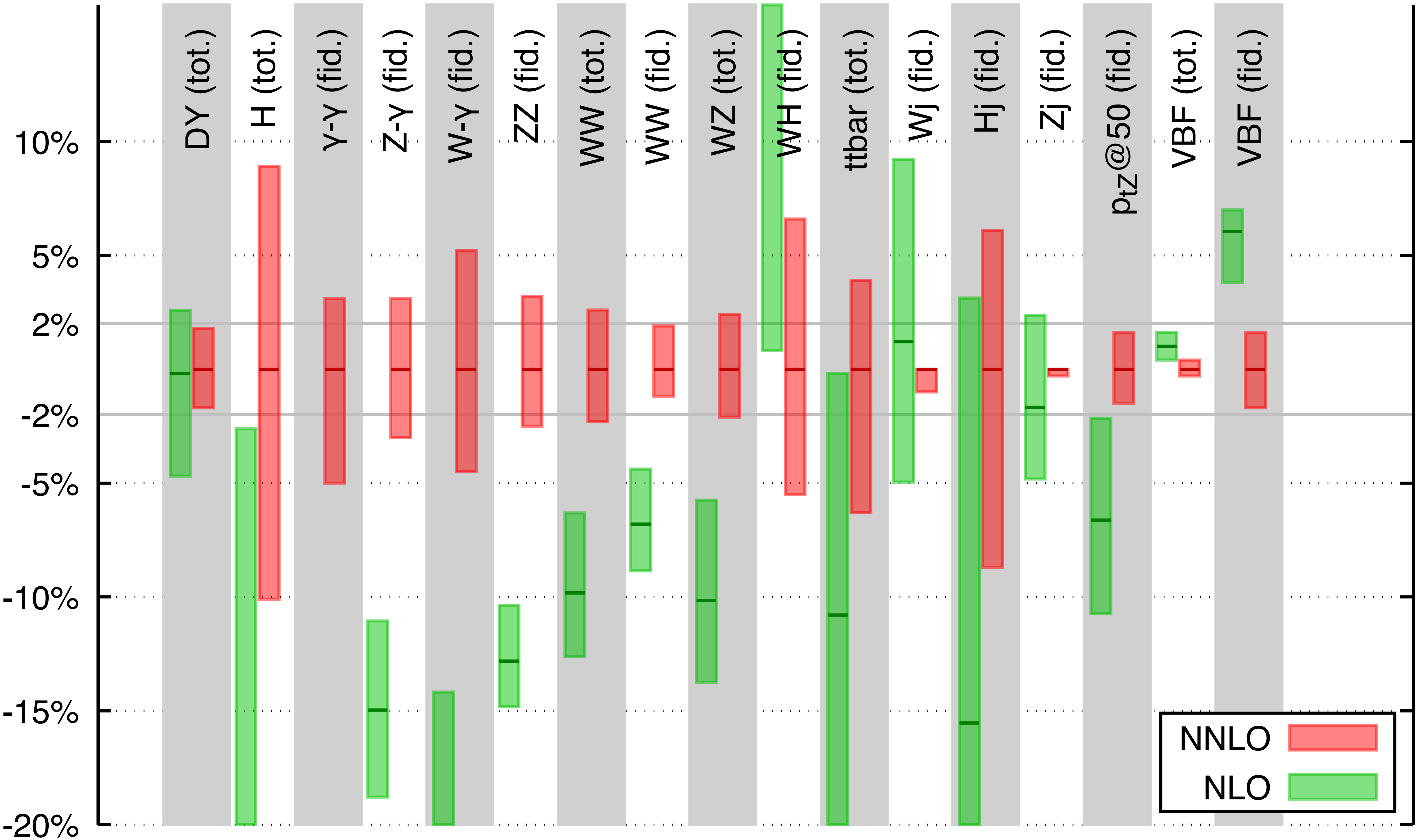
$$\sigma = c \cdot f(0) + \int_0^1 dz \left[\frac{f(z)}{z} - \frac{f(0)}{z} \right]$$

virtual & counterterm:
may need (tough)
analytic calc^n

real part:
MC integration is finite
even without cut

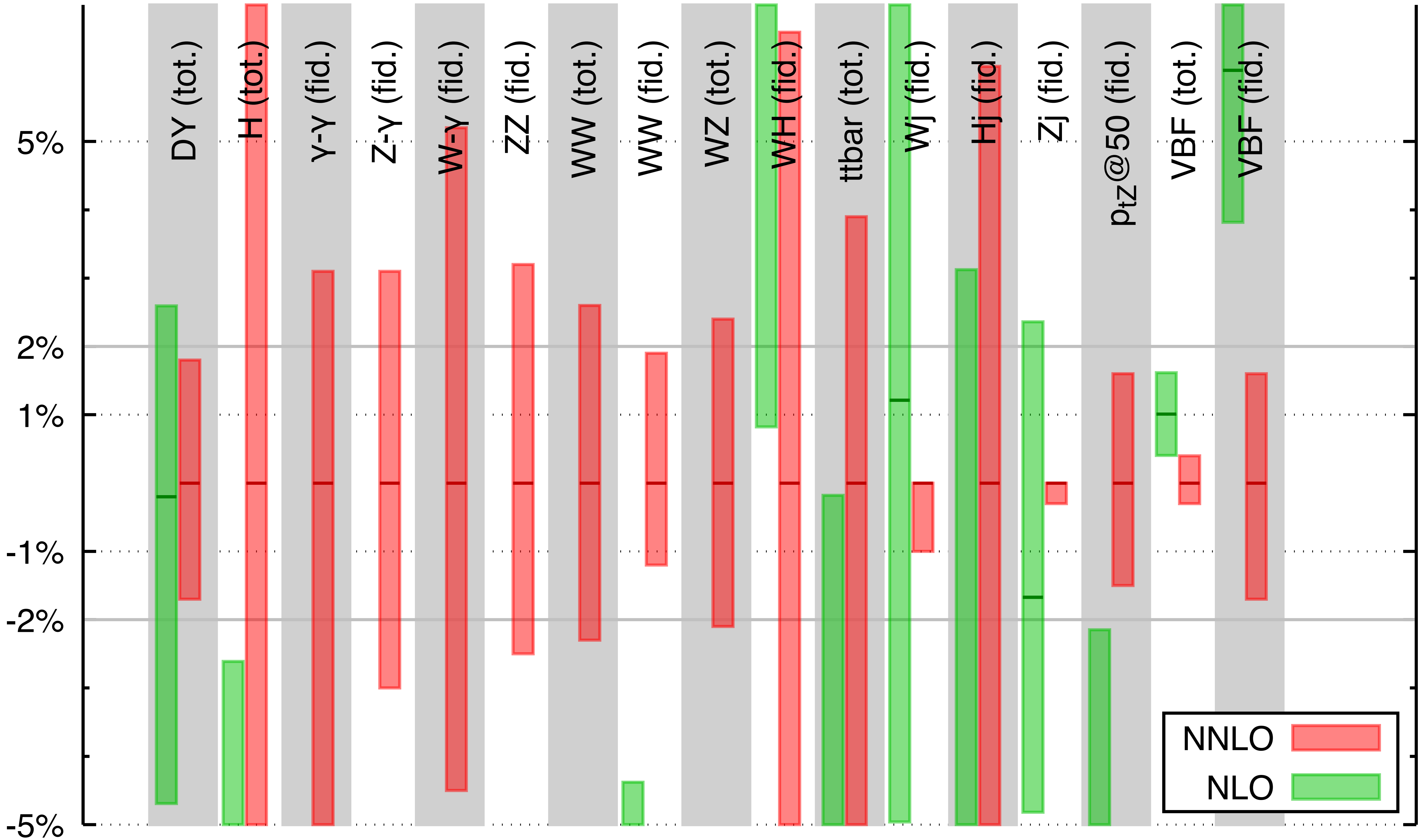
Sector decomposition: Anastasiou, Melnikov, Petriello; Binoth, Heinrich
Antennae subtraction: Kosower; Gehrmann, Gehrmann-de Ridder, Glover
Sector-improved residue subtraction: Czakon; Boughezal, Melnikov, Petriello
CoLorFul subtraction: Del Duca, Somogyi, Trocsanyi
Projection-to-Born: Cacciari, Dreyer, Karlberg, GPS, Zanderighi

WHAT PRECISION AT NNLO?



For many processes NNLO scale band is ~±2%
But only in 3/17 cases is NNLO (central) within NLO scale band…

WHAT PRECISION AT NNLO?



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But only in 3/17 cases is NNLO (central) within NLO scale band…

Processes currently known through NNLO

dijets	O(3%)	gluon-gluon, gluon-quark	PDFs, strong couplings, BSM
H+0 jet	O(3-5 %)	fully inclusive (N3LO)	Higgs couplings
H+1 jet	O(7%)	fully exclusive; Higgs decays, infinite mass tops	Higgs couplings, Higgs p_t, structure for the ggH vertex.
tT pair	O(4%)	fully exclusive, stable tops	top cross section, mass, p_t, FB asymmetry, PDFs, BSM
single top	O(1%)	fully exclusive, stable tops, t-channel	V_{tb}, width, PDFs
WBF	O(1%)	exclusive, VBF cuts	Higgs couplings
W+j	O(1%)	fully exclusive, decays	PDFs
Z+j	O(1-3%)	decays, off-shell effects	PDFs
ZH	O(3-5 %)	decays to bb at NLO	Higgs couplings (H-> bb)
ZZ	O(4%)	fully exclusive	Trilinear gauge couplings, BSM
WW	O(3%)	fully inclusive	Trilinear gauge couplings, BSM
top decay	O(1-2 %)	exclusive	Top couplings
H -> bb	O(1-2 %)	exclusive, massless	Higgs couplings, boosted

done ~ in past year

K. Melnikov @ KITP

n3lo

Higgs via gluon fusion

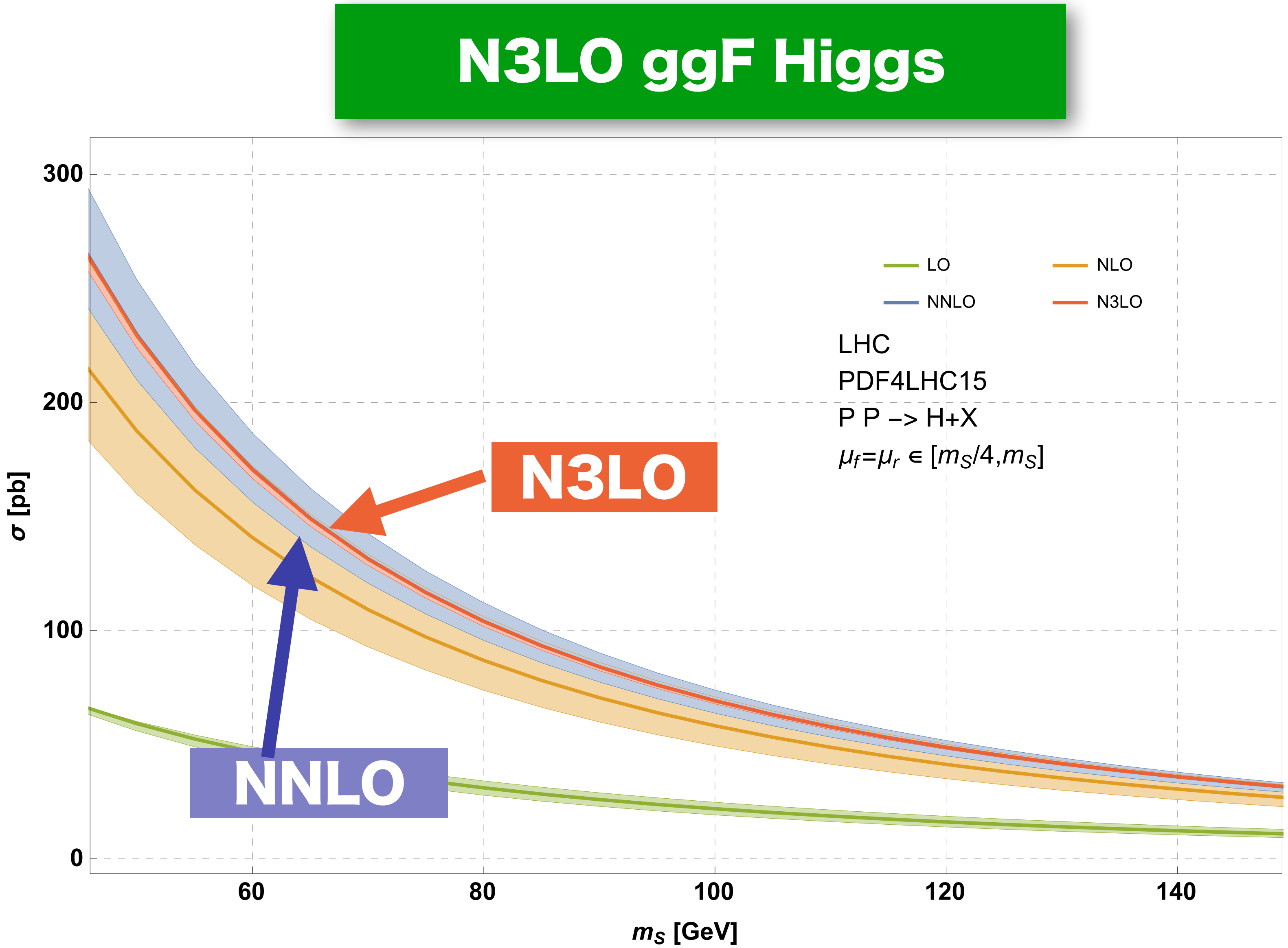
Higgs via weak-boson fusion

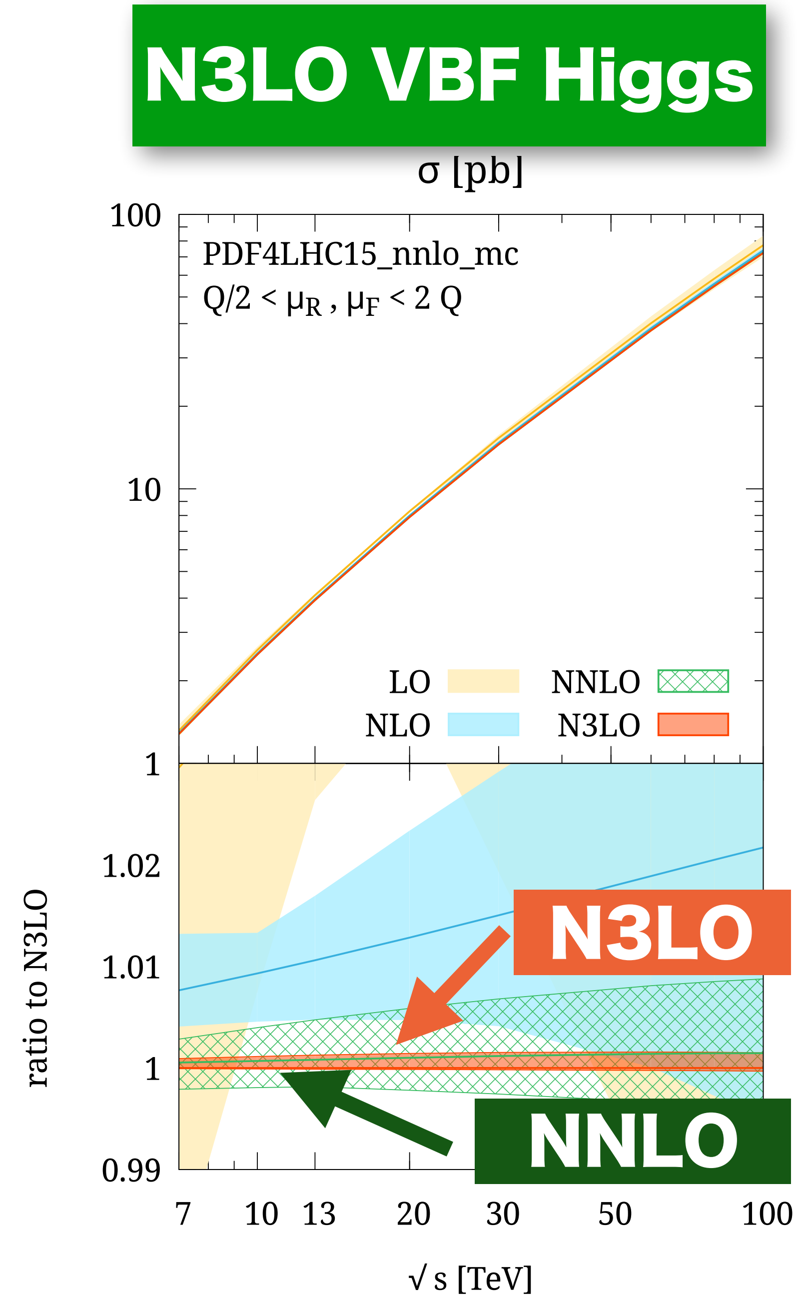
PDFs?

N3LO CONVERGENCE?

Anastasiou et al, 1602.00695

Dreyer & Karlberg, 1606.00840





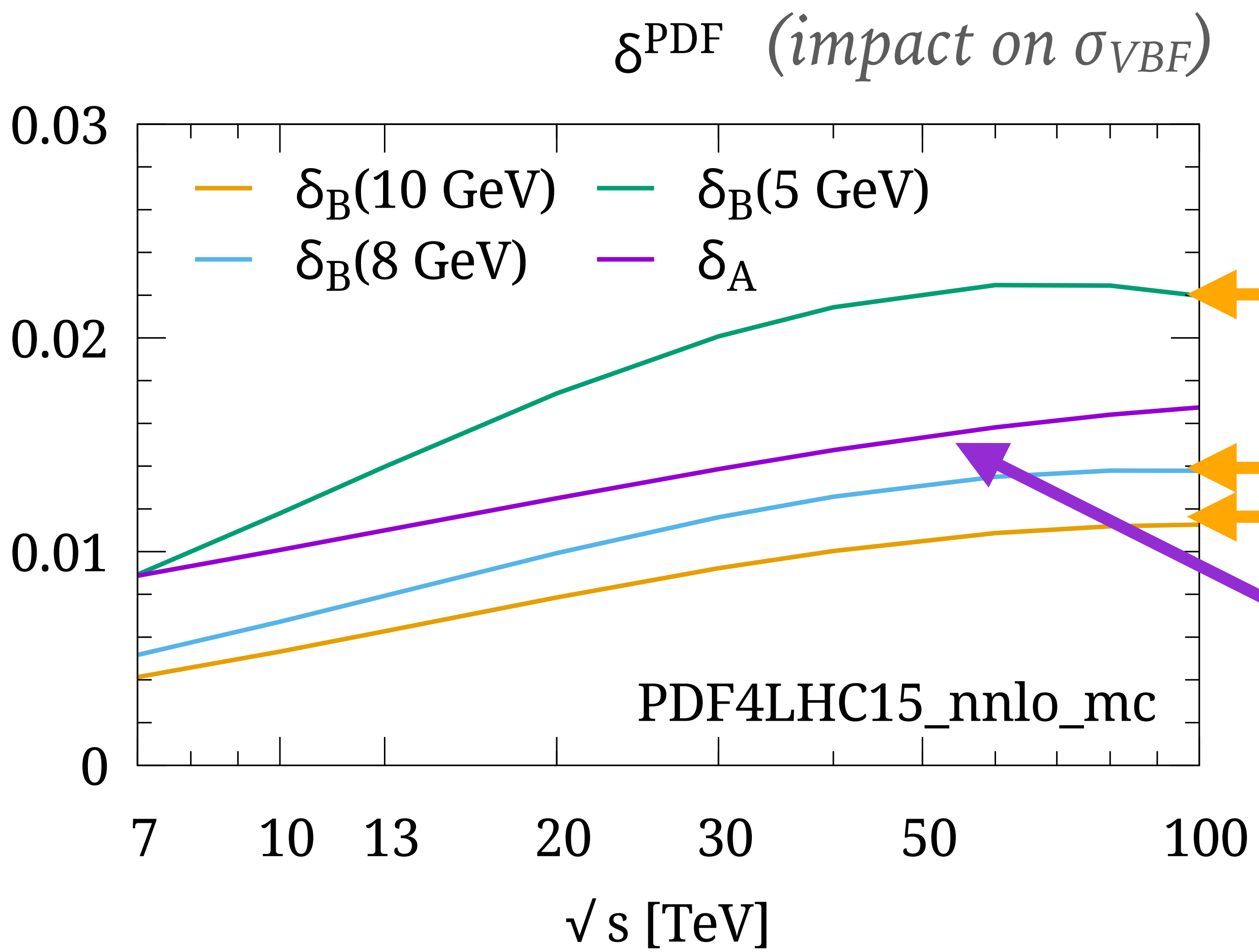
VBF converges much faster than ggF

But both calc[ns] share feature that NNLO fell outside NLO scale band, while N3LO (with good central scale choice) is very close to NNLO

N3LO PDFS ?

N3LO splitting functions not known. But N3LO DIS coefficient functions are known and their impact for quarks is >> NNLO splitting-function scale variation (~0.1%)

Dreyer & Karlberg, 1606.00840



impact of N3LO coefficient functions non-negligible on PDFs

$$\frac{1}{2}\left(\frac{\text{NNLO}}{\text{NLO}} - 1\right)$$

First results on N3LO splitting-fn moments

e-Print: arXiv:1605.08408

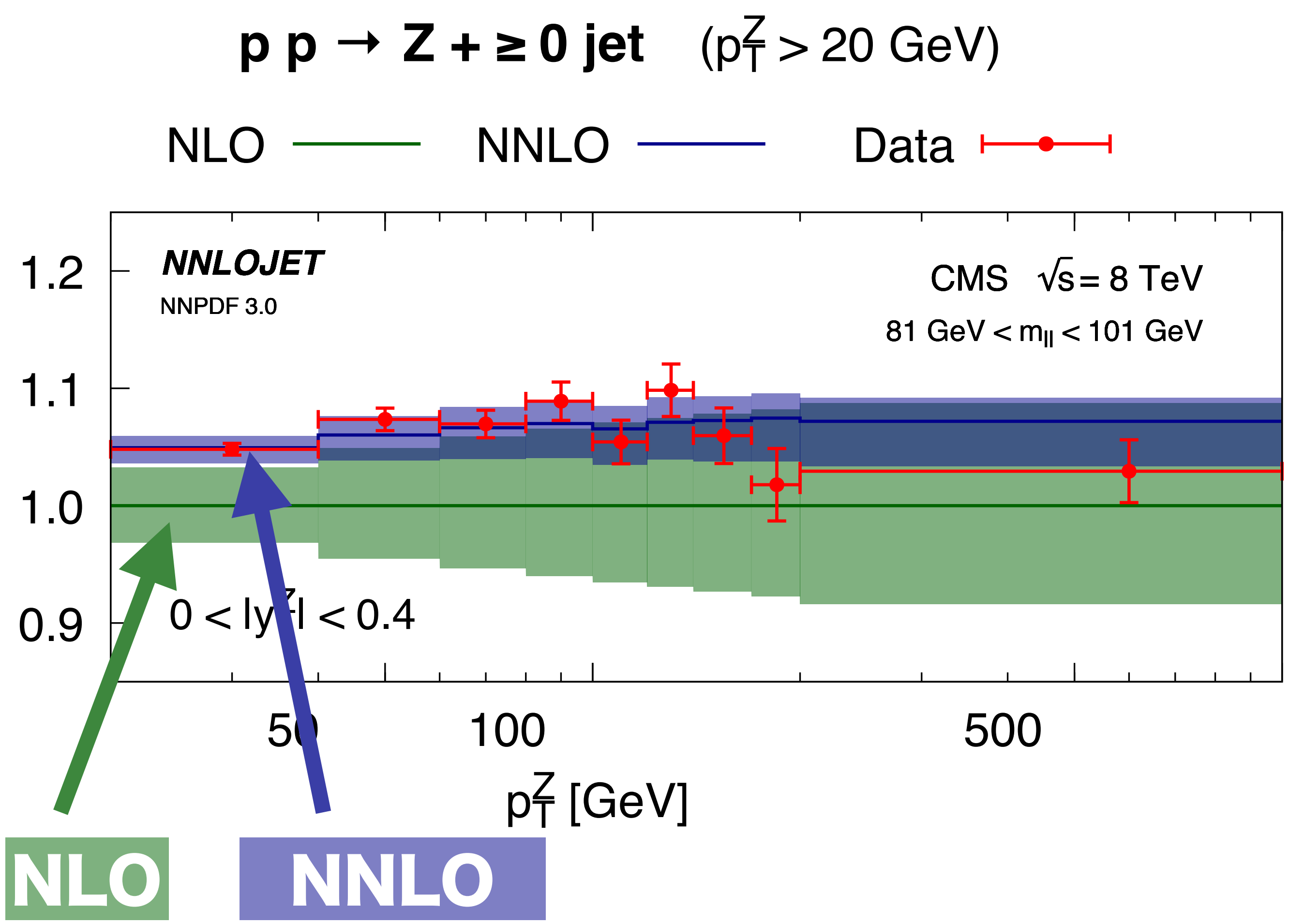
First Forcer results on deep-inelastic scattering and related quantities

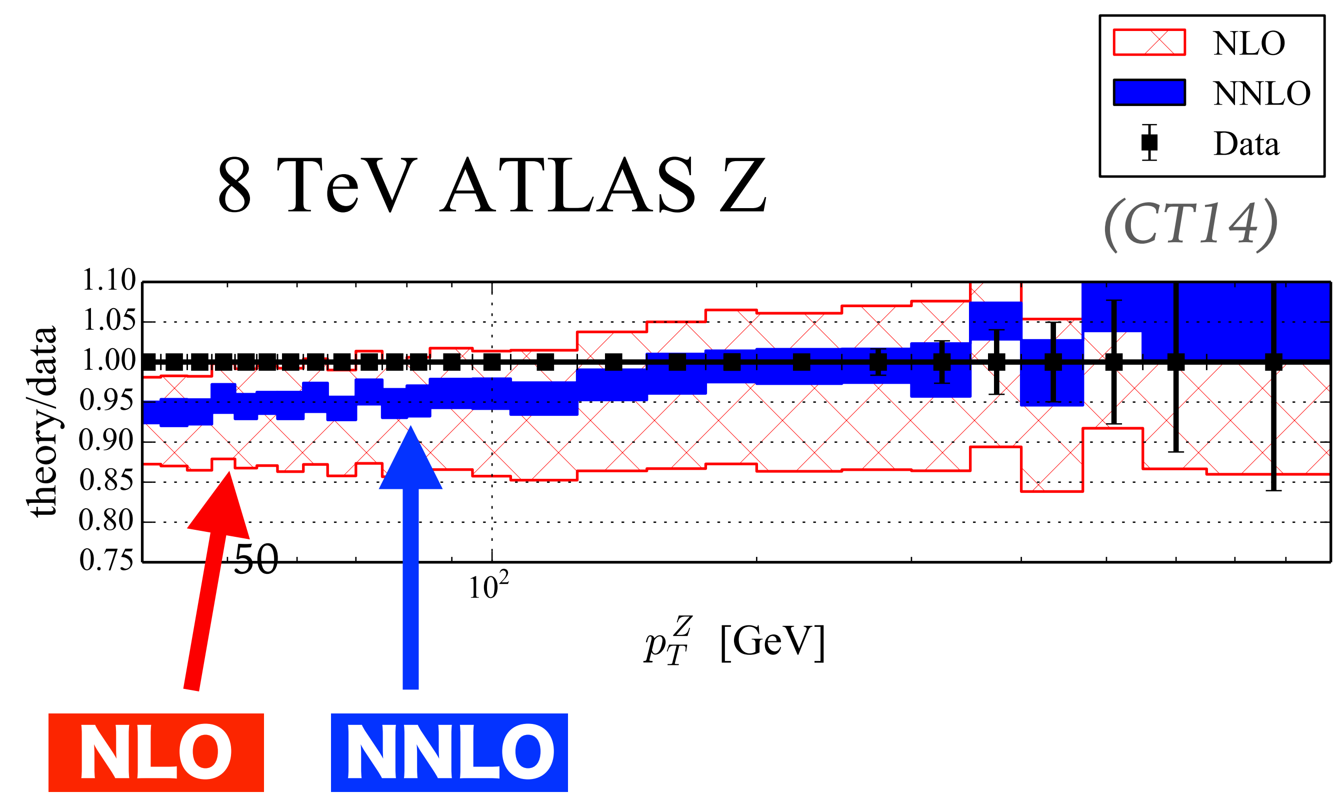
B. Ruijl, T. Ueda, J.A.M. Vermaseren (NIKHEF, Amsterdam), J. Davies, A. Vogt (Liverpool U., Dept. Math.).

Z p_T

Z p_T: Data v. two theory calculations

NNLO ~ ±1.5 %



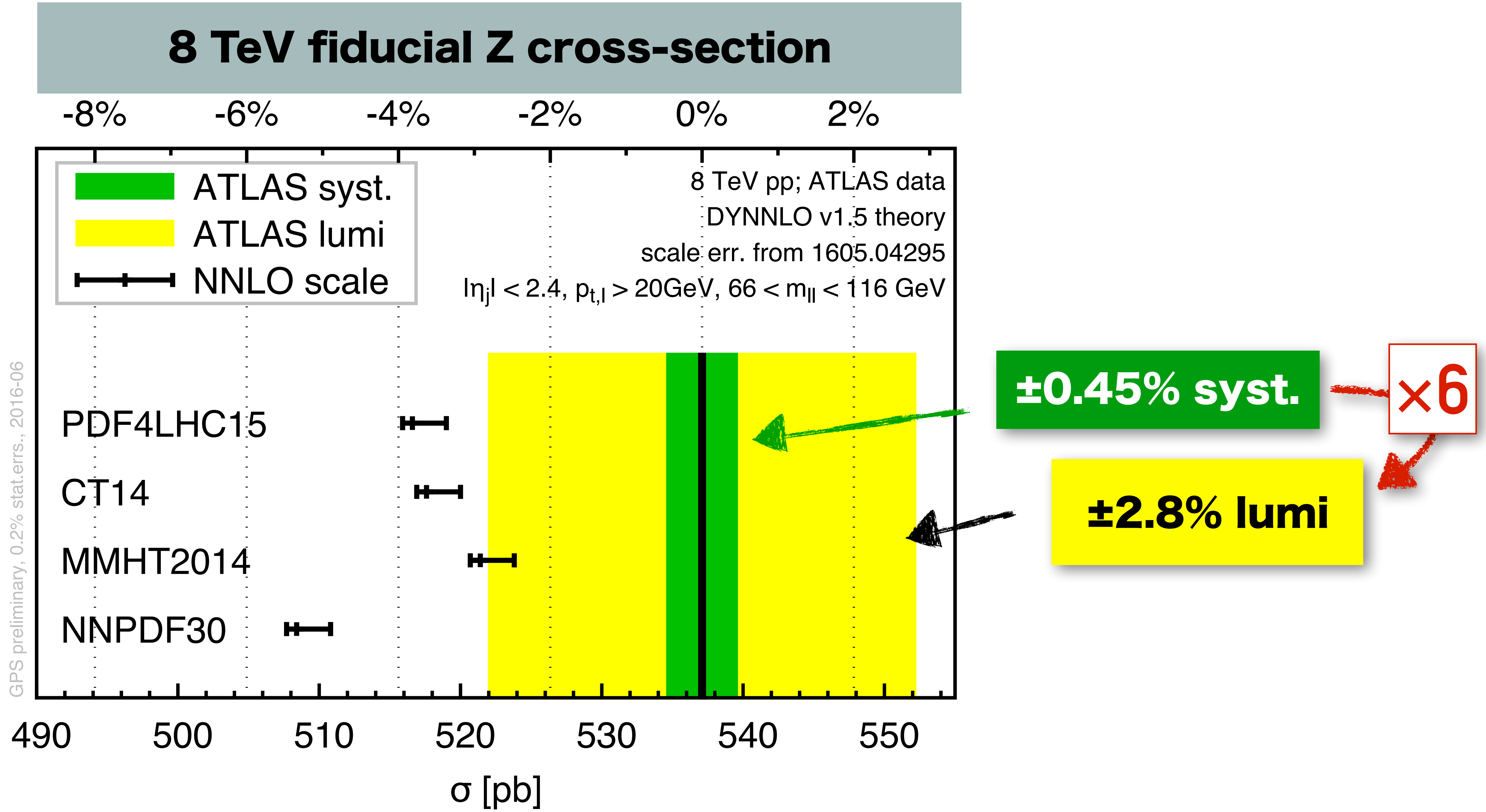


Gehrmann-de Ridder, Gehrmann
Glover, Huss & Morgan

arXiv:1605.04295

Boughezal, Liu & Petriello
'16 preliminary

X-sections normalised to Z are great, if we understand Z production



Up to 5% discrepancy?

Are NNLO scale errors (~0.5%) a reliable indicator of uncertainties?

Does it matter, given the large luminosity uncertainty?